Determining SUSY Lagrangian Parameters

by

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Talk outline

- LHC SUSY measurements
- Tools
- SUSY model fits

Please ask questions while I'm talking

The MSSM Lagrangian

$$W = (Y_E)_{ij} L_i H_1 \bar{E}_j + (Y_D)_{ij} Q_i H_1 \bar{D}_j + (Y_U)_{ij} Q_i H_2 \bar{U}_j + \mu H_2 H_1 \quad (1)$$

$$\tilde{Q}_{i_L} (U_A)_{ij} \tilde{u}_j H_2 + \tilde{Q}_{i_L} (D_A)_{ij} \tilde{d}_j H_1 + \tilde{L}_{i_L} (E_A)_{ij} \tilde{e}_j H_1 +$$

$$H.c. + m_{H_1}^2 {H_1}^* H_1 + m_{H_2}^2 {H_2}^* H_2 + \tilde{Q}_i^* (m_{\tilde{Q}}^2)_{ij} \tilde{Q}_j +$$

$$\tilde{L}_i^* (m_{\tilde{L}}^2)_{ij} \tilde{L}_j + \tilde{u}_i (m_{\tilde{u}}^2)_{ij} \tilde{u}_j^* + \tilde{d}_i (m_{\tilde{d}}^2)_{ij} \tilde{d}_j^* + \tilde{e}_i (m_{\tilde{e}}^2)_{ij} \tilde{e}_j^* +$$

$$[m_3^2 H_2 H_1 + \frac{1}{2} \left(M_1 \tilde{b} \tilde{b} + M_2 \tilde{w} \tilde{w} + M_3 \tilde{g} \tilde{g} \right) + H.c.]$$

$\mathcal{Q}$: How many parameters including $g_{1,2,3}$?

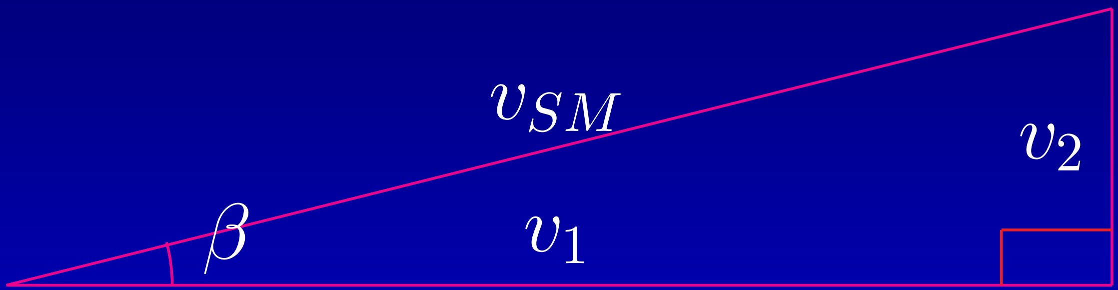
$\mathcal{A}$: ~ 105

Electroweak Breaking

Both Higgs get vacuum expectation values:

$$\begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} \rightarrow \begin{pmatrix} v_1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

and to get M_W correct, match with $v_{SM} = 246$ GeV:



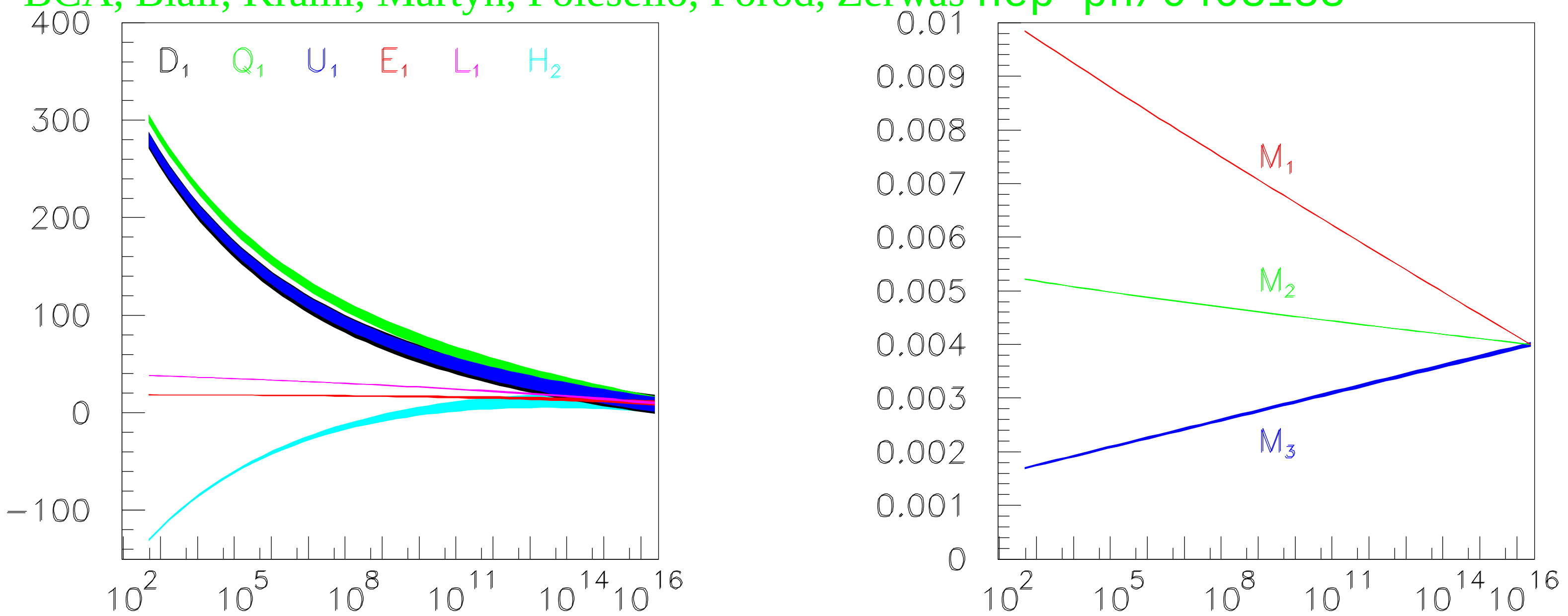
$\tan\beta = \frac{v_2}{v_1}$

$$\begin{aligned} \mathcal{L} &= h_t \bar{t}_L H_2^0 t_R + h_b \bar{b}_L H_1^0 b_R + h_\tau \bar{\tau}_L H_1^0 \tau_R \\ \Rightarrow \frac{m_t}{\sin\beta} &= \frac{h_t v_{SM}}{\sqrt{2}}, \qquad \frac{m_{b,\tau}}{\cos\beta} = \frac{h_{b,\tau} v_{SM}}{\sqrt{2}}. \end{aligned}$$

The Inverse Problem

Want to do this with LHC+ILC data:

BCA, Blair, Kraml, Martyn, Polesello, Porod, Zerwas hep-ph/0403133



Arkani-Hamed, Kane, Thaler, Wang, hep-ph/0512190

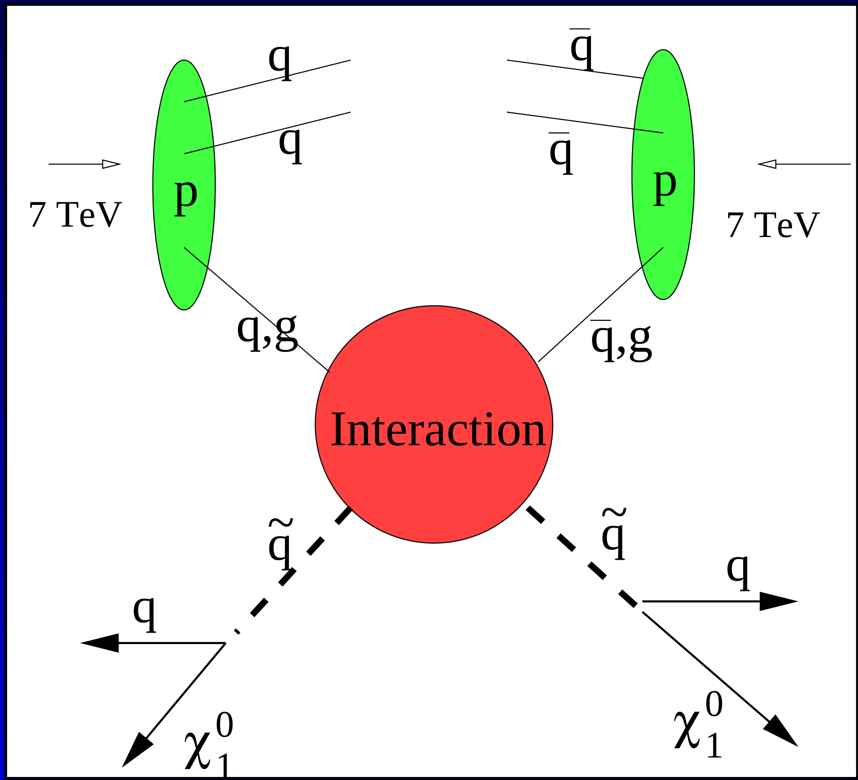
What the LHC can do

One can constrain some MSSM sparticle masses using *kinematic endpoints*. Since the mass spectrum depends on the SUSY breaking $\mathcal{L}_{soft}$, very difficult to constrain things in general. Each pattern of $\mathcal{L}_{soft}$ leads to very different decays of sparticles: many different possibilities. So: making the model constrained and doing a top-down fit is much easier.

Alternatively, one only considers a couple of sparticles (see later) and attempts to constrain these simple scenarios.

Collider SUSY Dark Matter Production

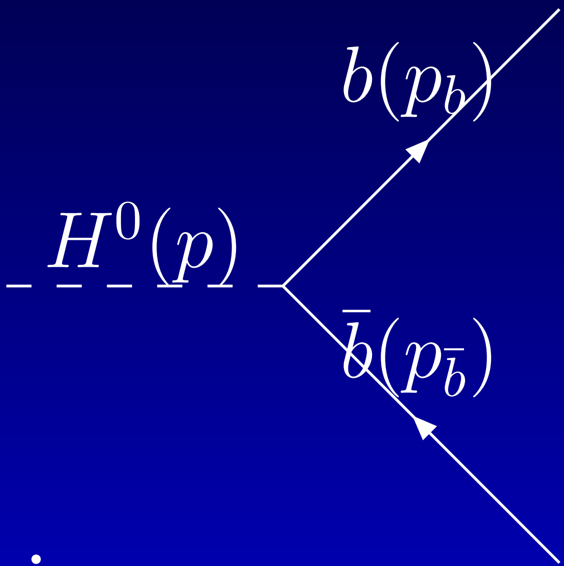
Strong sparticle production and decay to dark matter particles.



Any (light enough) dark matter candidate that couples to hadrons can be produced at the LHC

SUSY Kinematics: a Reminder

Take an *on-shell* particle decaying into 2 particles, eg $H^0 \to b\bar{b}$. We define the invariant mass of the $b\bar{b}$ pair such that:



$$p^\mu = (\sqrt{m_H^2 + p^2}, \underline{p}) = p_b^\mu + p_{\bar{b}}^\mu$$

$$\Rightarrow p^2 \equiv p^\mu p_\mu = m_H^2 = (p_b + p_{\bar{b}})^2$$

Is *invariant* in boosted frames

Question: What happens to invariant mass in SUSY cascade decays, where we miss the final particle?

Narrow Width Approximation

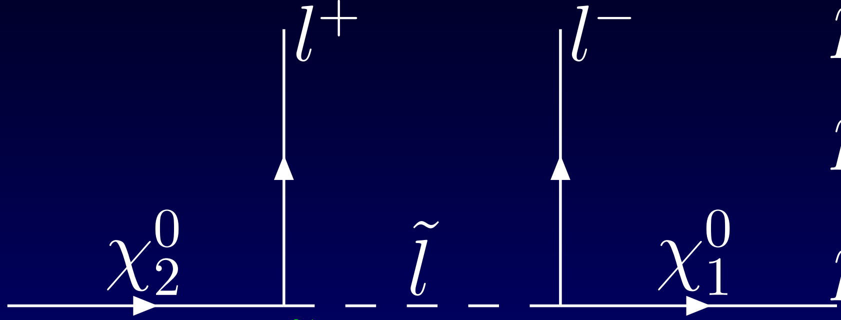
Take some scalar propagator mod-squared:

$$D(p^2) = \frac{1}{(p^2 - m^2)^2 + m^2\Gamma^2}.$$

$$\lim_{\Gamma/m\to 0} D(p^2) = \pi/(m\Gamma)\delta(p^2 - m^2).$$

Thus (as is often the case in the MSSM), for particles with narrow widths, we may approximate them assuming they have $p^2 = m^2$, ie they are *on-shell*. The next order in perturbation theory is $\mathcal{O}(m/\Gamma)$.

Cascade Decay



$$p^\mu_{\tilde{l}} = (m_{\tilde{l}}, \underline{0})$$
$$p^\mu_{l\pm} = (|\underline{p}_{l\pm}|, \underline{p}_{l\pm})$$
$$p^\mu_{\chi^0_{1,2}} = \left(\sqrt{{m_{\chi^0_{1,2}}}^2 + |\underline{p}_{\chi^0_{1,2}}|^2}, \underline{p}_{\chi^0_{1,2}}\right)$$

Work in $\tilde{l}$ rest frame.

