PreSUSY 10

Calculating Radiative Corrections

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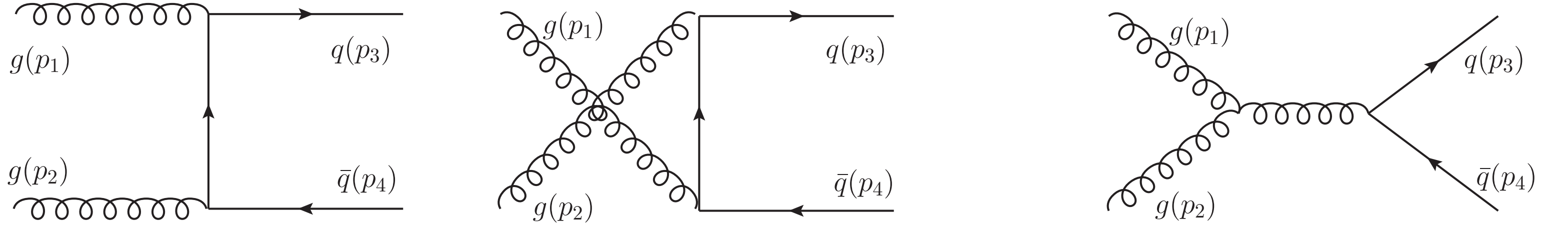
Statement of intent:

- this is about *calculating* radiative corrections not *looking at* radiative corrections
- no plots (with one exception)
- no motivational blah blah (if you're not convinced computing radiative corrections is important, there is still time to leave)
- discuss explicit example $gg \to q\bar{q}$ at one-loop in general context
- no technical details, but explain outline/structure of calculation
- because this is PreSUSY, more emphasis than usual on scheme dependence (SUSY preserving regularization)

introduction	• none
$gg \to q\bar{q}$ at tree-level	• matrix element squared
	• from matrix element to physical cross section
$gg \to q\bar{q}$ at one-loop	• overview / singularities
	• virtual corrections
	• real corrections
	• scheme dependence and assemble
beyond strict one-loop	• structure of two-loop calculation
	• resummation of (large) logarithms
	• parton showers
	• current state-of-the art

Compute matrix element squared $\mathcal{M}^{(0)} \equiv \mathcal{A}^{(0)} \mathcal{A}^{(0)\,*}$



$\sim (T^{a_1} T^{a_2})_{i_3\, i_4}$ $\quad\sim (T^{a_2} T^{a_1})_{i_3\, i_4}$ $\quad\sim (T^{a_1} T^{a_2})_{i_3\, i_4} - (T^{a_2} T^{a_1})_{i_3\, i_4}$

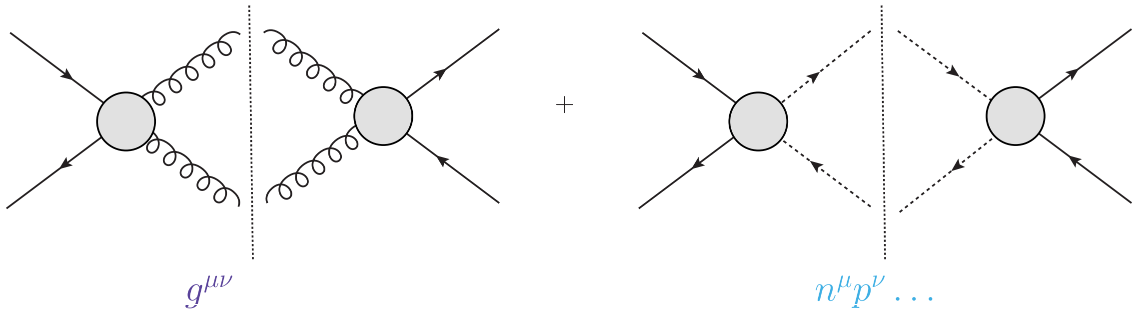
colour:

$$\mathcal{A}^{(0)} = (T^{a_1} T^{a_2})_{i_3\, i_4} A_{12}(s,t,u) + (T^{a_2} T^{a_1})_{i_3\, i_4} A_{21}(s,t,u)$$

$$\mathcal{M}^{(0)} = \underbrace{\frac{(N_c^2-1)^2}{4\,N_c}}_{\text{leading colour}} \left(|A_{12}|^2 + |A_{21}|^2\right) - \underbrace{\frac{(N_c^2-1)}{4\,N_c}}_{\text{subleading colour}} \left(A_{12}\, A_{21}^* + A_{12}^*\, A_{21}\right)$$

Structure of (sub)amplitude: $A_{\#\#} = \bar{u}_\alpha(p_3) v_\beta(p_4) \varepsilon^\mu(p_1) \varepsilon^\nu(p_2)\, (a_{\mu\nu})_{\alpha\beta}$

squaring the amplitude



conventional:

$$\sum_{\text{pols}} \varepsilon^\mu(p_i)\varepsilon^{\nu *}(p_i) \to -g^{\mu\nu} + \underbrace{\frac{n_i^\mu p_i^\nu + p_i^\mu n_i^\nu}{(n_i p_i)} - \frac{n_i^2 p_i^\mu p_i^\nu}{(n_i p_i)^2}}_{n_i^\mu \text{ arbitrary}}; \quad \sum_{\text{pols}} u_\alpha(p)\bar{u}_\beta(p) = (\not{p} + m)_{\alpha\beta};$$

QED: can drop n^μ parts, since $p^\mu_{3/4}\, a_{\mu\nu} = 0$

QCD: $p^\mu_{3/4}\, a_{\mu\nu} \neq 0$, but result independent of $n^\mu_{3/4}$.
alternatively, drop n^μ parts but include ghost diagrams in squaring the amplitude.

In D dimensions we get (including mass terms) e.g.

$$|a_{12}|^2 = -\frac{2\,\alpha_s^2}{s^2 t^2}\left((D-2)t(s+t)\left((D-2)s^2 + 4st + 4t^2\right) + 16m^4 s^2 + 16m^2 st(s+t)\right)$$

helicity method: (massless quarks)

fix helicities of external particles and express amplitude in terms of spionor inner products:

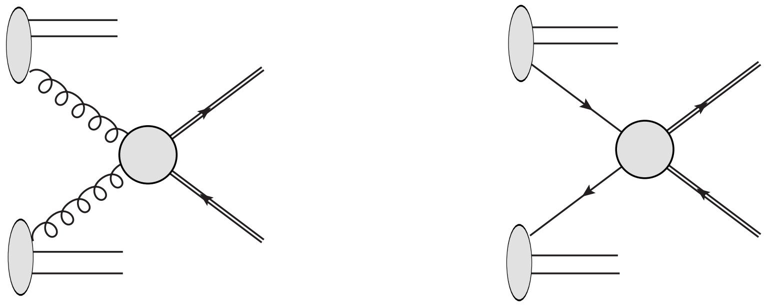
$\langle ij\rangle = \langle p_i - | p_j + \rangle \equiv \bar{u}(p_i,-)u(p_j,+)\,; \quad [ij] = \langle p_i + | p_j - \rangle \equiv \bar{u}(p_i,+)u(p_j,-)$) ;

for gauge bosons use $\varepsilon^\mu(p,\pm) = \pm \frac{\langle p \pm |\gamma^\mu| n\pm\rangle}{\sqrt{2}\langle n \mp | p\pm\rangle}$

