The Idea of F-theory GUTs

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(Bhg, A. Collinucci, B. Jurke, T. Grimm, T. Weigand)
Grand Unification
Grand Unification

- One-loop running of the three Standard Model gauge couplings with MSSM matter spectrum above the TeV scale, (Ellis, Kelley, Nanopoulos), (Amaldi, de Boer, Fürstenau), (Langacker, Luo)

- Evidence for a supersymmetric Grand Unification at $M_X = 2.1 \cdot 10^{16} \text{ GeV}$, as for instance:
  - Gauge group: $SU(5)$
  - chiral matter in $10 + \overline{5} + 1$
  - Higgs field: $5_H + \overline{5}_H$
  - Yukawa couplings: $10 \ 10 \ 5_H$, $10 \ \overline{5} \ 5_H$, $\overline{5} \ 1 \ 5_H$
Grand Unification from String Theory

Attempts to realize GUTs from String Theory

- Weakly coupled $E_8 \times E_8$ Heterotic String (heterotic orbifolds)
  - Need large threshold corrections at $M_X$
  - GUT breaking via discrete Wilson lines

- F-theory/Type IIB compactifications with $(p, q)$–7-branes:
  - Solves the $10\,10\,5_H$ Yukawa problem of orientifolds
  - GUT brane wraps a shrinkable 4-cycle
  - GUT breaking via $U(1)_Y$ flux
F-Theory
F-theory is a way of book-keeping of the positions of more general $(p, q)$-7-branes in Type IIB $\mathcal{N} = 1$ compactifications

elliptic fibration: $Y \to B_3$

Susy $\to Y$ Calabi-Yau 4-fold. Elliptic curve $y^2 = x^3 + f(u)x + g(u)$ with complex structure:

$$\tau = C_0 + i e^{-\varphi}$$

with $j(\tau) = \frac{4(24)^3 f^3}{4f^3 + 27g^2}$

Grand Unification from F-theory
Grand Unification from F-theory

Working hypothesis: Decoupling of GUT scale from Planck scale → localisation of GUT physics on del-Pezzo surfaces

(Beasley, Heckman, Vafa, arXiv:0806.0102)

Shortcomings

• Missing stringy global consistency conditions: landscape vs. swampland

• Physics of abelian gauge symmetries: Green-Schwarz mechanism, Freed-Witten anomalies, (Grimm, Weigand)

• Need local mechanism for Susy breaking → gauge mediated susy breaking

• closed string Moduli stabilisation, need to explain why susy breaking is subleading to gauge mediation

series of recent papers: (Beasley, Choi, Donagi, Hayashi, Heckman, Marsano, Saulina, Schäfer-Nameki, Vafa, Watari, Wijnholt+ ... )
Grand Unification from F-theory
Grand Unification from F-theory

Program:

- Embed the local ideas into a global framework: F-theory on elliptically fibered four-folds with shrinkable 4-cycles
- Derivation of the global consistency conditions,
- Lift and generalise Type IIB orientifold consistency conditions to genuine F-theory models
- Study of consequences of $U(1)_Y$ flux → gauge coupling unification
- Moduli stabilization via flux and instanton generated superpotentials
F-Theory
F-Theory

• Gauge symmetry on $D$: Degeneration of elliptic curve, ADE Kodaira classification

• Consistency condition: Degeneration loci can be described by a compact Calabi-Yau fourfold $Y$

• Four-form flux quantisation (chirality)

\[ G_4 + \frac{1}{2} c_2(Y) \in \mathbb{Z} \]

• D3-tadpole:

\[ N_{D3} + \frac{1}{2} \int_Y G_4 \wedge G_4 = \frac{\chi(Y)}{24} \]
Matter fields
Matter fields are generally localised on curves: \( C = D_a \cap D_b \)

**F-theory: Enhancement of the singularity over the intersection:** \( SU(5) \times U(1) \rightarrow SU(6) \)

\[
35 = 24_0 + 1_0 + 5_1 + 5_{-1}
\]

resp. \( SU(5) \times U(1) \rightarrow SO(10) \)

\[
45 = 24_0 + 1_0 + 10_2 + 10_{-2}
\]
Yukawa couplings
Yukawa couplings:

The Yukawa couplings which give masses to the MSSM fields after GUT and electroweak symmetry breaking are

\[ 10^{(2,0)} 10^{(2,0)} 5_H^{(1,-1)}, \quad 10^{(2,0)} \overline{5}^{(-1,-1)} \overline{5}_H^{(-1,1)}, \]

\[ 1^{(0,2)}_N \overline{5}^{(-1,-1)} 5_H^{(1,-1)} \]

Problem: The coupling \( 10^{(2,0)} 10^{(2,0)} 5_H^{(1,-1)} \) is perturbatively forbidden.

Solution: Arises in F-theory from an \( E_6 \) enhancement of the singularity.
GUT symmetry breaking
GUT symmetry breaking

Symmetry breaking via gauge flux $F_Y$: $c_1(L_Y) \in H^2(D)$ has to be trivial in $H^2(X)$, i.e. $\iota_*(L_Y) = 0$.