The invariant mass of the l^+l^- pair is

$$m^2_{ll} = (p_{l^+} + p_{l^-})^\mu (p_{l^+} + p_{l^-})_\mu = p^2_{l^+} + p^2_{l^-} + 2p_{l^+} \cdot p_{l^-}$$
$$= 2|\underline{p}_{l^+}||\underline{p}_{l^-}|(1 - \cos\theta) \leq 4|\underline{p}_{l^+}||\underline{p}_{l^-}|.$$

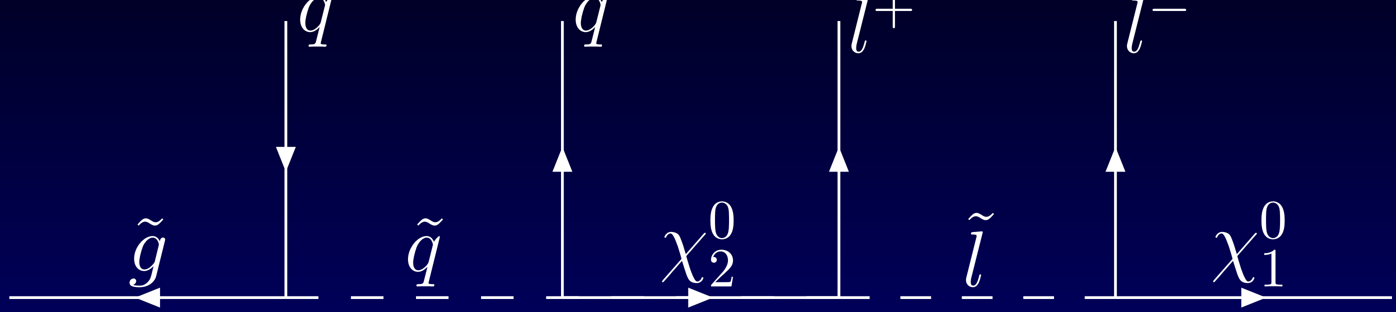
Momentum conservation:

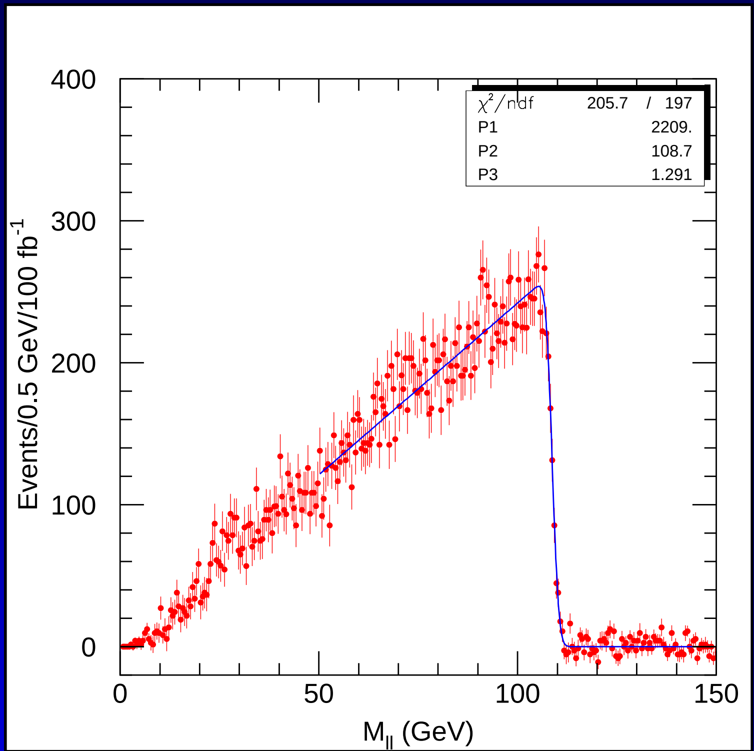
$$\Rightarrow \underline{p}_{\chi^0_2} + \underline{p}_{l^+} = \underline{0}, \qquad \underline{p}_{l^-} + \underline{p}_{\chi^0_1} = \underline{0}.$$

Energy conservation: $\sqrt{{m_{\chi^0_2}}^2 + |\underline{p}_{\chi^0_2}|^2} = m_{\tilde{l}} + |\underline{p}_{l^+}|$,

$$\Rightarrow |\underline{p}_{l^+}| = \frac{m^2_{\chi^0_2} - m^2_{\tilde{l}}}{2m_{\tilde{l}}}.$$ Similarly $$|\underline{p}_{l^-}| = \frac{m^2_{\tilde{l}} - m^2_{\chi^0_1}}{2m_{\tilde{l}}}.$$

LHC SUSY Measurements





$$m^2_{ll}(max) = \frac{(m^2_{\chi^0_2} - m^2_{\tilde{l}})(m^2_{\tilde{l}} - m^2_{\chi^0_1})}{m^2_{\tilde{l}}}$$

Q: Can we measure enough of these to pin SUSY[a] down?

[a] BCA, Lester, Parker, Webber, JHEP 0009 (2000) 004

Other Observables

Often more complicated, eg m_{llq} edge:

$$\max\left[\frac{(m^2_{\tilde q}-m^2_{\chi^0_2})(m^2_{\chi^0_2}-m^2_{\chi^0_1})}{m^2_{\chi^0_2}}, \frac{(m^2_{\tilde q}-m^2_{\tilde l})(m^2_{\tilde l}-m^2_{\chi^0_1})}{m^2_{\tilde l}}, \right.$$

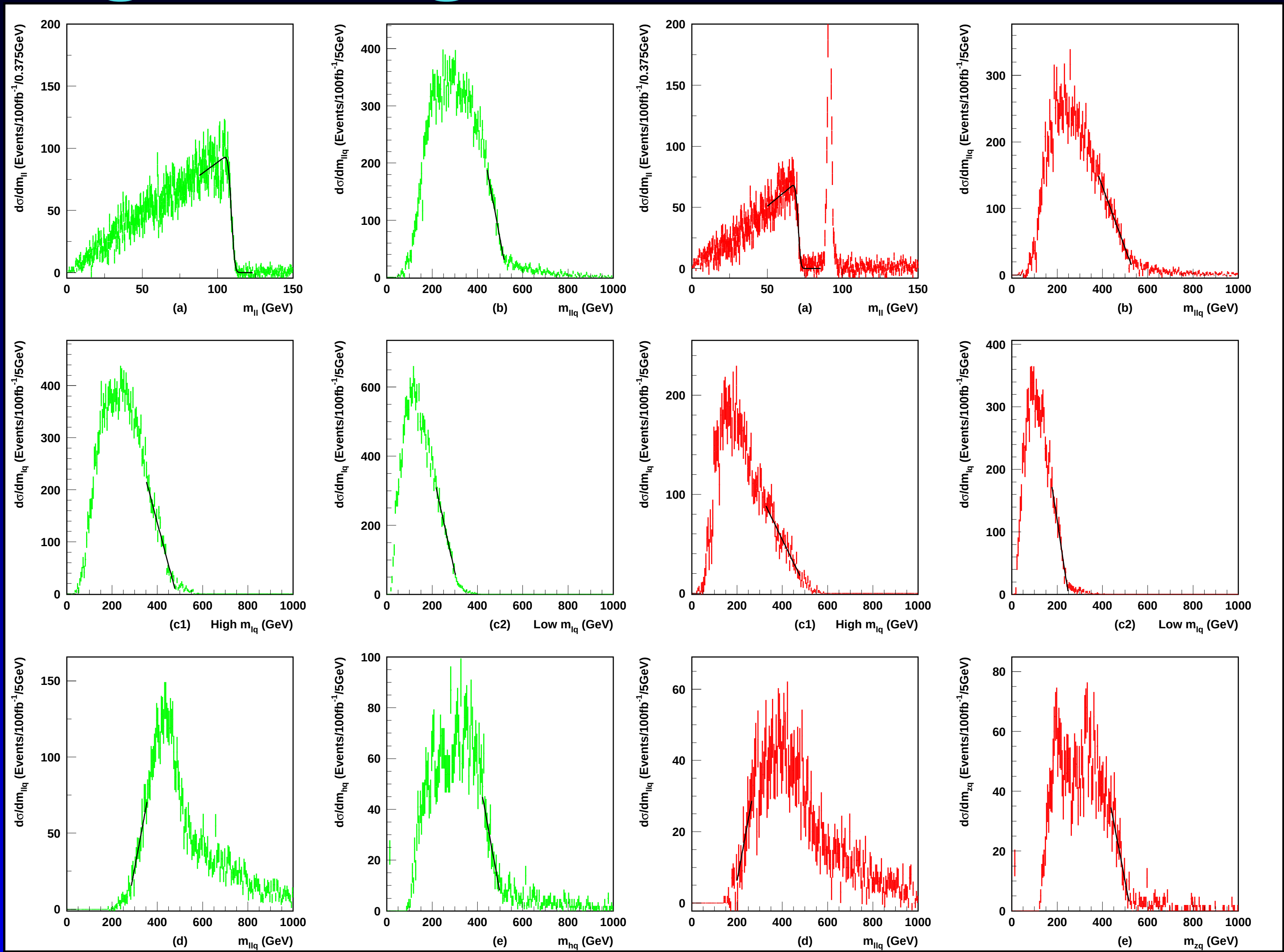
$$\left. \frac{(m_{\tilde q}m_{\tilde l}-m_{\chi^0_2}m_{\chi^0_1})(m^2_{\chi^0_2}-m^2_{\tilde l})}{m_{\chi^0_2}m_{\tilde l}}\right]$$

Also m_{lq}^{high}, m_{lq}^{low}, llq *threshold* [a], $M^2_{T_2}(m) =$

$$\min_{\not p_1+\not p_2=\not p_T}\left[\max\left\{m^2_T(p_T^{l_1},\not p_1,m), m^2_T(p_T^{l_2},\not p_2,m)\right\}\right],$$

$\max[M_{T_2}(m_{\chi^0_1})] = m_{\tilde l}]$ for dislepton production.

Edge Fitting at S5 and O1



Edge Positions

Do a fit: all scalars considered degenerate in mSUGRA at M_{GUT}, whereas for O1, squarks are massless there.

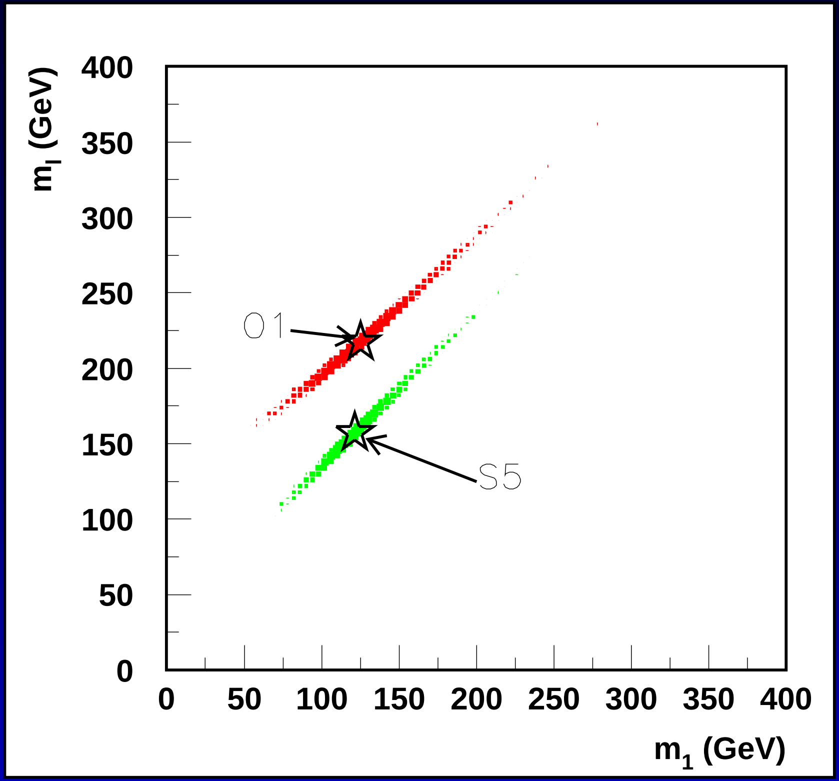
endpoint/GeV	S5 fit	O1 fit
m_{ll}	109.10±0.13	70.47±0.15
m_{llq} edge	532.1±3.2	544.1±4.0
lq high	483.5±1.8	515.8±7.0
lq low	321.5±2.3	249.8±1.5
llq thresh	266.0±6.4	182.2±13.5

Best case lepton mass measurements can be as accurate as 1 per mille, but jets are a few percent[a]

[a] See Barr, Lester, `arXiv:1004.2732` for a review of other mass measurement techniques

Edge to Mass Measurements

	width S5	width O1
χ_1^0	17	22
$\tilde{l}_R$	17	20
χ_2^0	17	20
$\tilde{q}$	22	20

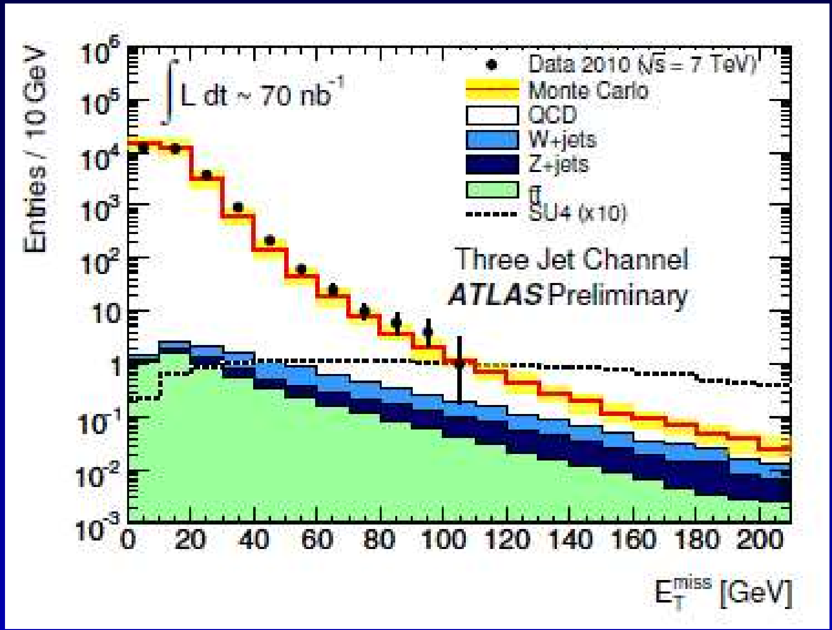


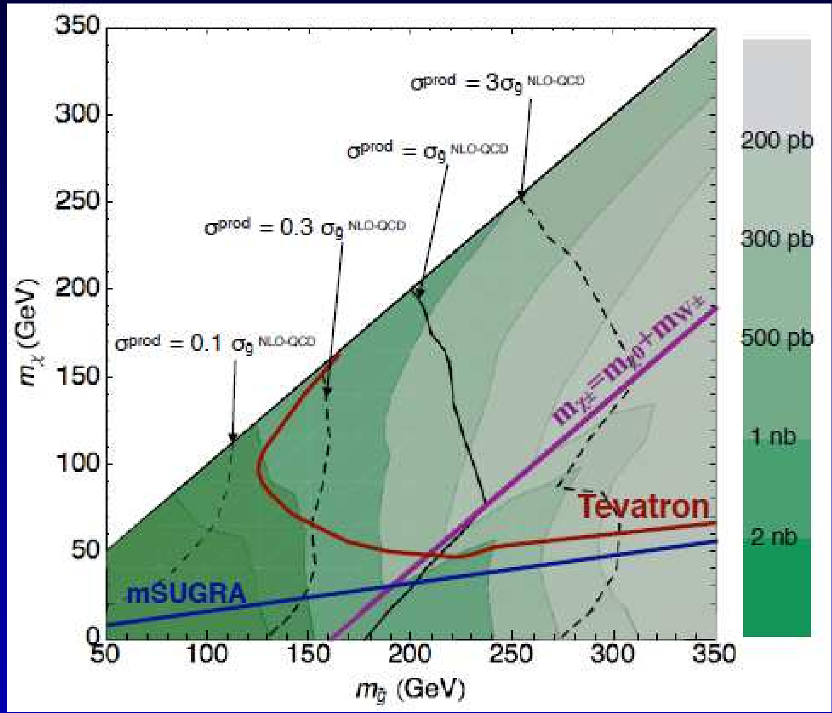
Mass differences well constrained, but overall mass scale not so well constrained by LHC[a]

[a] BCA, Lester, Parker, Webber, `hep-ph/0007009`

Simple Study

Can bound[a] $pp \to \tilde{g}\tilde{g}$, with $\tilde{g} \to 2j\not{E}_T$ from[b]:





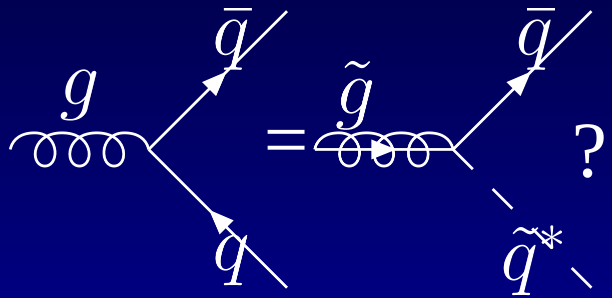
Very simple situation: depends only on $m_{\tilde{g}}$, $m_{\chi_1^0}$ and possibly $m_{\tilde{q}}$ through production matrix elements.