- lightlike reference momentum n^μ drops out for gauge invariant quantities
- very compact results, e.g: $a_{12}(g_1^-, g_2^+, q_3^-, \bar{q}_4^+) = i g^2 \frac{\langle 13\rangle^3 \langle 14\rangle}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$
- simplifications (due to gauge cancellations) at amplitude level
- sum over all (non-vanishing) helicity configurations

$$|a_{12}|^2 = \sum_{h_i} |a_{12}(g_1^{h_1}, g_2^{h_2}, q_3^{h_3}, \bar{q}_4^{h_4})|^2$$

- have to treat external particles in 4 dimensions (scheme dependence $\to$ see later) hadronic cross section



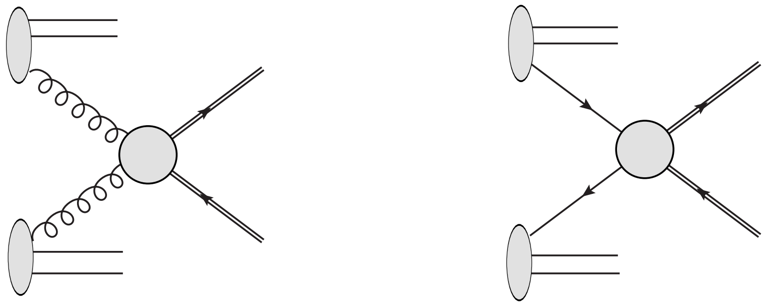
$$d\sigma_{H_1(P_1)H_2(P_2)\to t\bar{t}} =$$
$$\int_0^1 dx_1\, f_{g/H_1}(x_1,\mu_F) \int_0^1 dx_2\, f_{g/H_2}(x_2,\mu_F)\, d\hat{\sigma}_{g(x_1P_1)g(x_2P_2)\to t\bar{t}}(\alpha_s(\mu_R)\ldots) + \ldots$$

μ_F: factorization scale; μ_R: renormalization scale

$f_{g/H_1}(x_1,\mu_F)$: parton distribution functions

$d\hat{\sigma}$: hard partonic cross section, at tree level $d\hat{\sigma}^{(0)} = d\sigma^{(0)}$

hadronic cross section



$$d\sigma_{H_1(P_1)H_2(P_2)\to t\bar{t}} =$$

$$\int_0^1 dx_1\, f_{g/H_1}(x_1,\mu_F) \int_0^1 dx_2\, f_{g/H_2}(x_2,\mu_F)\, d\hat{\sigma}_{g(x_1P_1)g(x_2P_2)\to t\bar{t}}(\alpha_s(\mu_R)\ldots) + \ldots$$

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$f_{g/H_1}(x_1,\mu_F)$: parton distribution functions

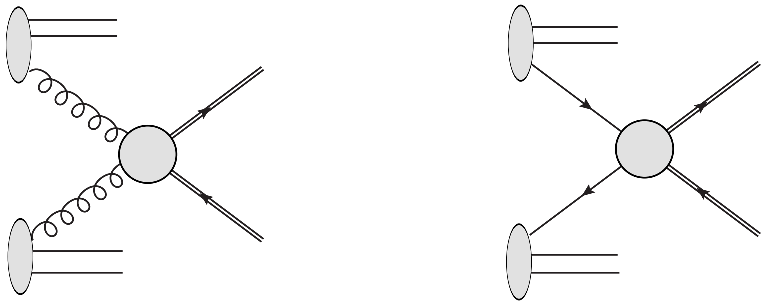
$d\hat{\sigma}$: hard partonic cross section, at tree level $d\hat{\sigma}^{(0)} = d\sigma^{(0)}$

there are additional partonic processes for $H_1H_2 \to t\bar{t}$

$$\begin{aligned} d\sigma_{H_1H_2\to t\bar{t}} &= \int_0^1 dx_1\, f_{g/H_1}(x_1) \int_0^1 dx_2\, f_{g/H_2}(x_2)\, d\hat{\sigma}_{gg\to t\bar{t}} \\ &+ \sum_{q\in\{u,d,c,s,(b)\}} \int_0^1 dx_1\, f_{q/H_1}(x_1) \int_0^1 dx_2\, f_{\bar{q}/H_2}(x_2)\, d\hat{\sigma}_{q\bar{q}\to t\bar{t}} + \{q \leftrightarrow \bar{q}\} \end{aligned}$$

tree level

hadronic cross section



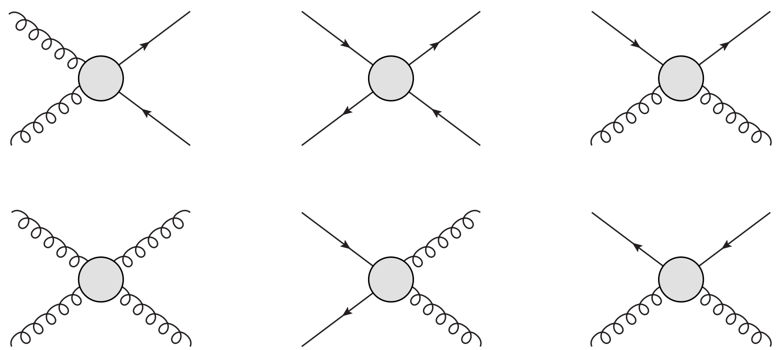
$$d\sigma_{H_1(P_1)H_2(P_2)\to t\bar{t}} =$$
$$\int_0^1 dx_1\, f_{g/H_1}(x_1,\mu_F) \int_0^1 dx_2\, f_{g/H_2}(x_2,\mu_F)\, d\hat{\sigma}_{g(x_1P_1)g(x_2P_2)\to t\bar{t}}(\alpha_s(\mu_R)\ldots) + \ldots$$

μ_F: factorization scale; μ_R: renormalization scale

$f_{g/H_1}(x_1,\mu_F)$: parton distribution functions

$d\hat{\sigma}$: hard partonic cross section, at tree level $d\hat{\sigma}^{(0)} = d\sigma^{(0)}$

and even more partonic processes
for $H_1H_2 \to JJ$



Tree-level: $d\hat{\sigma}^{(0)} = d\sigma^{(0)}$

1-loop: $d\hat{\sigma}^{(1)} = \underbrace{d\sigma^{(0)}}_{\mathcal{O}(\alpha_s^2)} + \underbrace{d\sigma^{\text{virt}} + d\sigma^{\text{real}} + d\sigma^{\text{coll}}}_{\mathcal{O}(\alpha_s^3)}$

- All $\mathcal{O}(\alpha_s^3)$ are (in general) divergent and only the sum is finite (for properly defined, i.e. infrared-safe observables).
- Regularize divergences by working in $D = 4 - 2\epsilon$ dimensions: $\int d^4k \to \mu_R^{2\epsilon} \int d^D k$; singularities $\to$ poles $1/\epsilon$ (dimensional regularization).
- Other possibilities in principle, but not in practice.
- Strictly speaking, only internal momenta have to be D dimensional. There is some freedom how to treat external particles (recall helicity method needs these to be 4 dimensional)
- different schemes (variant of dimensional regularization) possible (this also impacts on SUSY $\to$ later)

Virtual corrections

"measure of difficulty"

- number of external legs
- number of scales (masses, kinematic invariants)
- number of integration momenta in numerator

conventional solution

(in principle straightforward, in practice often very challenging to impossible)

