The Idea of F-theory GUTs

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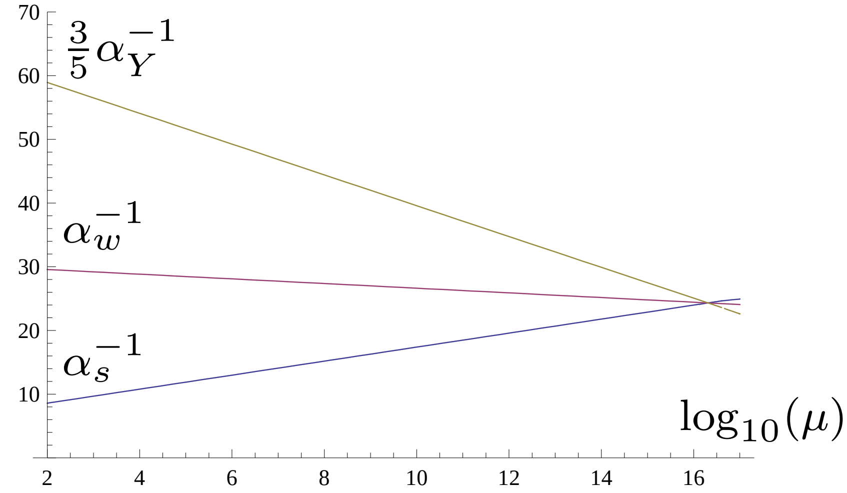
(Bhg, A. Collinucci, B. Jurke, T. Grimm, T. Weigand)

Grand Unification

Grand Unification

- One-loop running of the three Standard Model gauge couplings with MSSM matter spectrum above the TeV scale, (Ellis, Kelley, Nanopoulos), (Amaldi, de Boer, Fürstenau), (Langacker, Luo)



- Evidence for a supersymmetric Grand Unification at $M_X = 2.1 \cdot 10^{16}$ GeV, as for instance:
  - Gauge group: $SU(5)$
  - chiral matter in $\mathbf{10} + \overline{\mathbf{5}} + \mathbf{1}$
  - Higgs field: $\mathbf{5}_H + \overline{\mathbf{5}}_H$
  - Yukawa couplings: $\mathbf{10}\,\mathbf{10}\,\mathbf{5}_H$, $\mathbf{10}\,\overline{\mathbf{5}}\,\overline{\mathbf{5}}_H$, $\overline{\mathbf{5}}\,\mathbf{1}\,\mathbf{5}_H$

Grand Unification from String Theory

Grand Unification from String Theory

Attempts to realize GUTs from String Theory

- Weakly coupled $E_8 \times E_8$ Heterotic String (heterotic orbifolds)
  - Need large threshold corrections at M_X
  - GUT breaking via discrete Wilson lines

- F-theory/Type IIB compactifications with $(p, q)-$ 7-branes:
  - Solves the $\mathbf{10}\,\mathbf{10}\,\mathbf{5_H}$ Yukawa problem of orientifolds
  - GUT brane wraps a shrinkable 4-cycle
  - GUT breaking via $U(1)_Y$ flux