- exotic matter:

\[
24 \rightarrow (8, 1)_0 + (1, 3)_0 + (3, 2)_5 + (\overline{3}, 2)_{-5}
\]

i.e. $H^*(D, L^5_Y) = 0$.

- Solution: One defines fractional line bundles $\mathcal{L}_a$ and $\mathcal{L}_Y$ via

\[
L_a = \mathcal{L}_a \otimes \mathcal{L}^2_Y \quad L_Y = \mathcal{L}^1_Y
\]
Compact models
Compact models

Problem: Realisations of all these local features in genuine compact F-theory

Study manifolds using methods of toric geometry:

- Example: Elliptic fibration over $\mathbb{P}^3$
  The fourfold is given by the Weierstrass fibration:

\[
\begin{array}{cccccccc}
& y & x & z & u_1 & u_2 & u_3 & u_4 & p \\
q_1 & 3 & 2 & 1 & 0 & 0 & 0 & 0 & 6 \\
q_2 & 0 & 0 & -4 & 1 & 1 & 1 & 1 & 0 \\
\end{array}
\]

with Tate constraint

\[
y^2 = x^3 + xyz a_1 + x^2 z^2 a_2 + yz^3 a_3 + xz^4 a_4 + z^6 a_6
\]

and $a_n$ polynomials of degree $4n$ in $\vec{u}$.
Compact models
Compact models

roadmap:

- Perform transitions of these manifold leading new del-Pezzo type four-cycles
- Analyze the new elliptic fibration whether it allows for an $SU(5)$ GUT with the wanted matter curves and Yukawa couplings $\rightarrow$ tadpole conditions
- For chirality turn on extra gauge flux ($G_4$ form flux) and compute spectra and tadpoles

Gauge coupling unification
Gauge coupling unification

The gauge couplings at the string/GUT scale are changed due to the $U(1)_{Y}$ flux (Bhg, arXiv:0812.0248):

$$f_{SU(3)} = \tau_a - \frac{1}{2} S \int_{D_a} c_1^2(\mathcal{L}_a)$$

$$f_{SU(2)} = \tau_a - \frac{1}{2} S \int_{D_a} c_1^2(\mathcal{L}_a) + [c_1^2(\mathcal{L}_Y) + 2c_1(\mathcal{L}_Y)c_1(\mathcal{L}_a)]$$

$$\frac{3}{5} f_{U(1)_Y} = \tau_a - \frac{1}{2} S \int_{D_a} c_1^2(\mathcal{L}_a) + \frac{3}{5} [c_1^2(\mathcal{L}_Y) + 2c_1(\mathcal{L}_Y)c_1(\mathcal{L}_a)],$$

with $\tau_a = e^{-\varphi} \frac{1}{2} \int_{D_a} J \wedge J$. The MSSM gauge couplings satisfy the relation

$$\frac{1}{\alpha_Y(M_s)} = \frac{1}{\alpha_{W}(M_s)} + \frac{2}{3 \alpha_s(M_s)}.$$
Gauge coupling unification
Gauge coupling unification

Include the Higgs triplet above a scale $1 \text{TeV} < M_{3\bar{3}} < M_X$ in the running:

$$(b_3, b_2, b_1) = (3, -1, -11) \rightarrow (\tilde{b}_3, \tilde{b}_2, \tilde{b}_1) = (2, -1, -\frac{35}{3}) .$$

Choosing for instance $M_{3\bar{3}} = 10^{15}$ GeV, the running around the GUT scale changes as
Gauge coupling unification
The three MSSM gauge couplings satisfy the F-theory GUT relation at

\[ M_X = 2.1 \cdot 10^{16} \text{ GeV} \]

independent of the triplet mass scale \( M_{33} \).
Moduli stabilization
Moduli stabilization

Uplift of KKLT and LARGE volume scenario from Type IIB to F-theory

- $G_3$ flux becomes $G_4$ form flux with one leg on $T^2$
- D3-brane instantons $\rightarrow$ M5-brane instantons, (Bhg, Collinucci, Jurke, arXiv:1002.1894)
- technically challenging: dilaton $\tau$ varies over background

New scenario: gravity mediated susy breaking on shrinkable MSSM 4-cycle

- Soft masses are suppressed relative to the gravitino
  \[ M_{\text{soft}} = \frac{M_{3/2}^p}{V^p}, \quad p = 1/2, 1 \]  (Bhg, Conlon, Krippendorf, Moster, Quevedo, arXiv:0906.3297)
Conclusions

F-theory models can provide string theory realisations of $SU(5)$ GUTs on localized branes.

• Gauge fields arise from degenerations of elliptic fiber over surfaces
• Matter fields and Yukawa couplings arise from singularity enhancements over curves and points
• Global compact models showing many of the desired features can be constructed
• GUT breaking works via $U(1)_Y$ flux $\rightarrow$ “problems” with gauge coupling unification
• Determination of flavour structure and Yukawa hierarchies by subleading corrections $\rightarrow$ devil in the details
Further progress both on the technical and the phenomenological level is expected.

Thank you!