- tensor integrals $\rightarrow$ scalar integrals (Passarino Veltman)

$$\int \frac{d^D k \, k^\mu k^\nu}{k^2 (k-p)^2} = A p^\mu p^\nu + B g^{\mu\nu}$$

  fix A and B by contraction with $p_\mu p_\nu$ and $g_{\mu\nu}$.
- reduce pentagon (and higher-point) integrals to (sums of) box integrals
- plug in (known) box, triangle and bubble integrals and evaluate diagrams

problem: often huge expressions with numerical instabilities (Gram determinants)

On-shell methods

Write amplitude as

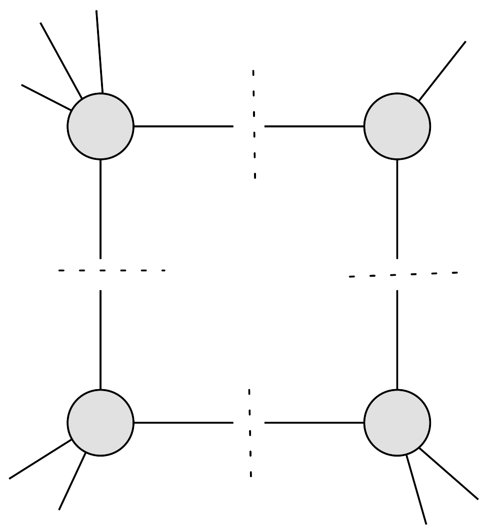
$$\mathcal{A}^{(0)} = \sum \left(d_i I_i^{\rm box} + c_i I_i^{\rm tri} + b_i I_i^{\rm bub} + a_i I_i^{\rm tad} \right) + \text{rational}$$

- identify coefficients of integrals by discontinuities $2\,\mathrm{Im} T_{fi} = (T^\dagger T)_{fi}$
- rational terms (i.e. non-$\log$ or Li_2 terms have to be obtained separately (e.g. recursion relations or D-dimensioal cuts)
- often numerical approach, i.e. for given (numerical) momenta obtain $1/\epsilon^i,\ i \in \{0, 1, 2\}$ coefficients of amplitude numerically

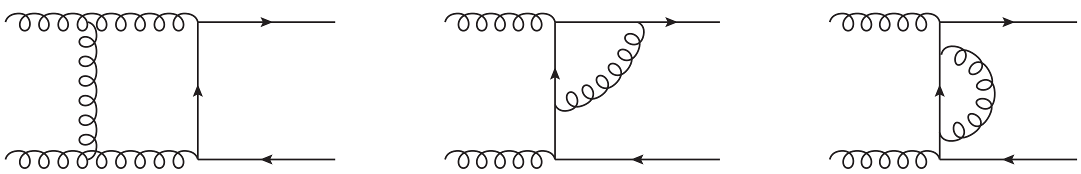
$$\frac{1}{(k-p_i)^2 + iO^+} \to -2\pi i \delta^+((k-p_i)^2)$$

propagators are now on-shell

$\to$ massive simplifications (gauge cancellations)



virtual corrections



do not (yet) include self-energy insertions on external legs (non-vanishing only for massive particles in Dim Reg)

$$\mathcal{A} = \underbrace{\mathcal{A}^{(0)}}_{\sim\alpha_s} + \underbrace{\mathcal{A}^{(1)}}_{\sim\alpha_s^2} + \ldots \Longrightarrow \mathcal{M}^{(0)} = |\mathcal{A}^{(0)}|^2 \sim \alpha_s^2 \;\text{ and }\; \mathcal{M}^{(1)} = 2\,\mathrm{Re}\left(\mathcal{A}^{(0)}\mathcal{A}^{(1)\,*}\right) \sim \alpha_s^3$$

colour:

$$\begin{aligned}
\mathcal{A}^{(1)} &= (T^{a_1}T^{a_2})_{i_3\,i_4}\left(\frac{N_c}{2}A^L_{12}(s,t,u) + \frac{1}{2N_c}A^S_{12}(s,t,u) + \frac{N_F}{2}A^F_{12}(s,t,u)\right) \\
&+ \{12 \leftrightarrow 21\} \\
&+ \delta_{i_3 i_4}\frac{1}{2}\mathrm{Tr}\,(T^{a_1}T^{a_2})\left(A_{\mathrm{tr}}(s,t,u) + \frac{N_F}{N_c}A^F_{\mathrm{tr}}(s,t,u)\right)
\end{aligned}$$

virtual corrections

- “result” of calculation

$$A_{12}^{L} = \frac{1}{\epsilon^2}\left[c_s\left(\frac{-s}{\mu^2}\right)^{-\epsilon} + c_t\left(\frac{-t}{\mu^2}\right)^{-\epsilon} + \ldots\right] + \frac{1}{\epsilon}\,\text{mess}(\log) + \text{finite mess}(\log^2, \text{Li}_2)$$

- UV singularities ($1/\epsilon$ per loop) $\Longrightarrow$ renormalization
- soft and final-sate collinear singularities ($1/\epsilon$ per loop) $\Longrightarrow$ combine with real corrections
- soft-collinear singularities ($1/\epsilon^2$ per loop) $\Longrightarrow$ combine with real corrections
- initial-sate collinear singularities ($1/\epsilon$ per loop) $\Longrightarrow$ combine with collinear counterterm $d\sigma^{\text{coll}}$

renormalization

recall: $|a_{12}|^2 = -\frac{2\,\alpha_0^2}{s^2 t^2}\left((D-2)t(s+t)\left((D-2)s^2+4st+4t^2\right)+16m_0^4 s^2+16m_0^2 st(s+t)\right)$

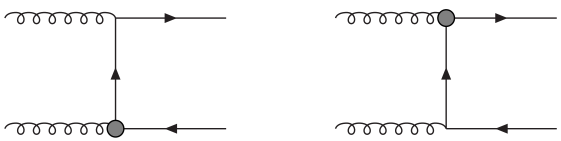
re-express bare quantities in terms of renormalized quantities:

$\overline{\text{MS}}$ scheme: $\alpha_0 = Z_\alpha\,\alpha_s = \left[1-\frac{\alpha_s}{4\pi}c_\Gamma\,\frac{\beta_0}{\epsilon}\right]\alpha_s$

on-shell scheme: $m_0 = Z_m\,m = \left[1+\frac{\alpha_s}{4\pi}c_\Gamma\,C_F\left(\frac{m^2}{\mu_R^2}\right)^{-\epsilon}\left(-\frac{3}{\epsilon}-4\right)\right]m$

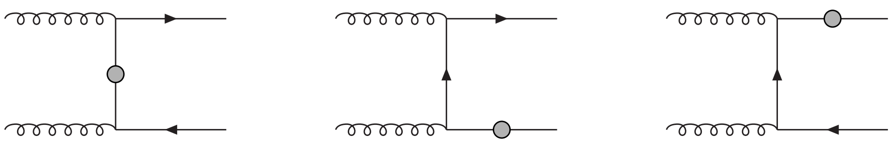
expressed in terms of couterterm diagrams:

α :



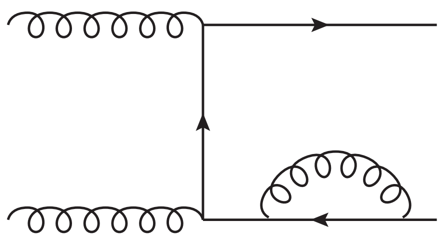
+ other diagrams

m :