F-Theory

F-Theory

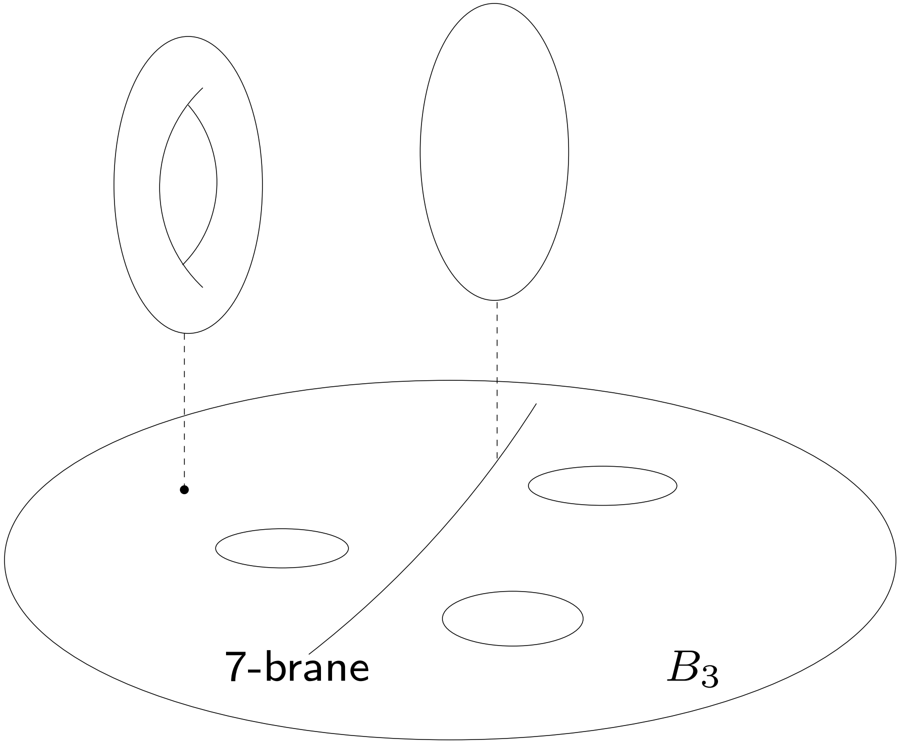
F-theory is a way of book-keeping of the positions of more general (p, q)-7-branes in Type IIB $\mathcal{N} = 1$ compactifications

$$\text{elliptic fibration}: \qquad Y \to B_3$$

Susy $\to$ Y Calabi-Yau 4-fold. Elliptic curve $y^2 = x^3 + f(u)\,x + g(u)$ with complex structure:

$$\tau = C_0 + i\,e^{-\varphi}$$

with $j(\tau) = \frac{4(24)f^3}{4f^3+27\,g^2}$



(Vafa,Nucl.Phys.B469:403,1996), (Beasley, Heckman, Vafa, arXiv:0802.3391+0806.0102), (Donagi, Wijnholt, arXiv:0802.29

Grand Unification from F-theory

Grand Unification from F-theory

Working hypothesis: Decoupling of GUT scale from Planck scale $\longrightarrow$ localisation of GUT physics on del-Pezzo surfaces (Beasley, Heckman, Vafa, arXiv:0806.0102)

Shortcomings

- Missing stringy global consistency conditions: landscape vs. swampland
- Physics of abelian gauge symmetries: Green-Schwarz mechanism, Freed-Witten anomalies, (Grimm, Weigand)
- Need local mechanism for Susy breaking $\longrightarrow$ gauge mediated susy breaking
- closed string Moduli stabilisation, need to explain why susy breaking is subleading to gauge mediation

series of recent papers: (Beasley, Choi, Donagi, Hayashi, Heckman, Marsano, Saulina, Schäfer-Nameki, Vafa, Watari, Wijnholt+ . . .)

Grand Unification from F-theory

Grand Unification from F-theory

Program:

- Embed the local ideas into a global framework: F-theory on elliptically fibered four-folds with shrinkable 4-cycles
- Derivation of the global consistency conditions,
- Lift and generalise Type IIB orientifold consistency conditions to genuine F-theory models
- Study of consequences of $U(1)_Y$ flux $\rightarrow$ gauge coupling unification
- Moduli stabilization via flux and instanton generated superpotentials

F-Theory

F-Theory

- Gauge symmetry on D: Degeneration of elliptic curve, ADE Kodaira classification
- Consistency condition: Degeneration loci can be described by a compact Calabi-Yau fourfold Y
- Four-form flux quantisation (chirality)