[a] Alves, Izaguirre, Wacker, `arXiv:1008.0407`

[b] ATLAS, ATLAS-CONF-2010-065

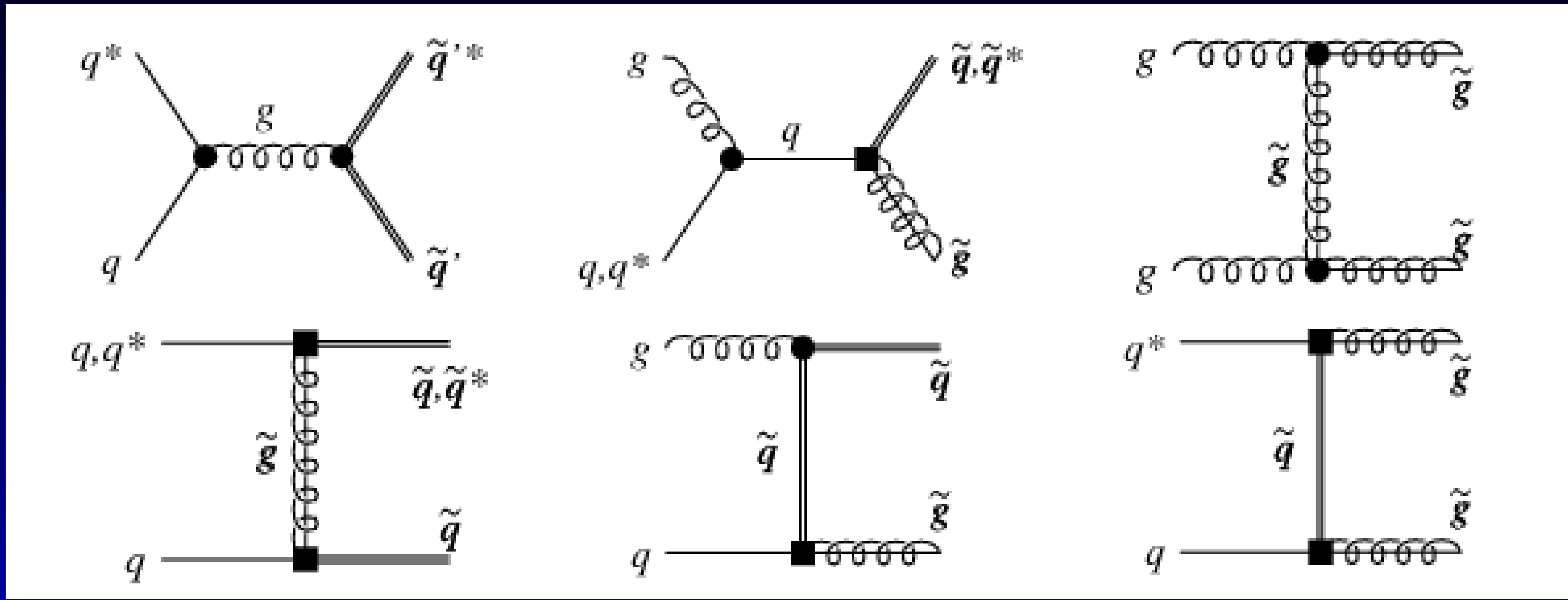
Other Things We Want to Check

- Do the spins correspond to SUSY?
- Do the couplings correspond to SUSY? Eg



All of these detailed checks are very difficult to do at the LHC. Really, one needs a future linear collider to do these things: with enough energy to produce the relevant sparticles.

Coupling Measurement

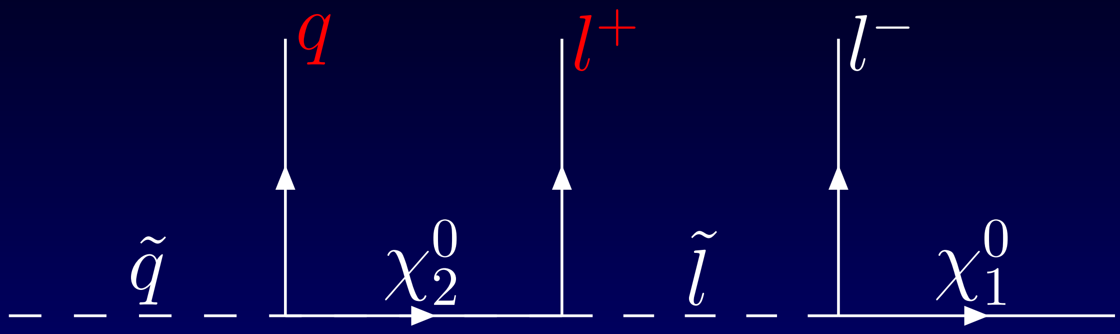


$$\tilde{u}_L \to d\chi_1^+ \to d l^+ \nu_l \chi_1^0,\ \tilde{u}_L^* \to \bar{d}\chi_1^- \to d l^- \bar{\nu}_l \chi_1^0$$

The idea is to use the lepton charge to tag the charge of the initial quark and look for $\tilde{q}_L \tilde{q}_L$ production. Assuming ILC data on BRs, can get $\sim 4\%$ accuracy for 100 fb^{-1}[a] at an easy point.

[a] Skands, Freitas, hep-ph/0606121. Zerwas *al et* arXiv:0806.3875

Spins at LHC



$$(m_{lq}^{\text{near}})^2 = 2|p_l||p_q|(1-\cos\theta_{lq}) = (m_{lq}^{\text{near}})^2_{\max} \sin^2(\theta_{lq}/2).$$

$$(m_{lq}^{\text{near}})^2_{\max} = \frac{(m_{\tilde{q}}^2 - m_{\chi_2^0}^2)(m_{\chi_2^0}^2 - m_{\tilde{l}}^2)}{m_{\chi_2^0}^2}$$

Consider $m \equiv m_{ql}/m_{ql}(max) = \sin\theta_{lq}/2$, for PS

$$\frac{dP_{PS}}{dm} = 2m.$$

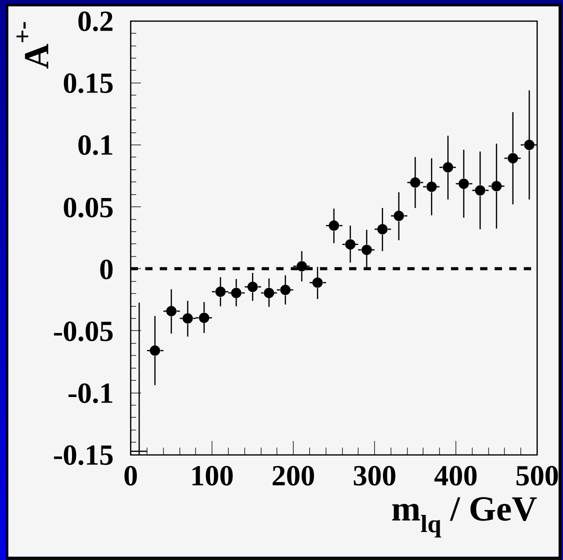
Spin correlations give different distributions!

Spins II

$$\frac{dP(l^+q/l^-\bar{q})}{dm} = 4m^3, \qquad \frac{dP(l^-q/l^+\bar{q})}{dm} = 4m(1-m^2),$$

Seems hopeless, since we cannot tag quarks vs anti-quarks (average is PS). But pp gives more $\tilde{q}$ than $\tilde{q}^*$! which leads to spin-generated lepton charge asymmetry

Barr, hep-ph/0405052



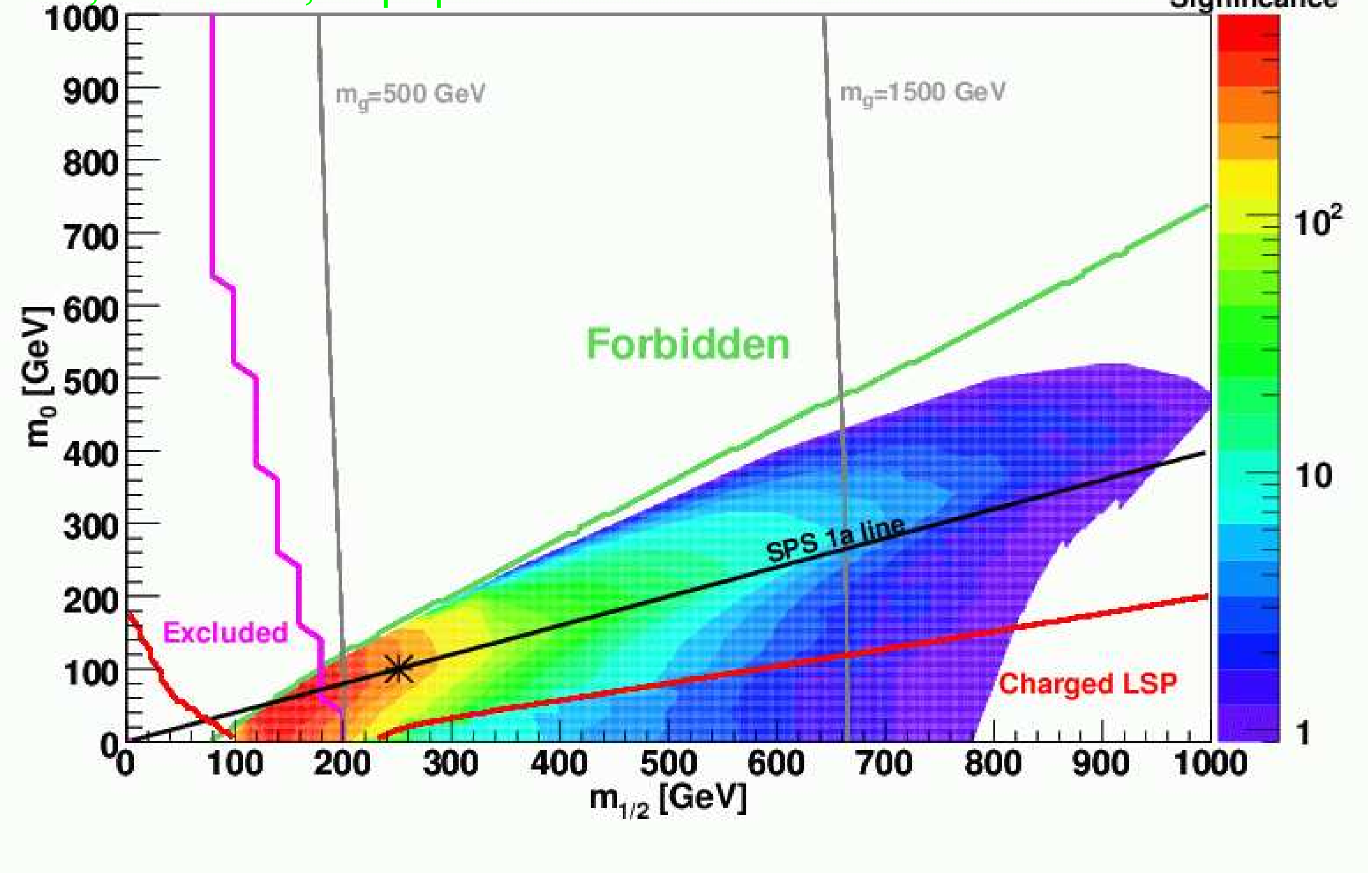
$$A^{+-} = \frac{s^+ - s^-}{s^+ + s^-}$$

$$s^{\pm} = \frac{d\sigma}{d(m_{l^{\pm}q})}$$

$$\mathcal{L} = 150 pb^{-1}$$

Region of Validity of Barr Method

BCA, Mahmoudi, hep-ph/0602198



For $\mathcal{L} = 150$ fb^{-1}, can discriminate against phase space in the orange and red regions *only*.

Universality

Reduces number of SUSY breaking parameters from 100 to 3:

- $\tan\beta \equiv v_2/v_1$
- m_0, the common scalar mass (flavour).
- $M_{1/2}$, the common gaugino mass (GUT/string).
- A_0, the common trilinear coupling (flavour).

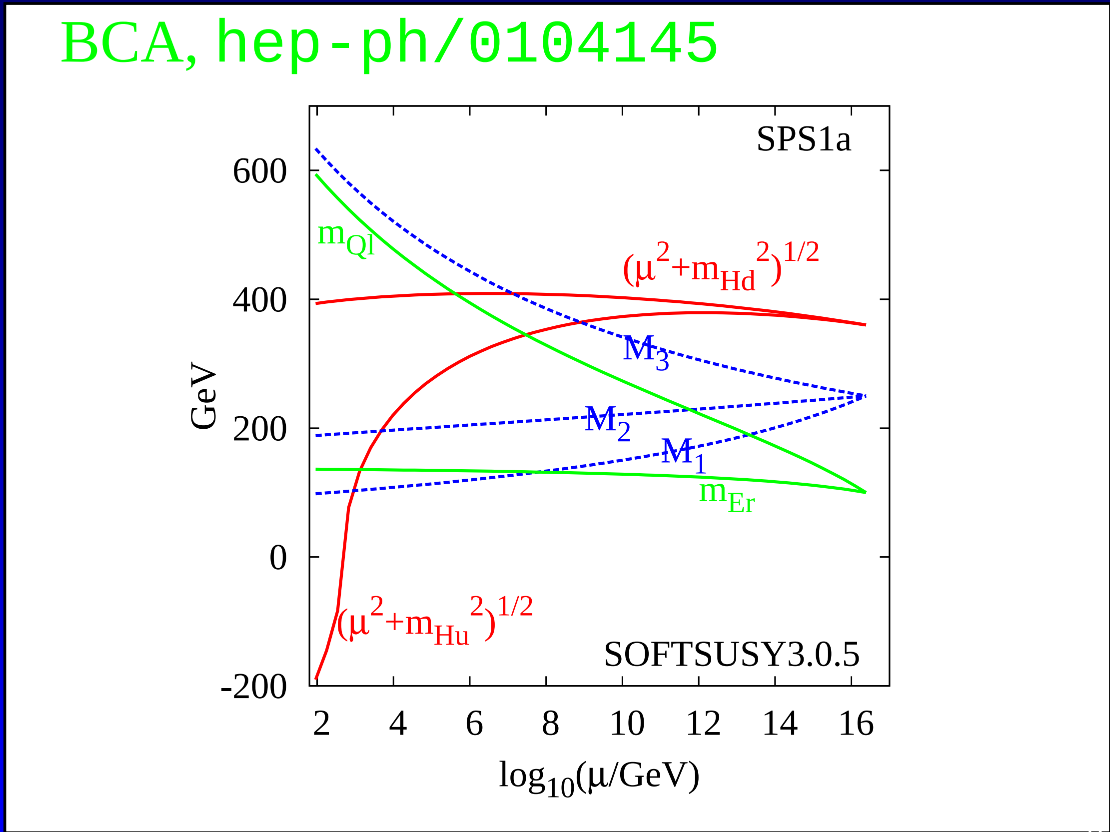
These conditions should be imposed at $M_X \sim O(10^{16-18})$ GeV and receive radiative corrections

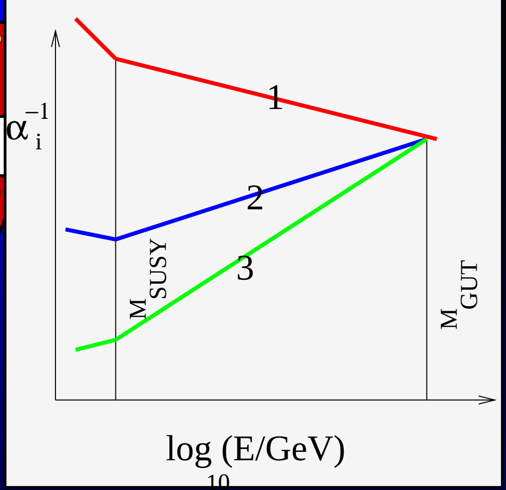
$$\propto \; 1/(16\pi^2)\ln(M_X/M_Z).$$

Also, Higgs potential parameter sgn(μ)=± 1.