+ other diagrams

virtual corrections

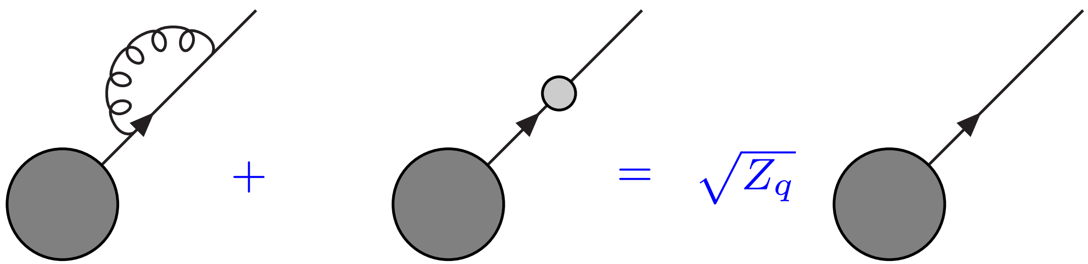


for massless external particles, self-energy vanishes (in dim reg): $\Longrightarrow$ only coupling renormalization

for massive quarks: intermediate propagator $1/(p^2 - m^2)$, but $p^2 \rightarrow m^2$.

massive quark: $-i\Sigma(p^2, m^2) = \underbrace{(\not{p} - m)\, B(m^2)}_{\text{wave func. ren}} + \underbrace{m\, A(m^2)}_{\text{mass ren}} + \ldots$

in on-shell scheme:



$\Longrightarrow$ mass counterterms only in internal lines renormalized virtual corrections: there is order in the apparent chaos; $(a_1 \dots a_n) = (g, g, q, \bar{q})$

$$\mathcal{M}^{(1)}_{\mathrm{RS}*}(a_1 \dots a_n) = \frac{\alpha_s}{2\pi} c_\Gamma \left[\mathcal{M}^{(0)}_{\mathrm{RS}*}(a_1 \dots a_n) \left(-\frac{1}{\epsilon} \sum_i \gamma_{\mathrm{RS}*}(a_i) \right) + \sum_{i,j} \mathcal{V}(i,j)\, \mathcal{M}^{ij}_{\mathrm{RS}*}(a_1 \dots a_n) + \mathcal{M}^{(1)}_{\mathrm{NS}}(a_1 \dots a_n) \right],$$

$$c_\Gamma \equiv (4\pi)^\epsilon \frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} = \left(\frac{e^{\gamma_E}}{4\pi} \right)^{-\epsilon} \left(1 - \frac{\epsilon^2 \pi^2}{12} + \mathcal{O}(\epsilon^3) \right)$$

$$\mathcal{V}(i,j) = -\frac{1}{2\epsilon^2} \mathrm{Re} \left(-\frac{s_{ij}}{\mu^2} \right)^{-\epsilon}$$ for massless particles

$\mathcal{M}^{ij}_{\mathrm{RS}*}$: colour “twisted” tree-level amplitudes

$$\gamma_{\mathrm{RS}}(q) = \frac{3C_F}{2} + \mathcal{O}_{\mathrm{RS}}(\epsilon)\,; \quad \gamma(Q) = C_F\,; \quad \gamma_{\mathrm{RS}}(g) = \frac{\beta_0}{2} + \mathcal{O}_{\mathrm{RS}}(\epsilon)\,;$$

$\mathcal{M}^{(1)}_{\mathrm{NS}}$: complicated but finite scheme dependence:

$$\mathcal{M}^{(1)}_{\mathrm{RS}*}(a_1 \ldots a_n) = \frac{\alpha_s}{2\pi} c_\Gamma \left[\mathcal{M}^{(0)}_{\mathrm{RS}*}(a_1 \ldots a_n) \left(-\frac{1}{\epsilon} \sum_i \gamma_{\mathrm{RS}*}(a_i) \right) + \sum_{i,j} \mathcal{V}(i,j)\, \mathcal{M}^{ij}_{\mathrm{RS}*}(a_1 \ldots a_n) + \mathcal{M}^{(1)}_{\mathrm{NS}}(a_1 \ldots a_n) \right],$$

- what precisely do we mean by e.g. $\mathcal{M}^{(0)}$, 4 dimensional ? D dimensional ?
- no problem if $\mathcal{M}^{(1)}$ is scheme dependent, but physical cross sections must be scheme independent !
- if we deal with helicity methods, how to consistently treat external 4-dimensional particles ?
- Having D dimensional gluon breaks SUSY (gluon and gluino have different number of degrees of freedom)
- Breaking a symmetry of a theory through regularization is not a disaster, but not very nice either.

Breaking of SUSY (or any other symmetry) by regularization requires additional symmetry restoring counterterms

- consider e.g. SUSY relation $m_e = m_{\tilde{e}}$, at one-loop $m^2(1L) = m^2 - \Sigma(p^2 = m^2)$
- in CDR :

$$\begin{aligned} m_e(1L) &= m_e\left[1 + \frac{\alpha}{4\pi}\,(2B_0 - 1)\right] \\ m_{\tilde{e}}(1L) &= m_e\left[1 + \frac{\alpha}{4\pi}\left(2B_0 + \frac{2}{3}\right)\right] \end{aligned}$$

- in DRED : $m_e(1L) = m_{\tilde{e}}(1L)$
- in CDR have to add additional counterterm: $\delta m_{\tilde{e}} = -\frac{\alpha}{4\pi}\frac{5m_e}{3}$
- DRED and FDH preserve SUSY (at least in this case)

scheme dependence:

Introduce three spaces: $\bar{g}$ (Quasi 4-dim) $\supset$ $\hat{g}$ (Quasi D-dim) $\supset$ $\bar{g}$ (Strictly 4-dim)

$$g^{\mu\nu}g_{\mu\nu} = 4; \quad \hat{g}^{\mu\nu}\hat{g}_{\mu\nu} = D; \quad \bar{g}^{\mu\nu}\bar{g}_{\mu\nu} = 4$$
$$g^{\mu\nu}\hat{g}_{\nu}{}^{\rho} = \hat{g}^{\mu\rho}; \quad g^{\mu\nu}\bar{g}_{\nu}{}^{\rho} = \bar{g}^{\mu\rho}; \quad \hat{g}^{\mu\nu}\bar{g}_{\nu}{}^{\rho} = \bar{g}^{\mu\rho}$$

- CDR (“conventional dimensional regularization"): Here internal and external gluons (and other vector fields) are all treated as D-dimensional.
- HV (“ ’t Hooft Veltman scheme"): Internal gluons are treated as D-dimensional but external ones are treated as strictly 4-dimensional.
- DRED (“original/old dimensional reduction"): Internal and external gluons are all treated as quasi-4-dimensional.
- FDH (“four-dimensional helicity scheme"): Internal gluons are treated as quasi-4-dimensional but external ones are treated as strictly 4-dimensional.