$$G_4 + \frac{1}{2} c_2(Y) \in \mathbb{Z}$$

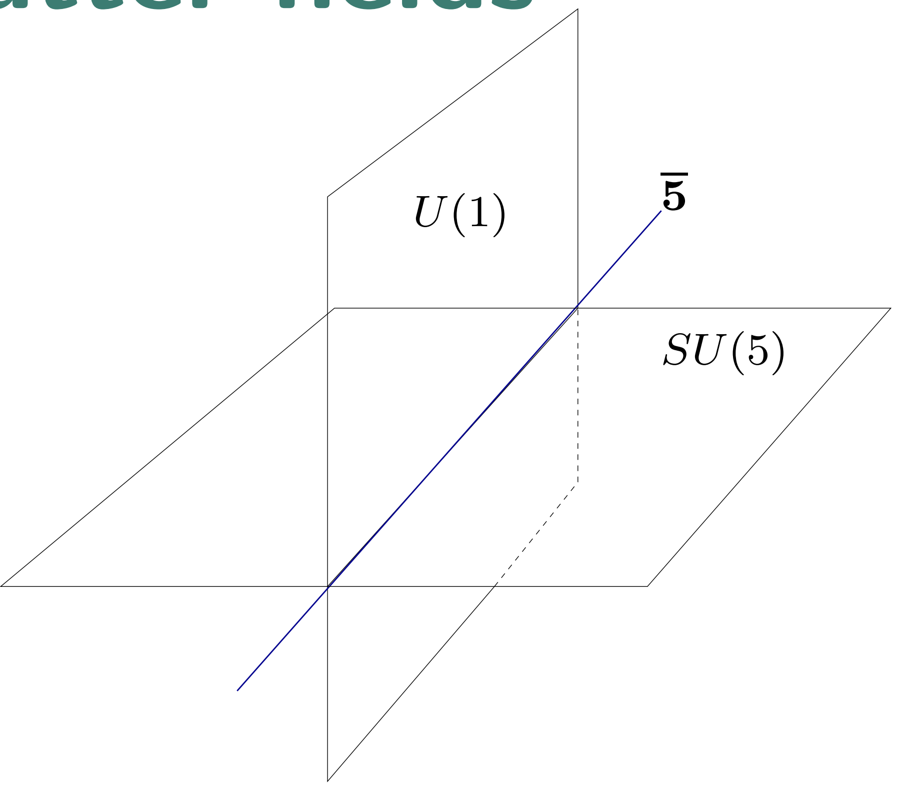
- D3-tadpole:

$$N_{\mathrm{D3}} + \frac{1}{2} \int_Y G_4 \wedge G_4 = \frac{\chi(Y)}{24}$$

Matter fields

Matter fields

Matter fields are generally localised on curves: $C = D_a \cap D_b$



F-theory: Enhancement of the singularity over the intersection: $SU(5) \times U(1) \to SU(6)$

$$\mathbf{35} = \mathbf{24}_0 + \mathbf{1}_0 + \mathbf{5}_1 + \overline{\mathbf{5}}_{-1}$$

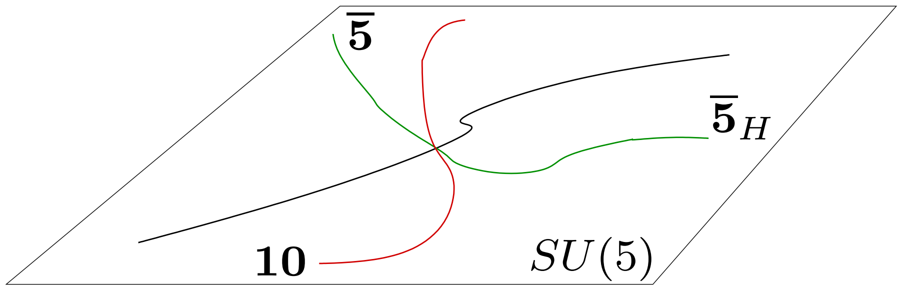
resp. $SU(5) \times U(1) \to SO(10)$

$$\mathbf{45} = \mathbf{24}_0 + \mathbf{1}_0 + \mathbf{10}_2 + \overline{\mathbf{10}}_{-2}$$

Yukawa couplings

Yukawa couplings

Yukawa couplings:



The Yukawa couplings which give masses to the MSSM fields after GUT and electroweak symmetry breaking are

$$\mathbf{10}^{(2,0)}\,\mathbf{10}^{(2,0)}\,\mathbf{5_H}^{(1,-1)}, \quad \mathbf{10}^{(2,0)}\,\overline{\mathbf{5}}^{(-1,-1)}\,\overline{\mathbf{5}}_\mathbf{H}^{(-1,1)},$$

$$\mathbf{1}_N^{(0,2)}\,\overline{\mathbf{5}}^{(-1,-1)}\,\mathbf{5_H}^{(1,-1)}$$

Problem: The coupling $\mathbf{10}^{(2,0)}\,\mathbf{10}^{(2,0)}\,\mathbf{5_H}^{(1,-1)}$ is perturbatively forbidden.
Solution: Arises in F-theory from an E_6 enhancement of the singularity.