SOFTSUSY

SOFTSUSY is an MSSM spectrum generator. Like 3 other public spectrum generators, it predicts MSSM masses and couplings consistent with weak-scale data and an assumed high-scale boundary condition on SUSY breaking.





SOFTSUSY

Get $g_i(M_Z)$, $h_{t,b,\tau}(M_Z)$.

↓

Run to M_S.

↓

REWSB, iterative solution of μ

↓

M_X. Soft SUSY breaking BC.

↓

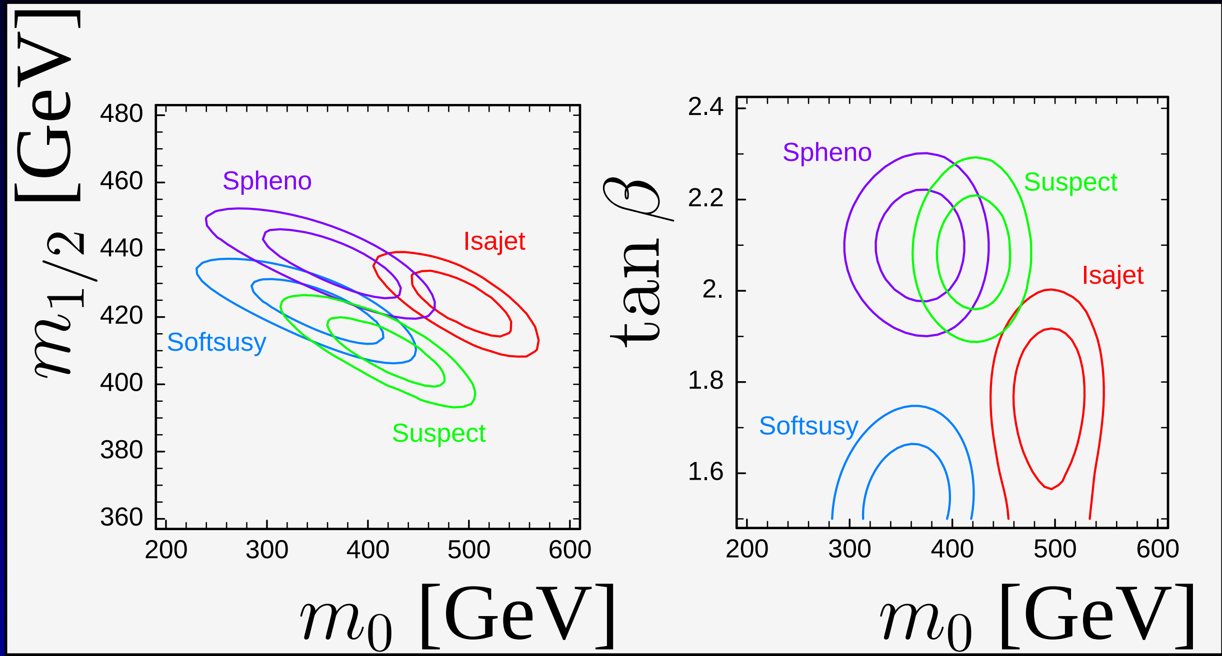
Run to M_S. Calculate[a] sparticle pole masses.

↓

Run to M_Z (→ back to "Get $g_i(M_Z)$, $h_{t,b,\tau}(M_Z)$.")

[a] BCA, Comp. Phys. Comm. 143 (2002) 305.

Fitting to SUSY Breaking Model

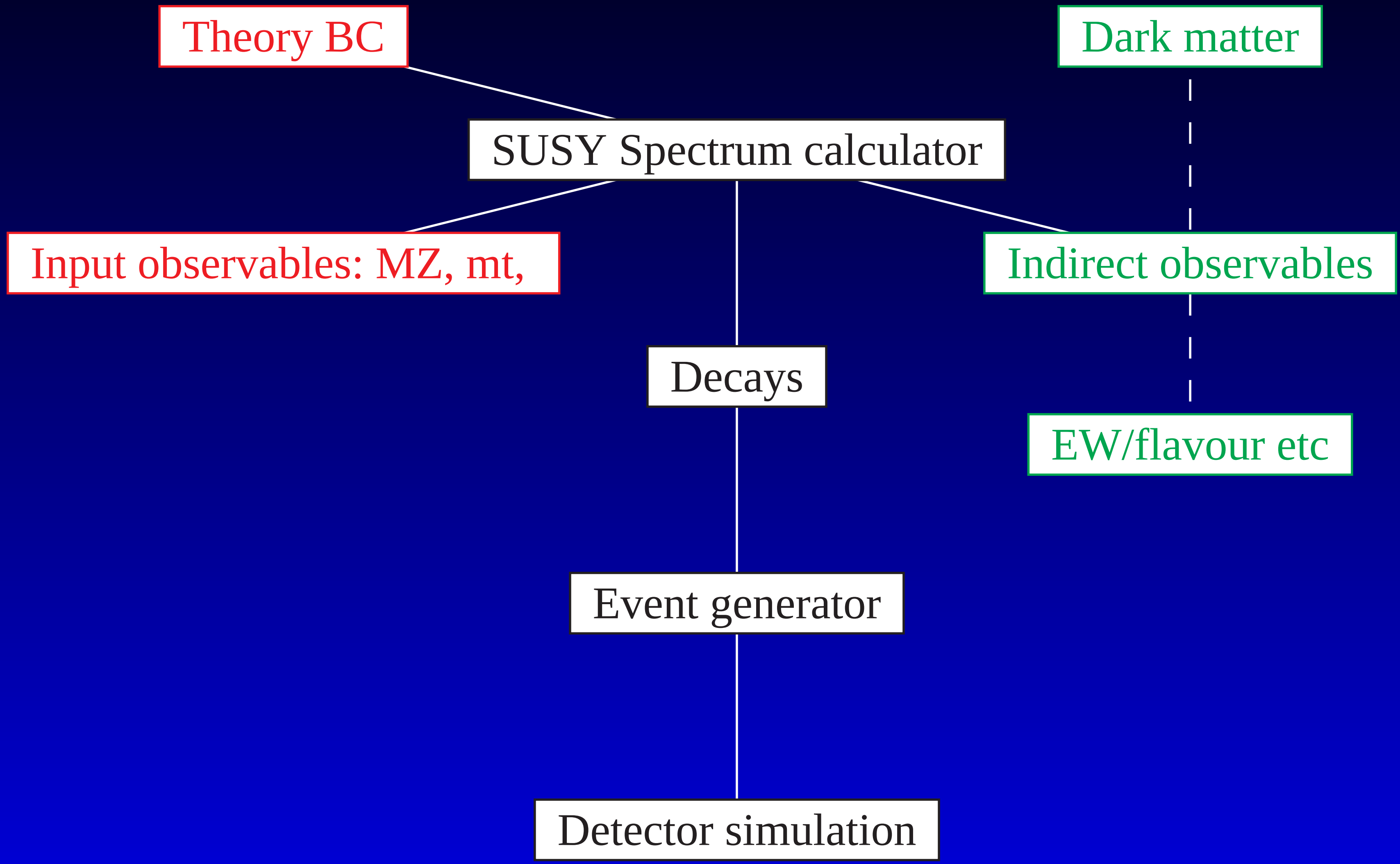


- Experimenters pick a SUSY breaking point
- They derive observables and errors after detector simulation
- We fit[a] this "data" with our codes

[a] BCA, S Kraml, W Porod, JHEP 0303 (2003) 016

See a review: BCA, arXiv:0805.2088

MSSM Tools



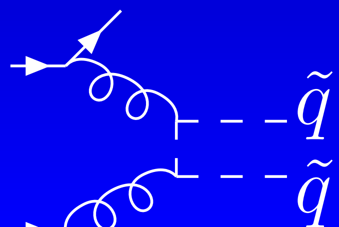
SLHA: Skands *et al*, `hep-ph/0311123`, SLHA2: BCA *et al*, `arXiv:0801.0045` (NMSSM, RPV, FV, CPV)

Spectrum and decays

- ISASUSY decouples particles at the mass thresholds but misses some finite terms in the matching: re-sums log splittings.
- SOFTSUSY, sPHENO, SUSPECT all catch the finite terms but do the splittings to leading log in RPC-MSSM.
- CPsuperH, FeynHiggs do Higgs mass spectrum and decays of CP violating MSSM
- NMSPEC does the CNMSSM spectrum, NMHDECAY gives the decays widths etc
- PYTHIA, HERWIG++, ISASUSY, sPHENO and SusyHIT do decays of Higgs and SUSY particles in MSSM.

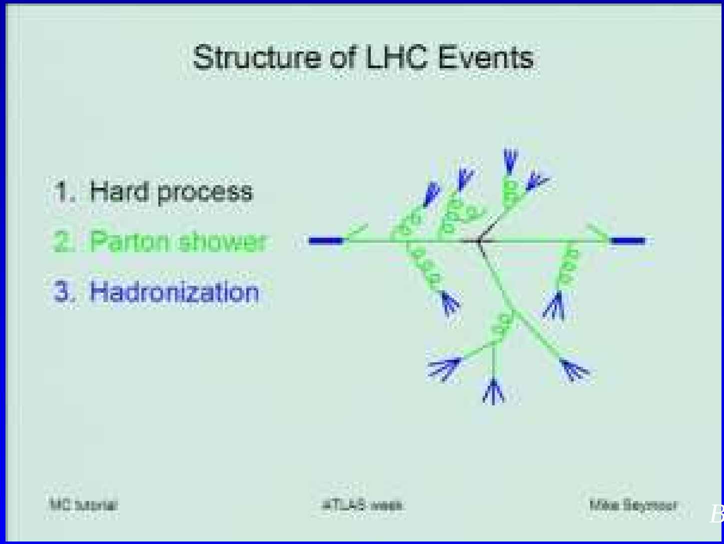
Matrix Element Generators

- Feyn Arts/Feyn Calc
- Additional hard jets *cannot* be modelled reliably using the parton shower - you need to simulate the matrix element.
- SMADGRAPH, compHEP, calcHEP, GRACE do SUSY and more general models at tree level. 2 to 4 possible. BRIDGE can be used to remember spin information in the decays.
- WHIZARD, SUSYGEN - polarisation included for e^+e^-
- PROSPINO does NLO-QCD sparticle production



Event Generation

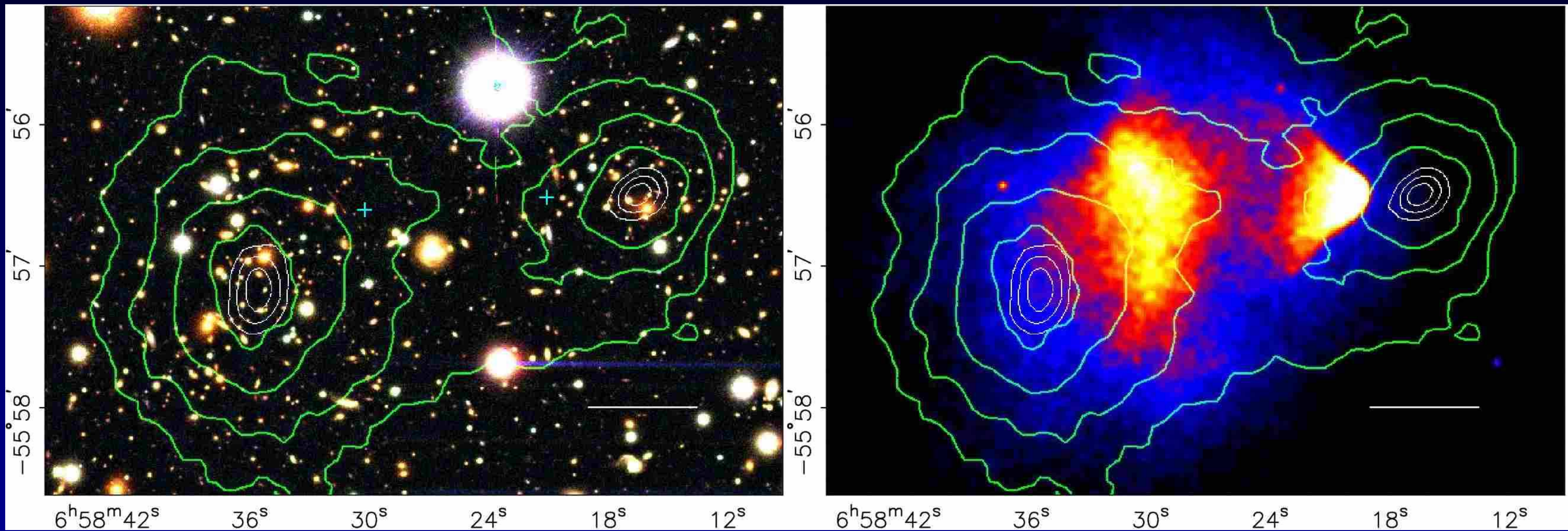
- Can pass matrix-element generated events to event generators with the (original) *Les Houches Accord*
- `PYTHIA` used extensively. Includes RPV. phase-space decays. `ISAJET` too.
- `HERWIG` maintains spin info down cascade decays. RPV too.
- `SHERPA` matches up ME with more standard event generation.
- Shift toward **C++**



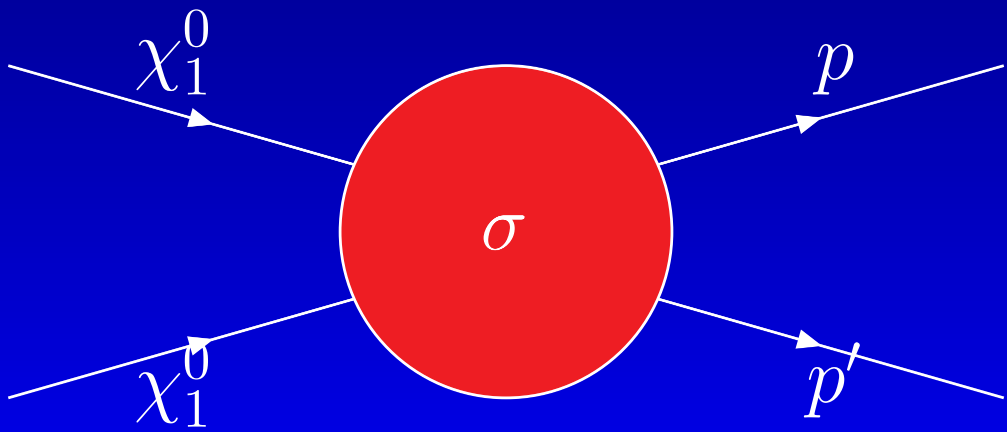
SUSY Prediction of Ωh^2

- Assume relic in thermal equilibrium with $n_{eq} \propto (MT)^{3/2} exp(-M/T)$.
- Freeze-out with $T_f \sim M_f/25$ once interaction rate $<$ expansion rate (t_{eq} critical)
- `microMEGAs` uses `calcHEP` to automatically calculate relevant Feynman diagrams for some given model Lagrangian: *flexible*.
- `darkSUSY`, `IsaRED` has MSSM annihilation channels hard-coded.
- Both `darkSUSY` and `micrOMEGAs` calculate (in-)direct predictions.