	CDR	HV	FDH	DRED
internal gluon	$\hat{g}^{\mu\nu}$	$\hat{g}^{\mu\nu}$	$g^{\mu\nu}$	$g^{\mu\nu}$
external gluon	$\hat{g}^{\mu\nu}$	$\bar{g}^{\mu\nu}$	$\bar{g}^{\mu\nu}$	$g^{\mu\nu}$

internal: in 1PI part of loop
(complication for real part! $\Longrightarrow$ later)

external: all others

scheme dependence: consistent use of DRED requires split $g \longrightarrow \hat{g} + \tilde{g}$ with
$\tilde{g}^{\mu\nu}\tilde{g}_{\mu\nu} = 4 - D = 2\epsilon,\ g^{\mu\nu}\tilde{g}_{\nu}{}^{\rho} = \tilde{g}^{\mu\rho},\ \hat{g}^{\mu\nu}\tilde{g}_{\nu}{}^{\rho} = 0$

$$\mathcal{M}^{(0)}_{\mathrm{DRED}}(g,g,t,\bar{t}) = \sum_{\{\breve{a}\}} \mathcal{M}^{(0)}_{\mathrm{DRED}}(\breve{a}_1 \dots \breve{a}_4)$$

$$= \mathcal{M}^{(0)}_{\mathrm{DRED}}(\hat{g},\hat{g},t,\bar{t}) + \mathcal{M}^{(0)}_{\mathrm{DRED}}(\hat{g},\tilde{g},t,\bar{t}) + \mathcal{M}^{(0)}_{\mathrm{DRED}}(\tilde{g},\hat{g},t,\bar{t}) + \mathcal{M}^{(0)}_{\mathrm{DRED}}(\tilde{g},\tilde{g},t,\bar{t})$$

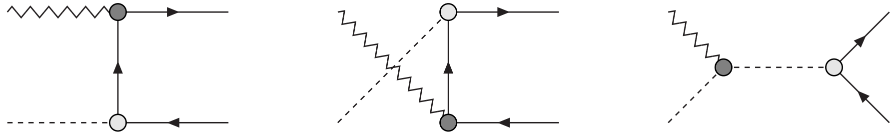
$$\begin{aligned} \mathcal{M}^{(1)}_{\mathrm{DRED}}(a_1 \dots a_n) &= \sum_{\{\breve{a}\}} \frac{\alpha_s}{2\pi} c_\Gamma \Bigg[\mathcal{M}^{(0)}_{\mathrm{DRED}}(\breve{a}_1 \dots \breve{a}_n) \left(-\frac{1}{\epsilon} \sum_i \gamma_{\mathrm{DRED}}(\breve{a}_i) \right) \\ &+ \sum_{i,j} \mathcal{V}(i,j)\, \mathcal{M}^{ij}_{\mathrm{DRED}}(\breve{a}_1 \dots \breve{a}_n) + \mathcal{M}^{(1)}_{\mathrm{NS}}(\breve{a}_1 \dots \breve{a}_n) \Bigg] . \end{aligned}$$

split is crucial for

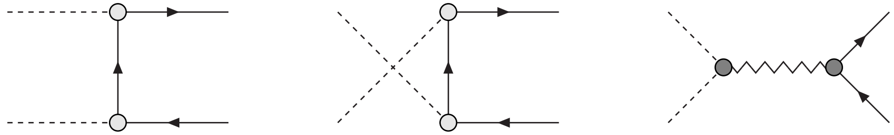
- $\gamma(\tilde{g}) = 2\,N_c - T_F\,N_F \neq \gamma(\hat{g})$
- renormalization: $\hat{g}t\bar{t}$ and $\tilde{g}t\bar{t}$ renormalize differently in QCD

renormalization

$$\mathcal{M}^{(1,\mathrm{ct})}_{\mathrm{DRED}}(\hat{g},\tilde{g};q,\bar{q}) =$$



$$\mathcal{M}^{(1,\mathrm{ct})}_{\mathrm{DRED}}(\tilde{g},\tilde{g};q,\bar{q}) =$$



$$\delta Z_g^{\mathrm{DRED}} = \frac{\alpha_s}{4\pi}\frac{c_\Gamma}{\epsilon}\frac{(-11+\epsilon)N_c + 4\,T_F N_F}{6}\,,$$

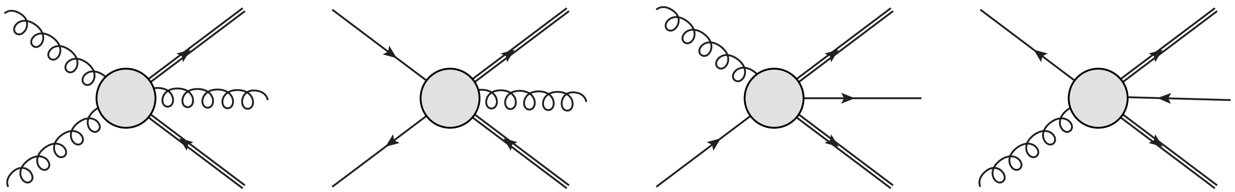
$$\delta \widetilde{Z}_g^{\mathrm{DRED}} = \frac{\alpha_s}{4\pi}\frac{c_\Gamma}{\epsilon}\left(\frac{1}{2\,N_c} - \frac{3\,N_c}{2} + T_F N_F + \epsilon\,\mathrm{Finite}\right)\,.$$

- Finite is irrelevant since corresponding $\mathcal{M}^{(0)} \sim \mathcal{O}(\epsilon)$. We do not need to fix a renormalization scheme for the $\tilde{g}t\bar{t}$ coupling
- there are $\mathcal{O}(\epsilon)$ terms in Z_g^{DRED} (even though we use the $\overline{\mathrm{MS}}$ scheme), because we use DRED and not CDR.

Real corrections

$$d\sigma^{\rm real}_{\rm RS*} = \sum_{\bar{a}_i} \int d\Phi_3(p_1,p_2;p_3,p_4,p_5)\langle \mathcal{M}^{(0)}_{\rm RS*}(a_1,a_2;\bar{a}_3,\bar{a}_4,\bar{a}_5)\rangle$$

processes: $\mathcal{M}^{(0)}_{\rm RS*}(g,g;t,\bar{t},g)$, but also new partonic channels $\mathcal{M}^{(0)}_{\rm RS*}(q,g;t,\bar{t},q)$ etc.
calculation of $\mathcal{M}^{(0)}_{\rm RS*}$ as for tree-level.



$\mathcal{M}^{(0)}_{\rm RS*}$ has no $1/\epsilon$ poles, but has (non-integrable) singularities in some regions of phase space.

$$\underbrace{\int d\Phi_{n-1}\left(\mathcal{M}^{(0)} - \sum_{\rm sing}\mathcal{M}^{\rm appr}\right)}_{\text{finite}} + \underbrace{\int d\Phi_{n-1}\sum_{\rm sing}\mathcal{M}^{\rm appr}}_{\text{use dim reg}}$$

Real corrections naive example (e.g. gluon g soft, $x \sim$ energy)

$$\mathcal{A}(g,g,t,\bar{t},g) \overset{g\to 0}{\sim} \frac{1}{\sqrt{x}}\mathcal{A}(g,g,t,\bar{t}) + \mathcal{A}^{\text{rem}}$$

$$\mathcal{M}(g,g,t,\bar{t},g) \sim \frac{1}{x}\mathcal{M}(g,g,t,\bar{t}) + \frac{1}{\sqrt{x}}\mathcal{M}^{\text{rem}}$$

$$\int d\Phi_3^D \mathcal{M}(g,g,t,\bar{t},g) = \underbrace{\int d\Phi_3^4 \left(\mathcal{M}(g,g,t,\bar{t},g) - \frac{1}{x}\mathcal{M}(g,g,t,\bar{t})\right)}_{\text{term 1}} + \underbrace{\int d\Phi_3^D \frac{1}{x}\mathcal{M}(g,g,t,\bar{t})}_{\text{term 1}}$$

term 1: evaluate numerically in 4 dimensions, square root singularities !