GUT symmetry breaking

GUT symmetry breaking

Symmetry breaking via gauge flux F_Y: $c_1(L_Y) \in H^2(D)$ has to be trivial in $H^2(X)$, i.e. $\iota_*(L_Y) = 0$.

- exotic matter:

$$\mathbf{24} \to (\mathbf{8}, \mathbf{1})_0 + (\mathbf{1}, \mathbf{3})_0 + (\mathbf{3}, \mathbf{2})_5 + (\overline{\mathbf{3}}, \mathbf{2})_{-5}$$

  i.e. $H^*(D, L_Y^5) = 0$.

- Solution: One defines fractional line bundles $\mathcal{L}_a$ and $\mathcal{L}_Y$ via

$$L_a = \mathcal{L}_a \otimes \mathcal{L}_Y^{\frac{2}{5}} \qquad L_Y = \mathcal{L}_Y^{\frac{1}{5}}$$

Compact models

Compact models

Problem: Realisations of all these local features in genuine compact F-theory

Study manifolds using methods of toric geometry:

- Example: Elliptic fibration over $\mathbb{P}^3$
  The fourfold is given by the Weierstrass fibration:

	y	x	z	u_1	u_2	u_3	u_4	p
q_1	3	2	1	0	0	0	0	6
q_2	0	0	-4	1	1	1	1	0

  with Tate constraint

$$y^2 = x^3 + xyza_1 + x^2z^2a_2 + yz^3a_3 + xz^4a_4 + z^6a_6$$

  and a_n polynomials of degree $4n$ in $\vec{u}$.

Compact models

Compact models

roadmap:

- Perform transitions of these manifold leading new del-Pezzo type four-cycles
- Analyze the new elliptic fibration whether it allows for an $SU(5)$ GUT with the wanted matter curves and Yukawa couplings $\rightarrow$ tadpole conditions
- For chirality turn on extra gauge flux (G_4 form flux) and compute spectra and tadpoles

More details in (Bhg, Grimm, Jurke, Weigand, arXiv:0908.1784), (Grimm, Krause, Weigand, arXiv:0912.3524), (Marsano, Saulina, Schäfer-Nameki, arXiv:0906.4672), (Chen, Knapp, Kreuzer, Mayrhofer, arXiv:1005.5735)

Gauge coupling unification

Gauge coupling unification

The gauge couplings at the string/GUT scale are changed due to the $U(1)_Y$ flux (Bhg, arXiv:0812.0248):

$$\begin{aligned}
f_{SU(3)} &= \tau_a - \frac{1}{2} S \int_{D_a} c_1^2(\mathcal{L}_a) \\
f_{SU(2)} &= \tau_a - \frac{1}{2} S \int_{D_a} c_1^2(\mathcal{L}_a) + \left[c_1^2(\mathcal{L}_Y) + 2c_1(\mathcal{L}_Y)\, c_1(\mathcal{L}_a)\right] \\
\frac{3}{5} f_{U(1)_Y} &= \tau_a - \frac{1}{2} S \int_{D_a} c_1^2(\mathcal{L}_a) + \frac{3}{5}\left[c_1^2(\mathcal{L}_Y) + 2c_1(\mathcal{L}_Y)\, c_1(\mathcal{L}_a)\right] ,
\end{aligned}$$

with $\tau_a = e^{-\varphi} \frac{1}{2} \int_{D_a} J \wedge J$. The MSSM gauge couplings satisfy the relation

$$\frac{1}{\alpha_Y(M_s)} = \frac{1}{\alpha_w(M_s)} + \frac{2}{3\,\alpha_s(M_s)} .$$