SUSY Dark Matter



astro-ph/0608407



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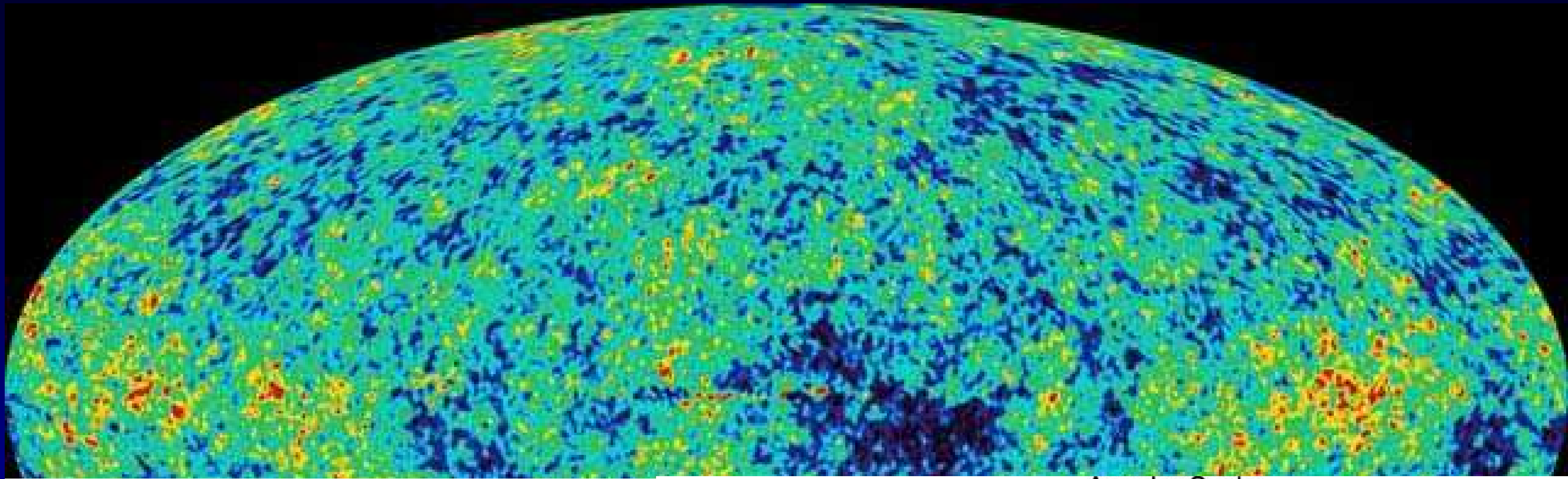
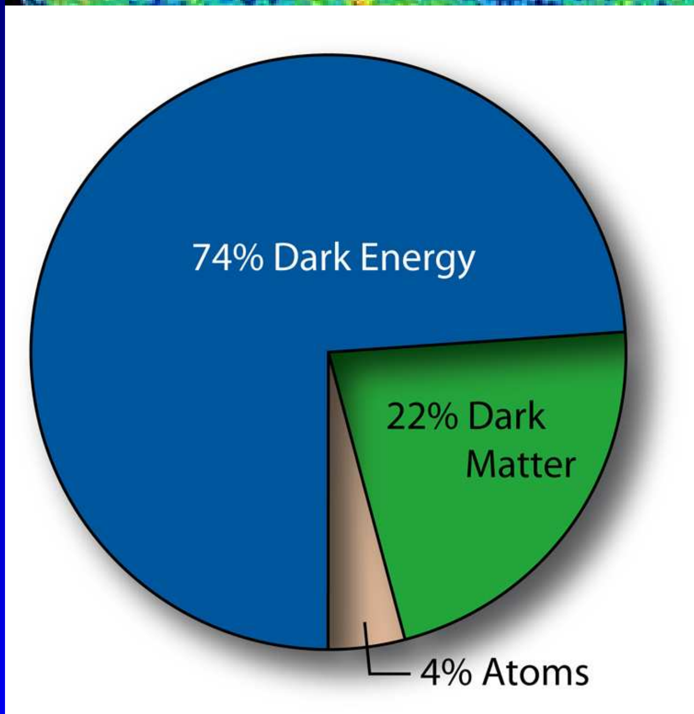
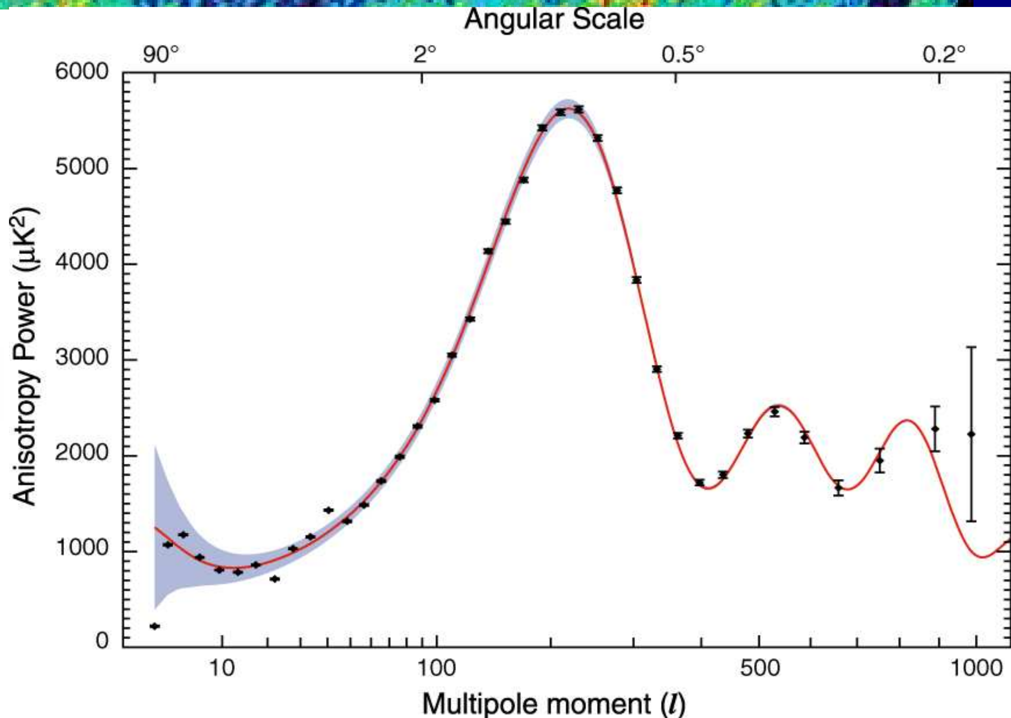
Caveats

- Implicitly assumed that LSP constitutes *all* of dark matter
- Assumed radiation domination in post-inflation era. No clear evidence between freeze-out+BBN that this is the case (t_{eq} changes).
- Examples of non-standard cosmology that would change the prediction:
  - Extra degrees of freedom
  - Low reheating temperature
  - Extra dimensional models
  - Anisotropic cosmologies
  - Non-thermal production of neutralinos (late decays?)

WMAP+BAO+Ia Fits







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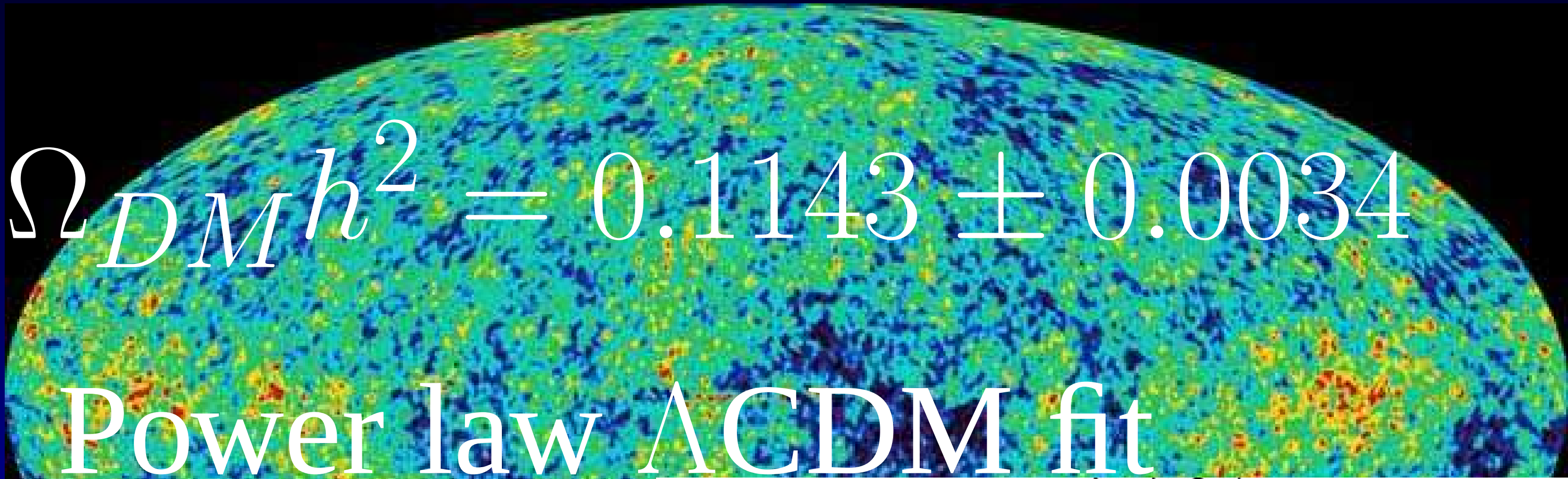
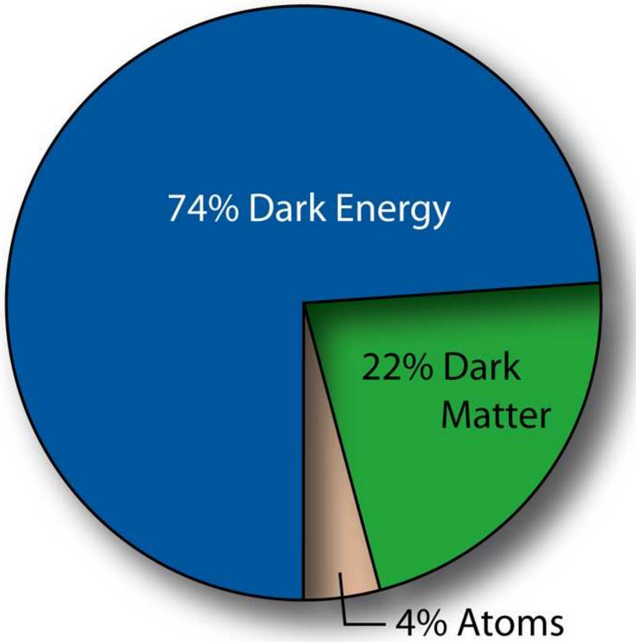
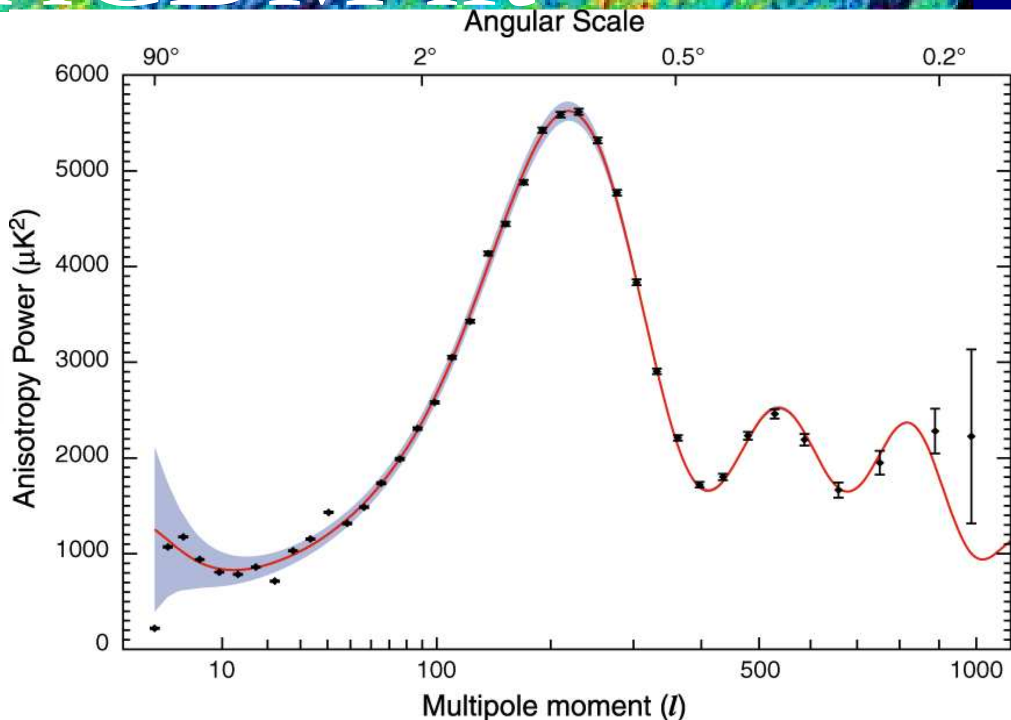


$$\Omega_{DM}h^2 = 0.1143 \pm 0.0034$$

Power law ΛCDM fit





Implementation

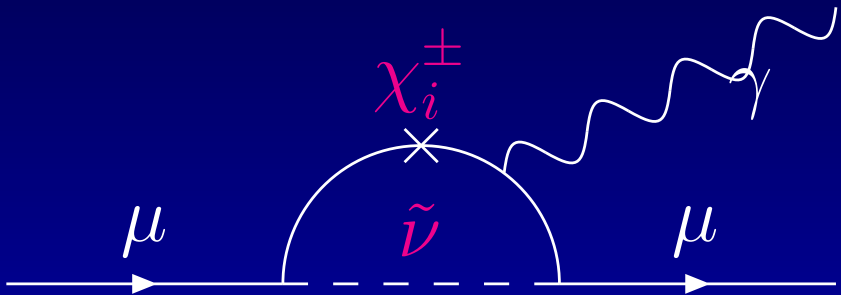
We use

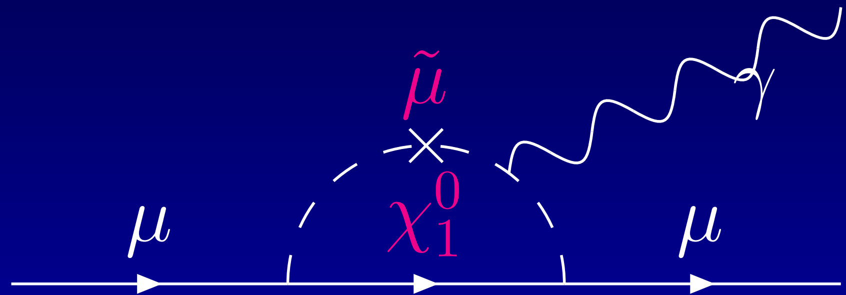
- *95% C.L. direct search constraints*
- $\Omega_{DM} h^2 = 0.1143 \pm 0.02$ Boudjema *et al*
- $\delta(g-2)_\mu/2 = (29.5 \pm 8.8) \times 10^{-10}$ Stöckinger *et al*
- $B-$physics observables including $BR[b \to s\gamma]_{E_\gamma > 1.6 \text{ GeV}} = (3.52 \pm 0.38) \times 10^{-4}$
- Electroweak data W Hollik, A Weber *et al*

$$2 \ln \mathcal{L} = -\sum_i \chi_i^2 + c = \sum_i \frac{(p_i - e_i)^2}{\sigma_i^2} + c$$