term 2: $\int x^{-\epsilon}\frac{1}{x}\int d\Phi_2^4\,\mathcal{M}(g,g,t,\bar{t}) = -\frac{1}{\epsilon}\int d\Phi_2^4\,\mathcal{M}(g,g,t,\bar{t})$

there are several well established general procedures

Real corrections, soft singularities

general structure of soft limit of matrix element squared:

$$\mathcal{M}^{(0)}_{\mathrm{RS}*}(a_1, a_2; \ldots g_k(p_k) \ldots \bar{a}_{n+1}) \stackrel{p_k \to 0}{=} g_s^2 \sum_{i,j} \frac{s_{ij}}{s_{ik} s_{jk}} \mathcal{M}^{ij}_{\mathrm{RS}*}(a_1 \ldots a_n)$$

$\int d\Phi_k \frac{s_{ij}}{s_{ik} s_{jk}} \sim \mathcal{V}(i,j)$ cancels corresponding virtual singularity

(contains also soft collinear $i \| k$ and $j \| k$ singularities)

only scheme dependence in $\mathcal{M}^{ij}$

CDR: D dimensional

DRED, HV, FDH: 4 dimensional

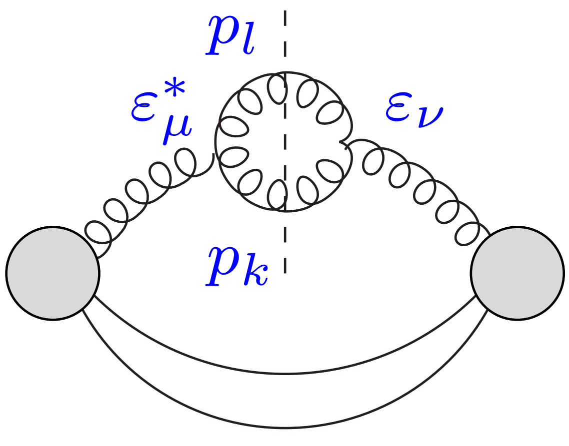
Real corrections, final-state collinear singularities

general structure of (two final state massless particles) collinear limit of matrix element squared:

$p_k \longrightarrow z(p_k + p_l)$ and $p_l \longrightarrow (1-z)(p_k+p_l)$

$$\mathcal{M}^{(0)}_{\mathrm{RS}*}(a_1, a_2; \ldots \bar{a}_l(p_l) \ldots \bar{a}_k(p_k) \ldots \bar{a}_{n+1}) \stackrel{p_k \| p_l}{=} \frac{2\,g_s^2}{s_{kl}} P^{<\,\mathrm{RS}*}_{(kl)^* \to kl}(z)\, \mathcal{M}^{(0)}_{\mathrm{RS}*}(a_1, a_2; \ldots a_{(kl)}(p_k+p_l) \ldots a_n)$$

scheme dependence $\mathcal{M}^{(0)}$ and $P^{<\,\mathrm{RS}*}_{(kl)^* \to kl}(z)$



treat partons k and l as internal

unitarity requires unresolved particles to be treated the same in virtual and real contributions

internal: particles in 1PI part of loop or unresolved initial/final state particles

origin of $\gamma(a_i)$ scheme dependence in real corrections

do NOT take $P^{<\,\mathrm{CDR}}_{(kl)^* \to kl}(z)|_{D \to 4}$ for HV and/or FDH.

Altarelli-Parisi kernels

$$P^{<\,\mathrm{CDR}}_{g^*\to gg} = P^{<\,\mathrm{DRED}}_{\hat{g}^*\to\hat{g}\hat{g}} = 2N_c\left(\frac{z}{1-z}+\frac{1-z}{z}+z(1-z)\right) ,$$

$$P^{<\,\mathrm{FDH}}{}_{g^*\to gg} = P^{<\,\mathrm{DRED}}{}_{\hat{g}^*\to gg} = 2N_c\left(\frac{z}{1-z}+\frac{1-z}{z}+\frac{2}{D-2}\,z(1-z)\right) ,$$

$$P^{<\,\mathrm{CDR}}_{g^*\to q\bar{q}} = P^{<\,\mathrm{FDH}}_{g^*\to q\bar{q}} = P^{<\,\mathrm{DRED}}_{\hat{g}^*\to q\bar{q}} = T_F\left(1-\frac{4}{D-2}z(1-z)\right) ,$$

$$P^{<\,\mathrm{CDR}}_{q^*\to qg} = P^{<\,\mathrm{DRED}}_{q^*\to q\hat{g}} = C_F\left(\frac{2z}{1-z}+\frac{D-2}{2}(1-z)\right) ,$$

$$P^{<\,\mathrm{FDH}}_{q^*\to qg} = P^{<\,\mathrm{DRED}}_{q^*\to qg} = C_F\left(\frac{2z}{1-z}+(1-z)\right) .$$

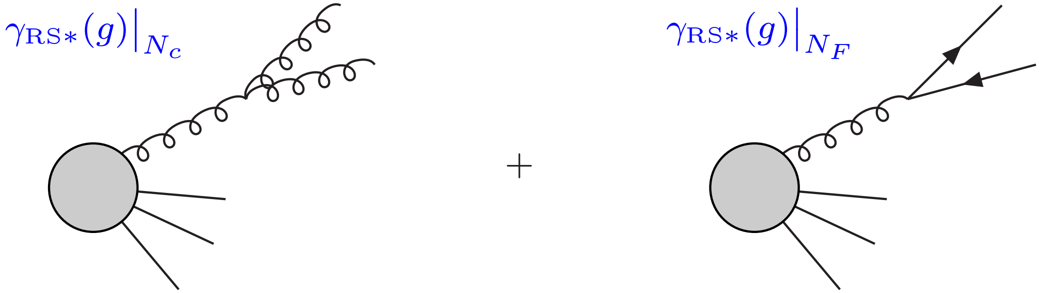
$$P^{<\,\mathrm{CDR}}_{i^*\to kl} = P^{<\,\mathrm{HV}}_{i^*\to kl}$$

$$P^{<\,\mathrm{DRED}}_{\hat{g}^*\to\tilde{g}\tilde{g}} = 2N_c\,\frac{4-D}{D-2}\,z(1-z),$$

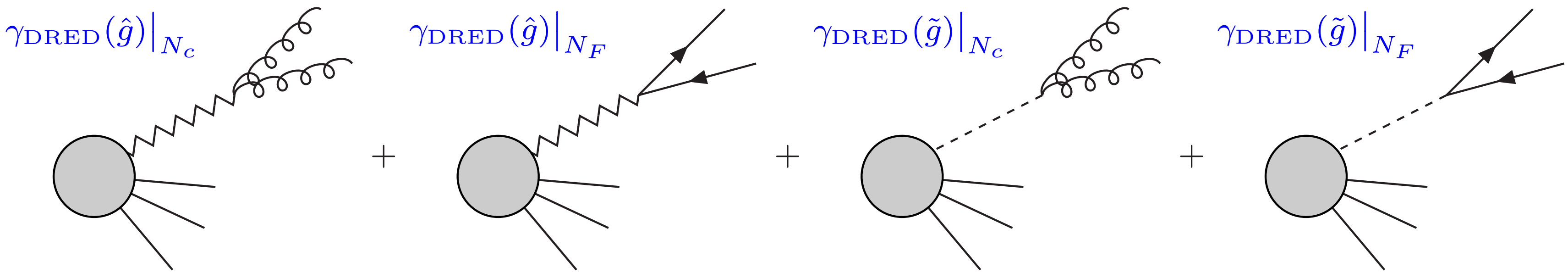
$$P^{<\,\mathrm{DRED}}_{\tilde{g}^*\to q\bar{q}} = T_F,$$