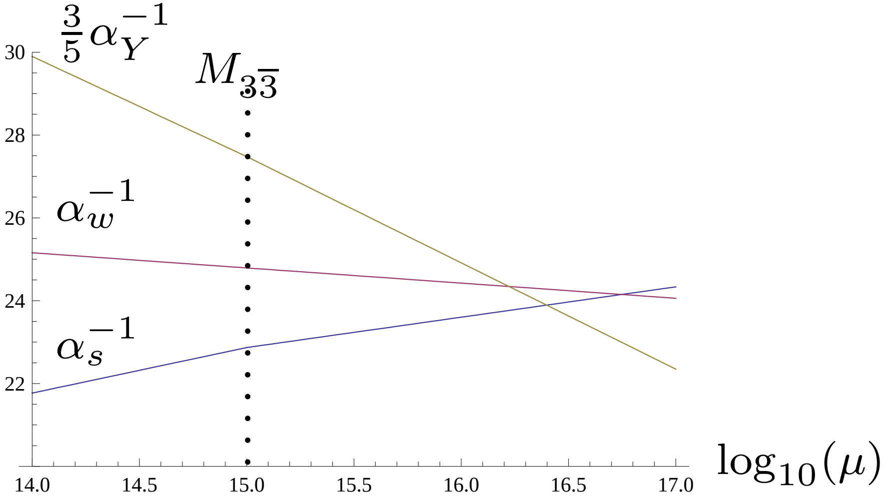
Gauge coupling unification

Gauge coupling unification

Include the Higgs triplet above a scale $1\mathrm{TeV} < M_{3\bar{3}} < M_X$ in the running:

$$(b_3, b_2, b_1) = (3, -1, -11) \to (\tilde{b}_3, \tilde{b}_2, \tilde{b}_1) = (2, -1, -\frac{35}{3}) \ .$$

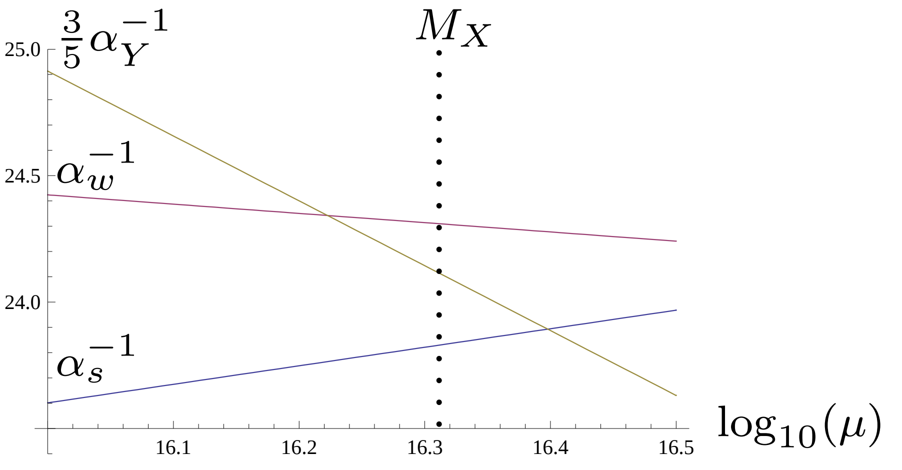
Choosing for instance $M_{3\bar{3}} = 10^{15}$ GeV, the running around the GUT scale changes as



Gauge coupling unification

Gauge coupling unification

zooming further in



The three MSSM gauge couplings satisfy the F-theory GUT relation at

$$M_X = 2.1 \cdot 10^{16}\,\mathrm{GeV}$$

independent of the triplet mass scale $M_{3\overline{3}}$.

Moduli stabilization

Moduli stabilization

Uplift of KKLT and LARGE volume scenario from Type IIB to F-theory

- G_3 flux becomes G_4 form flux with one leg on T^2
- D3-brane instantons $\longrightarrow$ M5-brane instantons, (Bhg, Collinucci, Jurke, arXiv:1002.1894)
- technically challenging: dilaton τ varies over background

New scenario: gravity mediated susy breaking on shrinkable MSSM 4-cycle

- Soft masses are suppressed relative to the gravitino mass: $M_{\rm soft} = \frac{M_{3/2}}{\mathcal{V}^p}$, $p = 1/2, 1$ (Bhg, Conlon, Krippendorf, Moster, Quevedo, arXiv:0906.3297)

Conclusions

Conclusions

F-theory models can provide string theory realisations of $SU(5)$ GUTs on localized branes.

- Gauge fields arise from degenerations of elliptic fiber over surfaces
- Matter fields and Yukawa couplings arise from singularity enhancements over curves and points
- Global compact models showing many of the desired features can be constructed
- GUT breaking works via $U(1)_Y$ flux $\rightarrow$ "problems" with gauge coupling unification
- Determination of flavour structure and Yukawa hierarchies by subleading corrections $\rightarrow$ devil in the details

Further progress both on the technical and the phenomenological level is expected

Thank you!