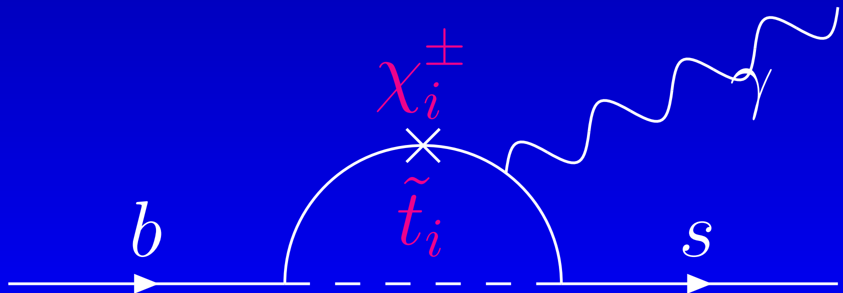
Additional observables

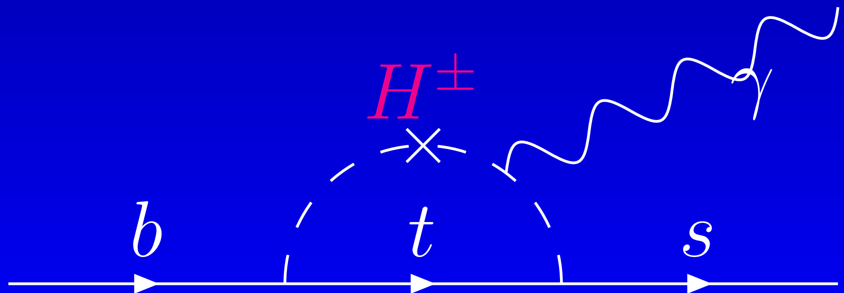
$$\delta \frac{(g-2)_\mu}{2} \sim 13 \times 10^{-10} \left(\frac{100 \text{ GeV}}{M_{SUSY}} \right)^2 \tan\beta$$





$$BR[b \to s\gamma] \propto \tan\beta (M_W/M_{SUSY})^2$$





Application of Bayes'

$\mathcal{L} \equiv p(\underline{d}|\underline{m}, H)$ is pdf of reproducing data $\underline{d}$ assuming pMSSM hypothesis H and model parameters $\underline{m}$

$$p(\underline{m}|\underline{d}, H) = p(\underline{d}|\underline{m}, H)\frac{p(\underline{m}, H)}{p(\underline{d}, H)}$$

$p(\underline{m}|\underline{d}, H)$ is called the posterior pdf. We will compare $p(\underline{m}, H) = c$ with a *different* prior.

$$p(m_0, M_{1/2}|\underline{d}, H) = \int d\underline{o}\, p(m_0, M_{1/2}, \underline{o}|\underline{d}, H)$$

Called *marginalisation*.

Likelihood and Posterior

$\mathcal{Q}$: What's the chance of observing someone to be pregnant, given that they are female?

d=pregnant, m=female

Likelihood

p(pregnant | female, human) = 0.01

Posterior

p(female | pregnant, human) = 1.00

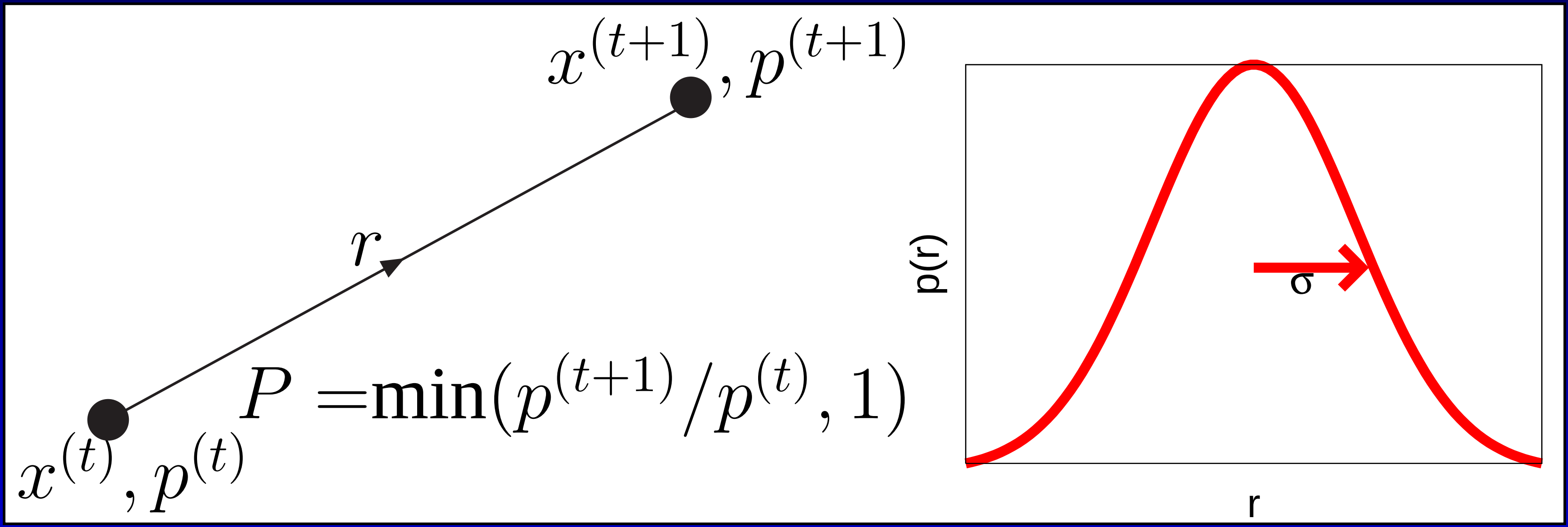
More obvious what to do in discrete cases like this one

Volume Effects

Can't rely on a good χ^2 in non-Gaussian situation

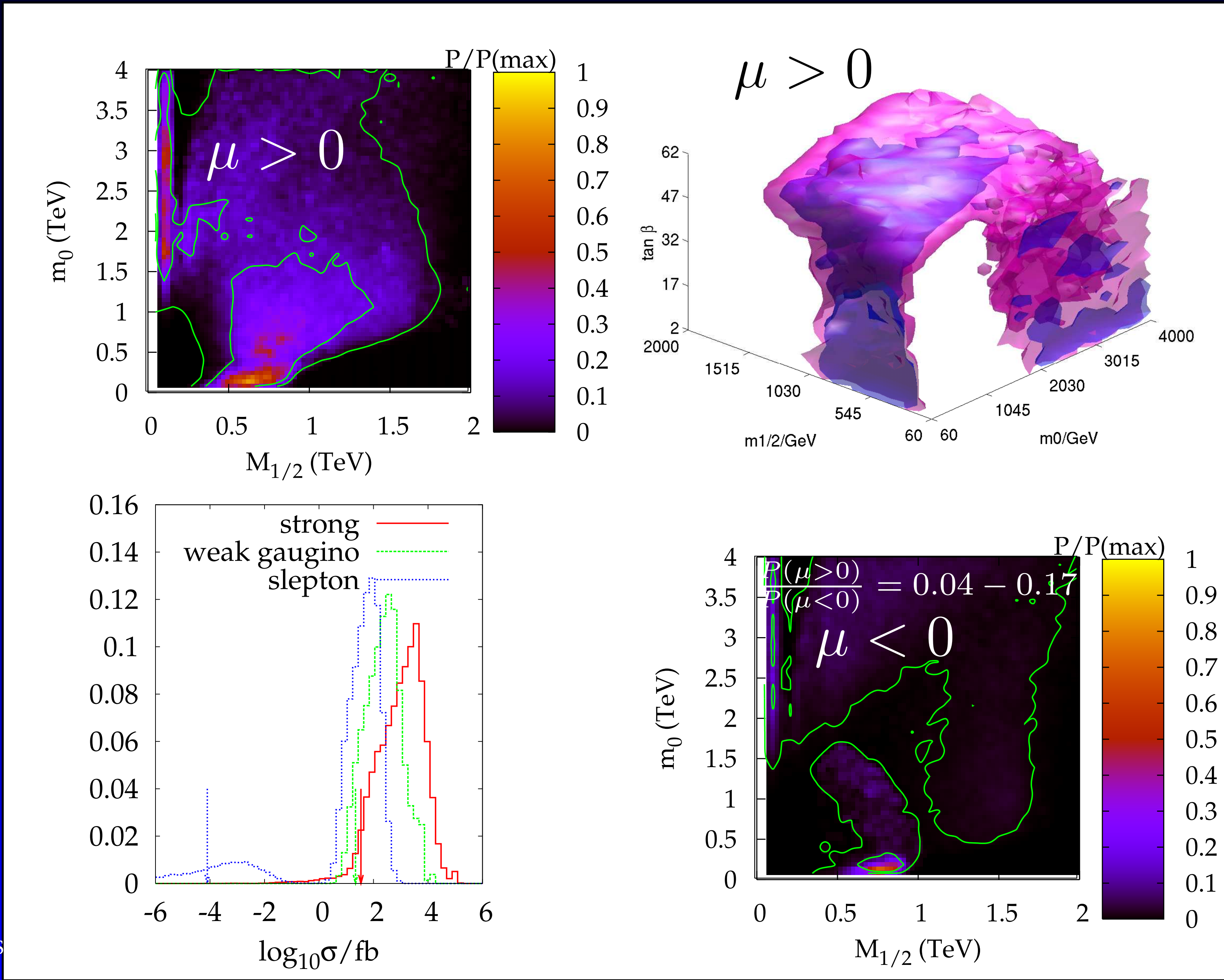
Markov-Chain Monte Carlo

Metropolis-Hastings Markov chain sampling consists of list of parameter points $x^{(t)}$ and associated posterior probabilities $p^{(t)}$.



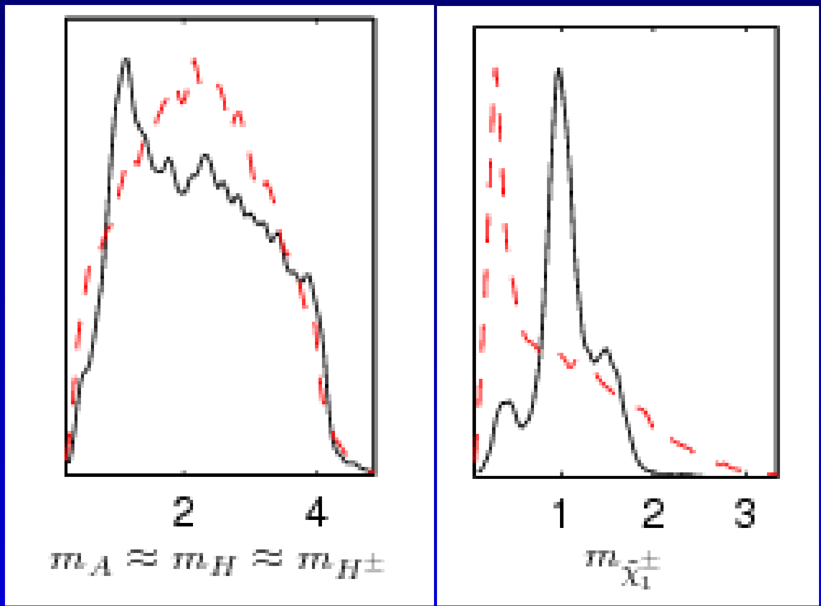
Final density of x points $\propto p$. Required number of points goes *linearly* with number of dimensions.

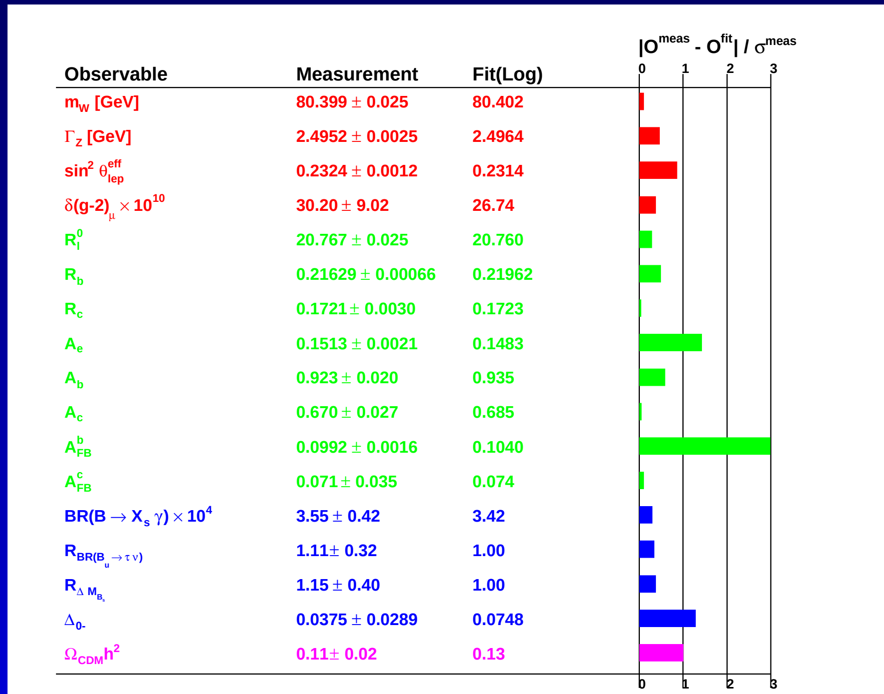
Global Fits II



pMSSM Fits

25 pMSSM input parameters are: $M_{1,2,3}$, $A_{t,b,\tau,\mu}$, $m_{H_{1,2}}$, $\tan\beta$, $m_{\tilde{d}_{R,L}} = m_{\tilde{s}_{R,L}}$, $m_{\tilde{u}_{R,L}} = m_{\tilde{c}_{R,L}}$, $m_{\tilde{e}_{R,L}} = m_{\tilde{\mu}_{R,L}}$, $m_{\tilde{t},\tilde{b},\tilde{\tau}_{R,L}}$ m_t, $m_b(m_b)$ $\alpha_s(M_Z)^{\overline{MS}}$, $\alpha^{-1}(M_Z)^{\overline{MS}}$, M_Z. Combined Bayesian fit[a]:



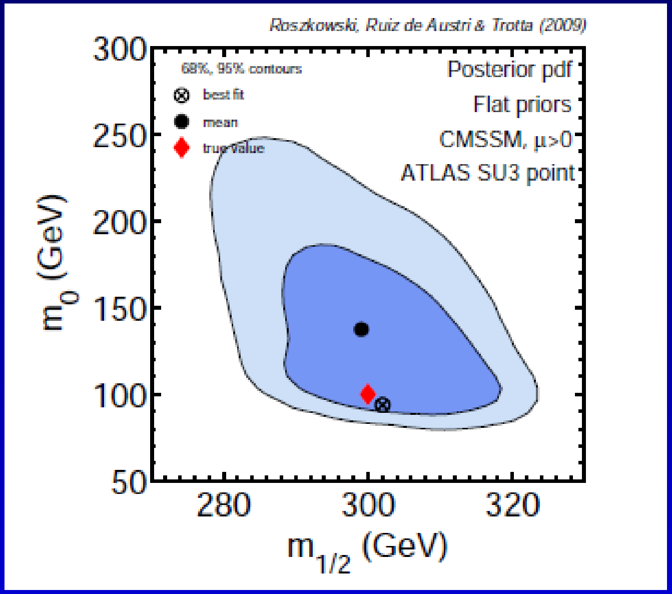


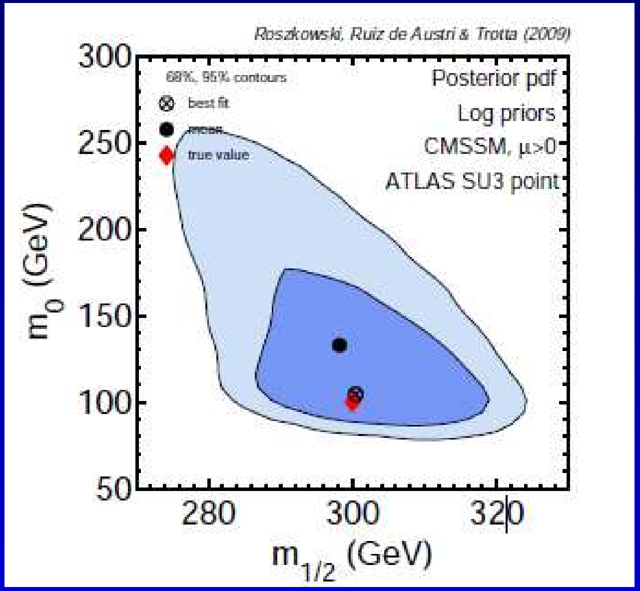
Observable	Measurement	Fit(Log)
m_W [GeV]	80.399 ± 0.025	80.402
Γ_Z [GeV]	2.4952 ± 0.0025	2.4964
$\sin^2\theta^{eff}_{lep}$	0.2324 ± 0.0012	0.2314
$\delta(g-2)_\mu \times 10^{10}$	30.20 ± 9.02	26.74
R_l^0	20.767 ± 0.025	20.760
R_b	0.21629 ± 0.00066	0.21962
R_c	0.1721 ± 0.0030	0.1723
A_e	0.1513 ± 0.0021	0.1483
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.685
A_{FB}^b	0.0992 ± 0.0016	0.1040
A_{FB}^c	0.071 ± 0.035	0.074
$BR(B \to X_s\gamma) \times 10^4$	3.55 ± 0.42	3.42
$R_{BR(B_u \to \tau\nu)}$	1.11 ± 0.32	1.00
$R_{\Delta M_{B_s}}$	1.15 ± 0.40	1.00
Δ_{0-}	0.0375 ± 0.0289	0.0748
$\Omega_{CDM}h^2$	0.11 ± 0.02	0.13

[a] S.S. AbdusSalam, BCA, F. Quevedo, F. Feroz, M. Hobson, arXiv:0904.2548

Prior Independence

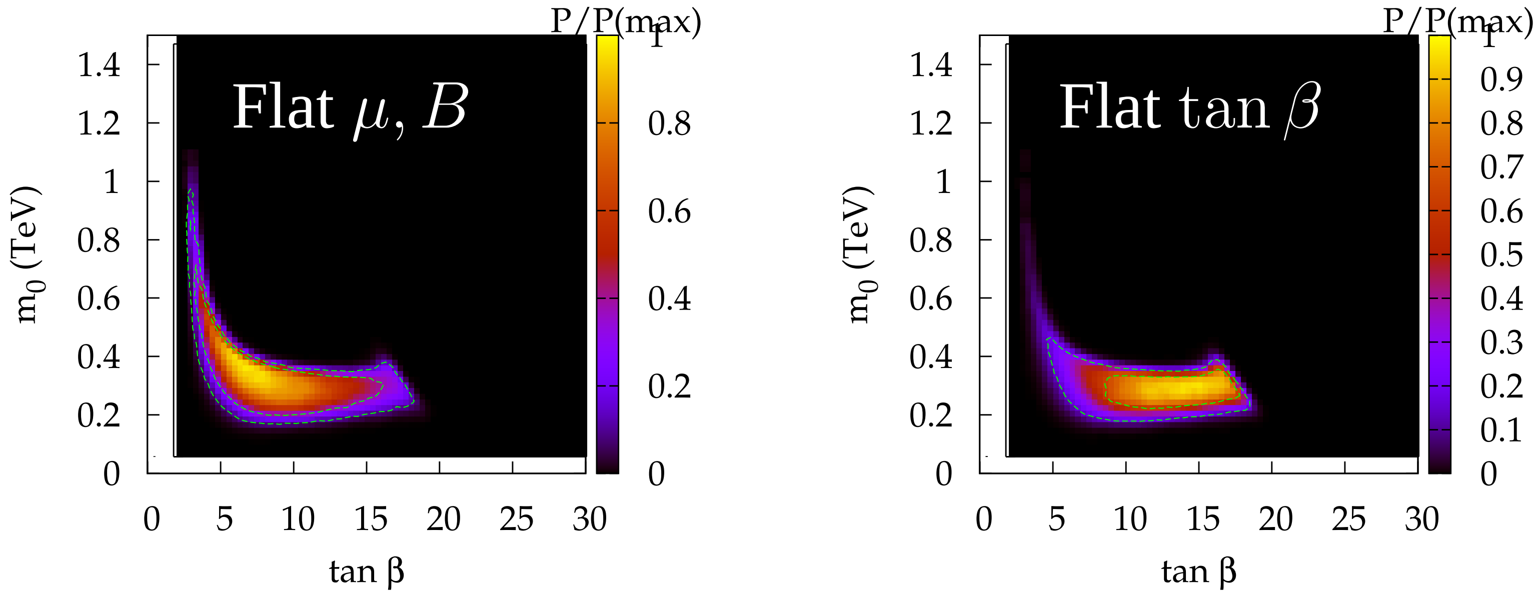
Once LHC data on sparticle production is included, prior dependence in mSUGRA decreases:
Roszkowski, Ruiz de Austri, Trotta, `arXiv:0907.0594`





Large Volume String Models

BCA, Dolan, `arXiv:0806.1184`



$$M_{1/2} = -A_0 = m_0/\sqrt{3}$$

$$M_X = 10^{11}\ \mathrm{GeV}$$

Two constraints enough!

Model Comparison

Calculate the *Bayesian evidence* of each model

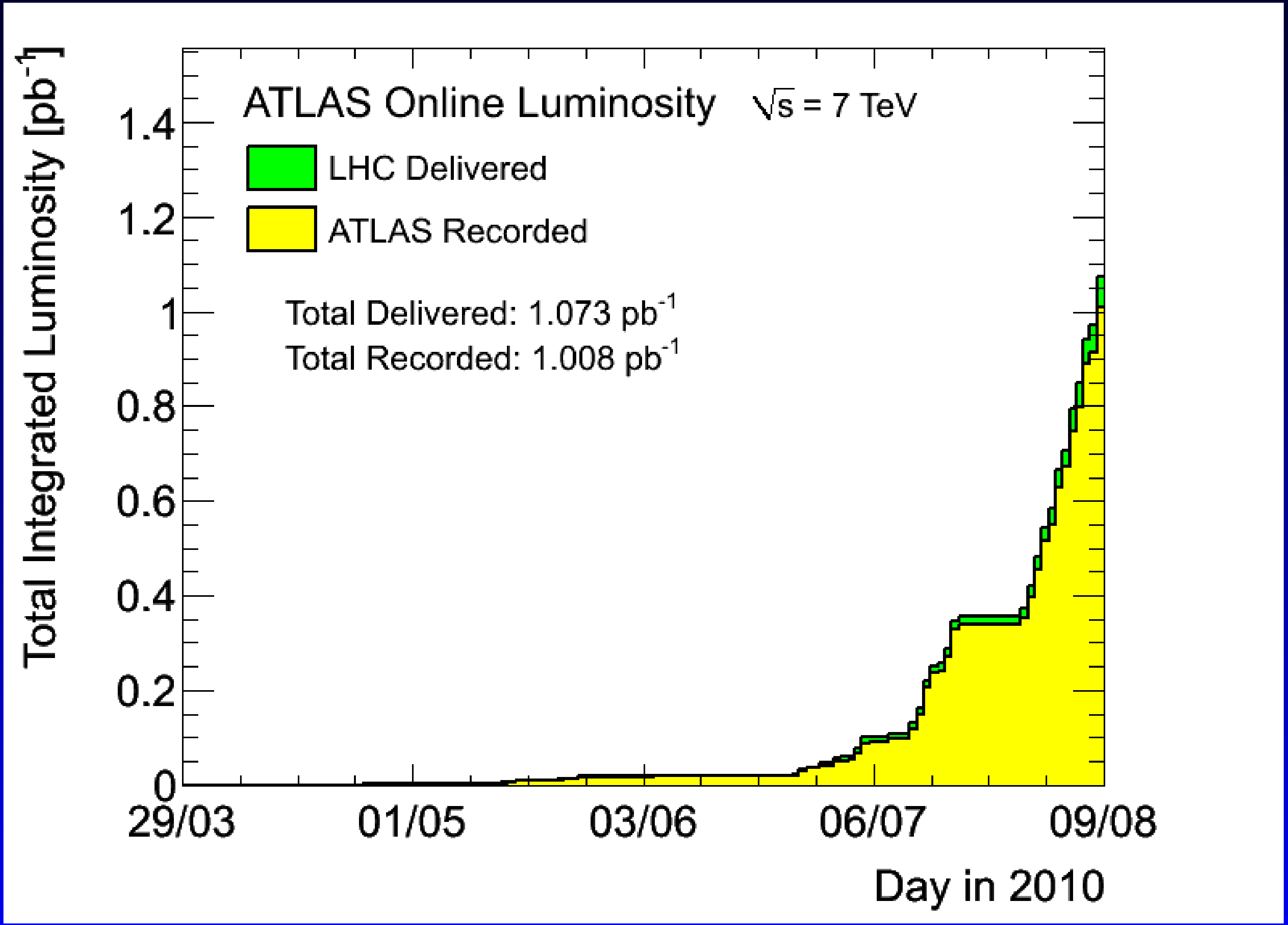
$$\mathcal{Z}_i = \int p(\underline{d}|\underline{m}, H_i)\, p(\underline{m}|H_i)\, d\underline{m}$$

$$\frac{p(H_1|\underline{d})}{p(H_0|\underline{d})} = \frac{p(\underline{d}|H_1)p(H_1)}{p(\underline{d}|H_0)p(H_0)} = \frac{\mathcal{Z}_1}{\mathcal{Z}_0}\frac{p(H_1)}{p(H_0)},$$

$p_i/p^{lin}_{\rm mSUGRA}$	asymmetric[a] $\mathcal{L}_{\rm DM}$		
Model/Prior	linear	log	flat μ, B
mSUGRA	1	3	4
mAMSB	164	403	148
LVS	18	20	22

[a] AbdusSalam, BCA, Dolan, Feroz, Hobson, arXiv:0906.0957

Summary


ATLAS Online Luminosity $\sqrt{s}$ = 7 TeV
LHC Delivered
ATLAS Recorded
Total Delivered: 1.073 pb^{-1}
Total Recorded: 1.008 pb^{-1}
Total Integrated Luminosity [pb^{-1}]
0
0.2
0.4
0.6
0.8
1
1.2
1.4
29/03
01/05
03/06
06/07
09/08
Day in 2010

Supplementary Material

MSSM Neutral Higgs Potential

$$V = (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2)$$
$$-\mu B(H_u^0 H_d^0 + c.c.)$$
$$+\frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2,$$

$$\frac{\partial V}{\partial H_u^0} = \frac{\partial V}{\partial H_d^0} = 0$$

$$\Rightarrow \mu B = \frac{\sin 2\beta}{2}(\bar{m}_{H_d}^2 + \bar{m}_{H_u}^2 + 2\mu^2),$$

$$\mu^2 = \frac{\bar{m}_{H_d}^2 - \bar{m}_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} - \frac{M_Z^2}{2}.$$

Natural Prior

We have assumed a flat prior in $\tan\beta$, implies a measure:

$$p(m_0, M_{1/2}|\text{data}) = \int dA_0 \, d\tan\beta \, ds \; p(m_0, M_{1/2}, A_0, \tan\beta, s|\text{data}).$$

Change variables: $\int d\mu dB \delta(M_Z - M_Z^{cen}) \rightarrow \int dM_Z d\tan\beta |J| \delta(M_Z - M_Z^{cen})$

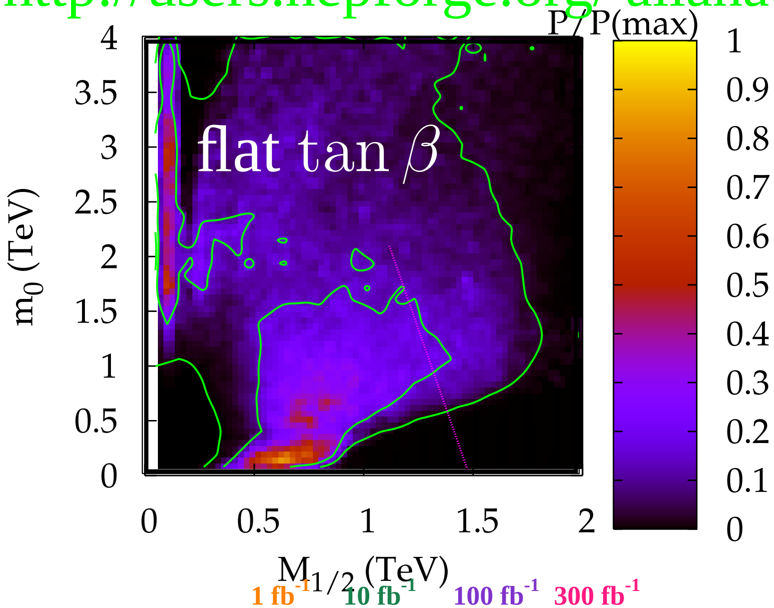
$$J = \frac{B}{\mu \tan\beta} \frac{\tan^2\beta - 1}{\tan^2\beta + 1} \frac{1}{\sin\beta}$$

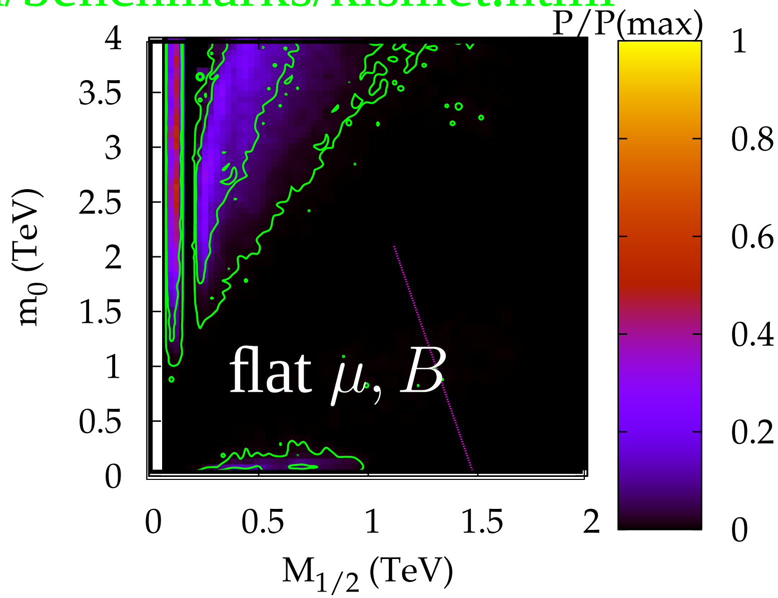
Cabrera, Casas, de Austri, arXiv:0812.5316 have considered $\{\mu, B, \lambda_t\} \rightarrow \{M_Z, \tan\beta, m_t\}$.