Real corrections, final-state collinear singularities

for CDR, HV and FDH:



for DRED:



Real corrections, initial-state collinear singularities

general structure of collinear limit (of final-state to initial-state particle) of matrix element squared:

$p_k \to (1-z)p_1$

$$\langle \mathcal{M}^{(0)}_{\mathrm{RS}*}(a_1(p_1), a_2; \dots \bar{a}_k(p_k) \dots \bar{a}_{n+1})\rangle \overset{p_k \| p_1}{=} \frac{2\,g_s^2}{s_{1k}} P^{<\,\mathrm{RS}*}_{1\to(1k)^* k}(z)\, \langle \mathcal{M}^{(0)}_{\mathrm{RS}*}(a_{(1k)}(z\, p_1), a_2; \dots a_n)\rangle$$

scheme dependence $\mathcal{M}^{(0)}$ and $P^{<\,\mathrm{RS}*}_{(kl)^*\to kl}(z)$

after (partial) phase-space integration

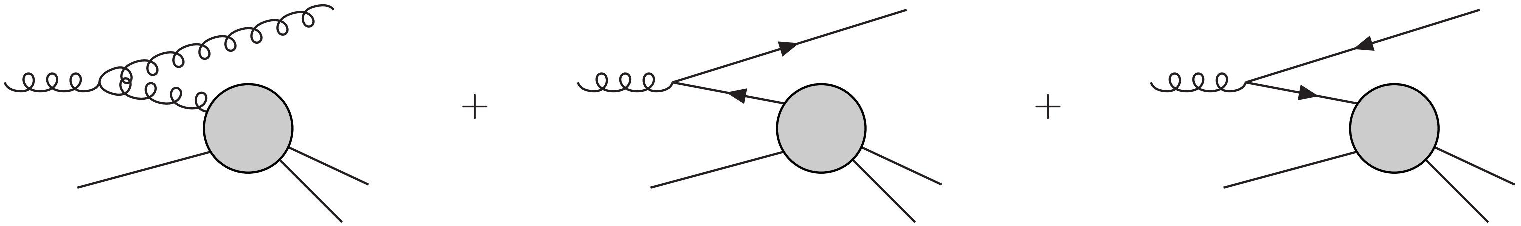
$$d\sigma^{\mathrm{real},1}_{\mathrm{RS}*}(a_1 \dots a_n) = \frac{\alpha_s}{2\pi}\frac{c_\Gamma}{\epsilon}\Big[\gamma_{\mathrm{RS}*}(a_1)\, d\sigma^{(0)}_{\mathrm{RS}*}(a_1(p_1), a_2; \dots a_n) - \sum_{a_k}\int dz\, \left(P^{\mathrm{RS}*}_{1\to(1k)k}\right) d\sigma^{(0)}_{\mathrm{RS}*}(a_{(1k)}(z\, p_1), a_2; \dots a_n)\Big]$$

singularity in first line cancells corresponding virtual singularity

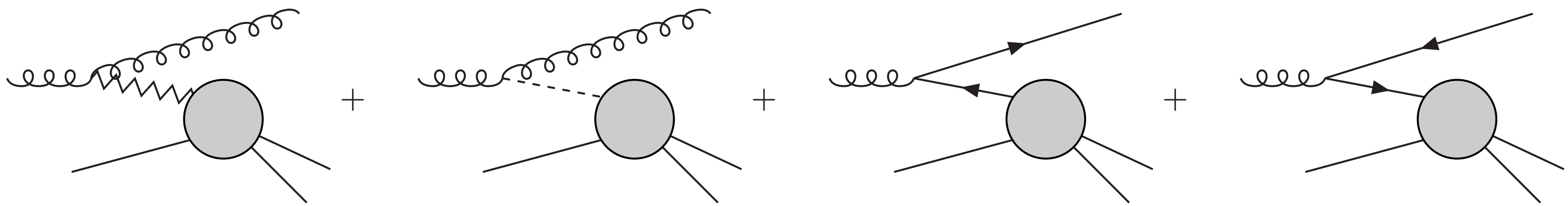
singularity in last line left over (will be cancelled by $d\sigma^{\mathrm{coll}}$)

Real corrections, initial-state collinear singularities

for CDR, HV and FDH:



for DRED:



Collinear counterterm

for CDR, HV and FDH: (for DRED additional summation over $\hat{g}$ and $\tilde{g}$)

$$d\sigma^{\text{coll}}_{\text{RS}*,\text{FS}}(a_1, a_2; \dots a_n) = \frac{\alpha_s}{2\pi} \frac{c_\Gamma}{\epsilon} \sum_{a_k} \int dz$$

$$\times \quad \Big[\big(P^{\text{RS}*}_{1\to ik}(z) + \epsilon X^{\text{FS}}_{1\to ik}(z)\big)\, d\sigma^{(0)}_{\text{RS}*}(a_i(z\, p_1), a_2(p_2); \dots a_n)$$

$$+ \big(P^{\text{RS}*}_{2\to ik}(z) + \epsilon X^{\text{FS}}_{2\to ik}(z)\big)\, d\sigma^{(0)}_{\text{RS}*}(a_1(p_1), a_i(z\, p_2); \dots a_n) \Big],$$

factorization scheme fixed by $X^{\text{FS}}_{1\to ik}(z)$.

in practice virtually always $\overline{\text{MS}}$: $\epsilon X^{\overline{\text{MS}}}_{1\to ik}(z) = -P^{\text{CDR}}_{1\to ik}(z) + \big[P^{\text{CDR}}_{1\to ik}(z)\big]_{D\to 4}$
i.e. remove $\mathcal{O}(\epsilon)$ parts of $P_{1\to ik}$.

can use $\overline{\text{MS}}$ factorization scheme i.e. “normal” pdf with any regularization scheme
do not need $\tilde{g}$ pdf

assemble

- terms $\frac{1}{\epsilon^2}\left(\frac{-s}{\mu^2}\right)^{-\epsilon}$ etc cancel completely between real and virtual corrections
- UV singularities cancel $\frac{1}{\epsilon}\left(\frac{-s}{\mu_R^2}\right)^{-\epsilon} - \frac{1}{\epsilon} \Longrightarrow$ residual μ_R dependence $\sim \log(s/\mu_R)$ partially compensating $\alpha_s(\mu_R)$ dependence.
- initial state collinear singularities cancel with residual μ_F dependence partially compensating $f_{g/H}(\mu_F)$ dependence.
- $d\hat{\sigma}^{(1)}$ is finite and scheme independent (up to the order we have computed, i.e. $\mathcal{O}(\alpha_s^3)$), but there are (hopefully numerically small) implicit $\mathcal{O}(\alpha_s^4)$ differences between various schemes. Split $g = \hat{g} + \tilde{g}$ in DRED is only needed for UV ren and initial state counter term.
- μ_F and μ_R have nothing to do with each other, do not vary them together ! (though everybody does because we're lazy)
- μ_F and μ_R dependence is not a reliable indicator of theoretical uncertainty, do not use this to determine theoretical error ! (though everybody does, because there is nothing much that is really better.)

Structure of two-loop calculation

virtual: $\int d\Phi(2)\left(2\mathrm{Re}\left[\mathcal{A}^{(0)}(g,g,t,\bar{t})\mathcal{A}^{(2)\,*}(g,g,t,\bar{t})\right]+|\mathcal{A}^{(1)}(g,g,t,\bar{t})|^2\right)$
virtual singularities $1/\epsilon^4\ldots$; phase space integration finite

real-virtual: $\int d\Phi(3)\,2\mathrm{Re}\left[\mathcal{A}^{(0)}(g,g,t,\bar{t},g)\mathcal{A}^{(1)\,*}(g,g,t,\bar{t},g)\right]$
virtual singularities $1/\epsilon^2\ldots$; phase space integration $\longrightarrow 1/\epsilon^2\ldots$;

double real: $\int d\Phi(4)\,|\mathcal{A}^{(0)}(g,g,t,\bar{t},g,g)|^2$
no virtual singularities; phase space integration $\longrightarrow 1/\epsilon^4\ldots$;

plus many additional processes: $qq'\longrightarrow t\bar{t}qq'$ plus many more

also needs pdf consistently at two-loops (available)

structure of singularities is known at two-loops (and higher) as well

Resummation of large logs

so far $d\sigma = \alpha_s^n \, d\sigma^{(n)} + \alpha_s^{n+1} \, d\sigma^{(n+1)} + \ldots$

this is fine if $d\sigma^{(n)} \sim d\sigma^{(n+1)}$, but very often $d\sigma^{(n+1)} \gg d\sigma^{(n)}$.

if problem has two (or more) widely different scales $q_L \gg q_S$ we can get one or two powers of $\log(q_L/q_S)$ per α_s.

$\alpha_s \ll 1$, but $\alpha_s \log(q_L/q_S) \ll 1$? $\alpha_s \log^2(q_L/q_S) \ll 1$????

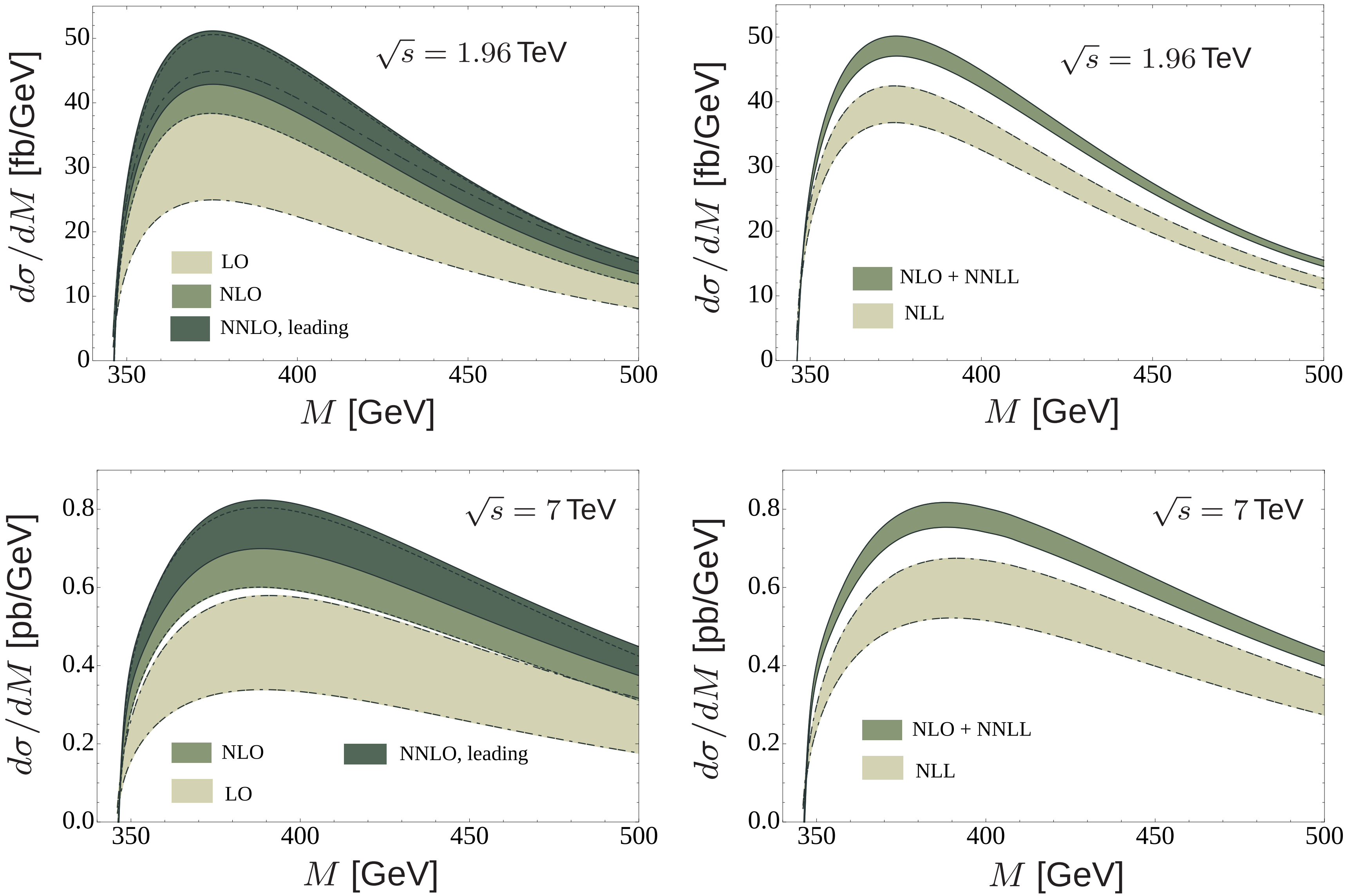
need to resum logarithms, i.e. count $\alpha_s \log(q_L/q_S) \sim 1$

logarithms are "remnants" of $1/\epsilon$ poles $\Longrightarrow$ they are simpler to predict

use renromalization-group equations to resum logarithms

from LO, NLO, NNLO etc to LL, NLL, NNLL etc.

threshold logs in $t\bar{t}$ case For invariant mass of $t\bar{t}$ pair M close to partonic centre-of-mass $\sqrt{\hat{s}}$ there are large logarithms $\log(1 - M/\hat{s})$. [plots from Ahrens et.al. arXiv:1003.5827]



Parton showers

- Can use perturbation theory only if there is (at least) on large scale Q
- collinear emission $\longrightarrow$ small relative p_T $\longrightarrow$ small relative p_T $\Longrightarrow$ large logarithm $\log(Q^2/p_T^2)$.
- resummation of these logarithms done by parton-showers.
- multiparton amplitudes have simple structure in (multiple) collinear limit
- to minimize deficiencies of using (collinear) approximation, combine parton showers with exact matrix elements (where possible) and one-loop corrections (where possible)
- because of its massive importance, this is a huge “industry” (don’t dare to mention programs, as I would miss some...)

This is NOT meant to be a complete list, just to give a flavour !!

real radiation, including subtraction terms

largely automated and fairly straightforward [Sherpa, MadGraph, Helac/Phegas . . .]

virtual one-loop

either analytic or semi-numerical [BlackHat, Golem, Rocket, CutTools . . .]

$gg \to gggg, \ldots$ and more gluons for special helicity configurations

complete one-loop

$2 \to 4$ (limit) [e.g. $t\bar{t}b\bar{b}$, $VJJJ$]
$2 \to 3$ (“standard”) [e.g. $t\bar{t}Z$, VVJ, HJJ, $\ldots$]

virtual two-loop

$2 \to 2$ massless parton processes (for 2 jet production); $gg \to t\bar{t}$

complete two-loop

$e^+e^- \to JJJ$, $gg \to H$