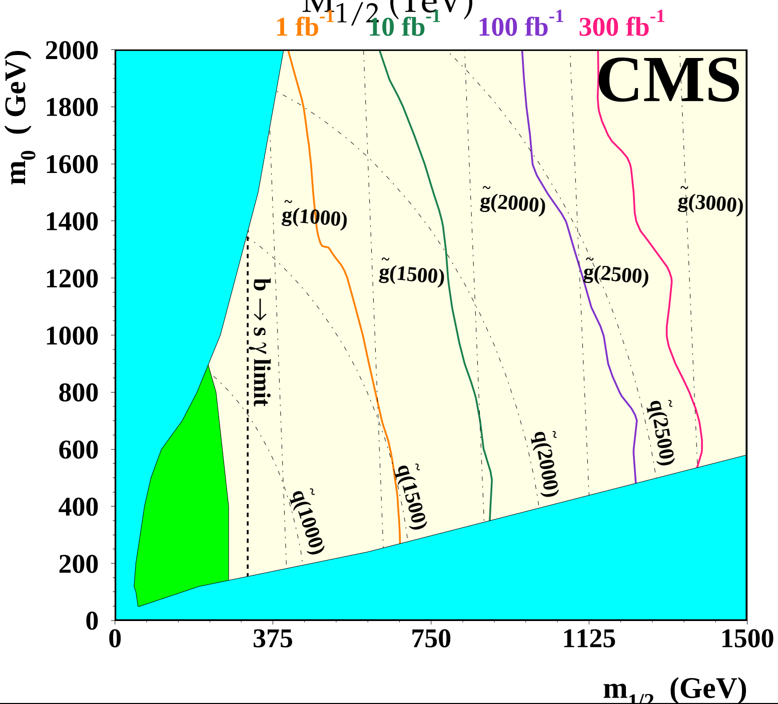
Killer Inference for Susy METeorology

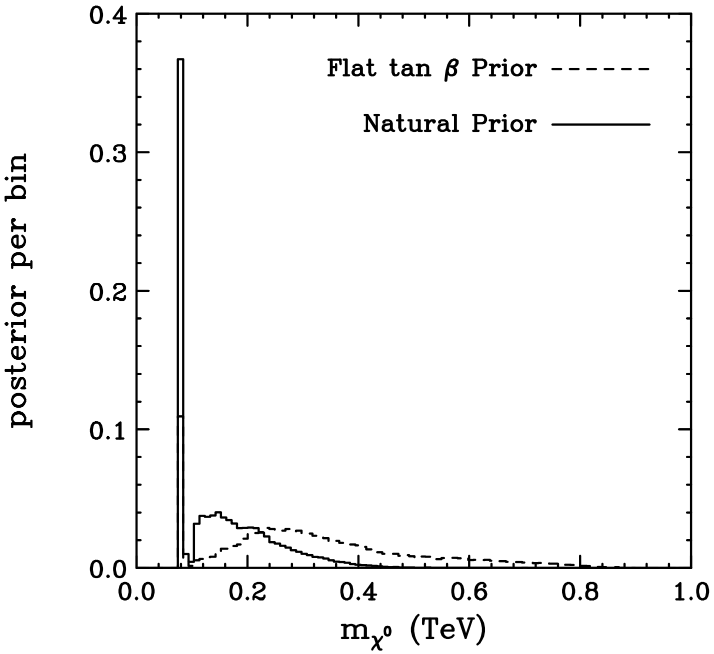
BCA, Cranmer, Weber, Lester, arXiv:0705.0487

http://users.hepforge.org/~allanach/benchmarks/kismet.html



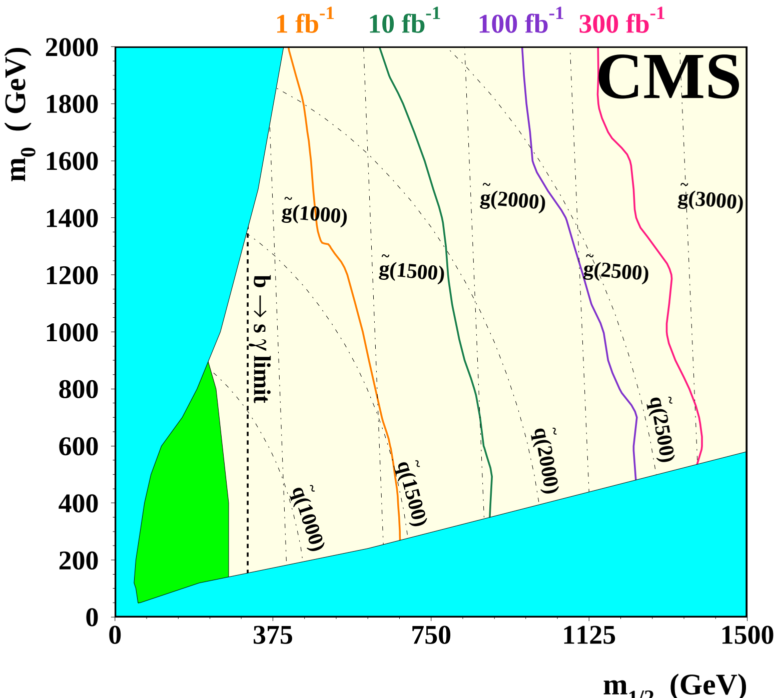


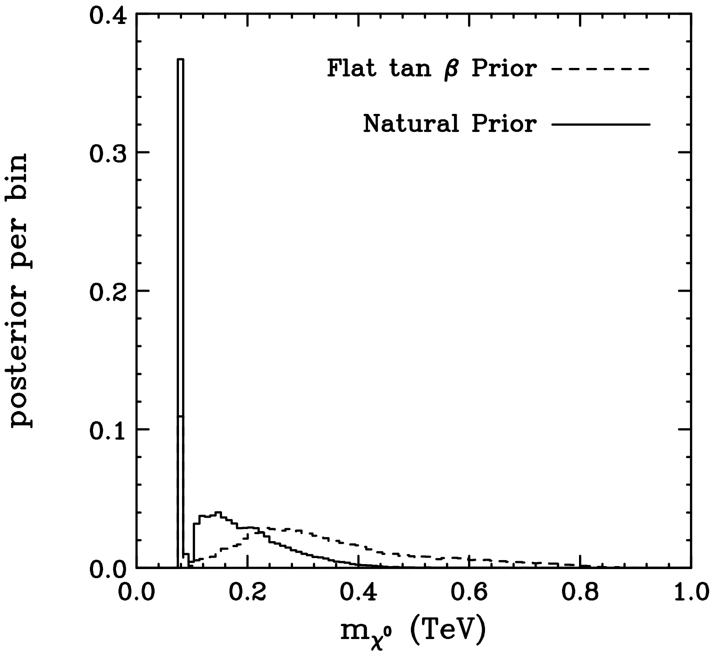




Killer Inference for Susy METeorology

BCA, Cranmer, Weber, Lester, arXiv:0705.0487





The Sign of μ

In order to calculate $p(d|H_1)/p(d|H_2)$, we calculate the Bayesian *evidence* ratio:

$$
\begin{aligned}
p(d|H_i) &= \int dm\, p(d|m, H_i)p(m|H_i) \\
\Rightarrow p(H_i|d) &= p(H_i)p(d|H_i)
\end{aligned}
$$

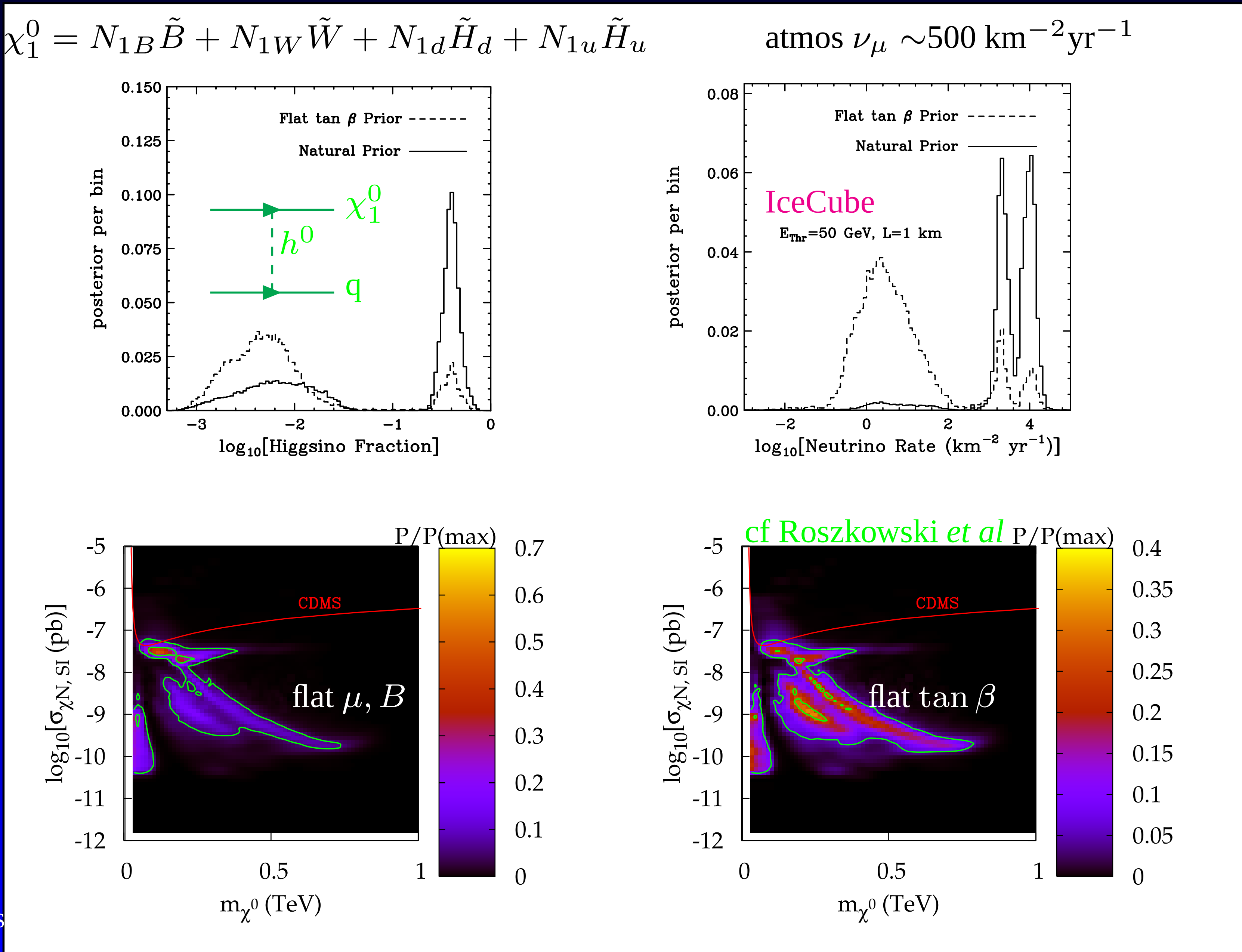
So, put $H_1 = \mu > 0$, $H_2 = \mu < 0$ to find:

Prior	P_+/P_-(2 TeV)	P_+/P_-(4 TeV)
flat	15.6	5.9
log	61.6	24.0

Requires multi-modal ellipsoidal nested sampling[a]

[a] Feroz, BCA, Hobson, AbdusSalam, Trotta, Weber, JHEP 10 (2008)

Dark Matter Detection



Ice Cube

Neutralinos can become trapped in the sun $\tilde{h}^0 - Z$ coupling $\sigma_{\chi^0 \mathrm{p,SD}} \propto [|N_{1d}|^2 - |N_{1u}|^2]^2$ dominates.
$A^{\odot} \equiv \sigma v / V$:

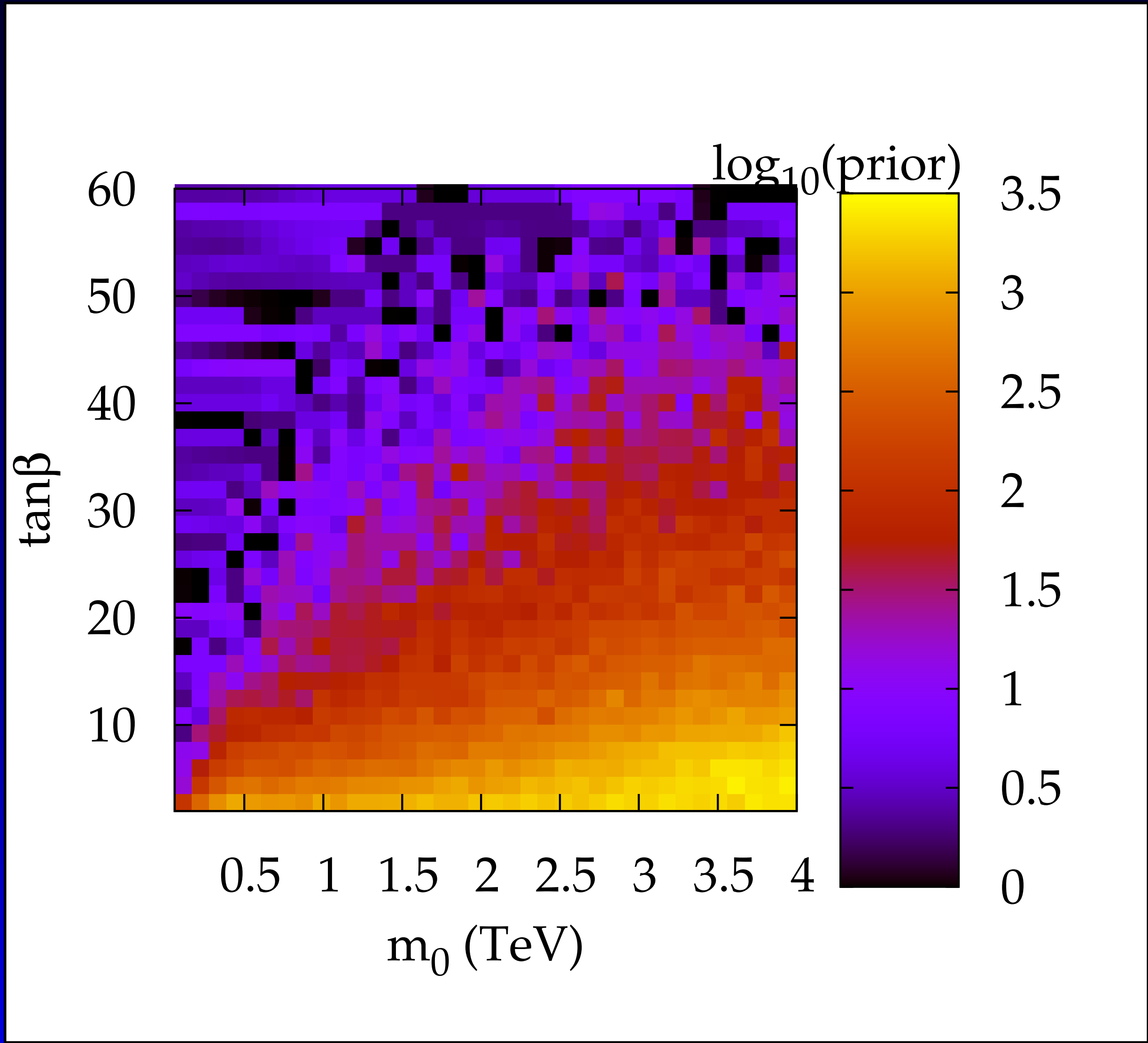
$$\dot{N} = C^{\odot} - A^{\odot} N^2,$$

$$\Gamma = \frac{1}{2} A^{\odot} N^2 = \frac{1}{2} C^{\odot} \tanh^2 \left(\sqrt{C^{\odot} A^{\odot}}\, t_{\odot} \right)$$

$$\frac{dN_{\nu_\mu}}{dE_{\nu_\mu}} = \frac{C_{\odot} F_{\mathrm{Eq}}}{4\pi D_{\mathrm{ES}}^2} \left(\frac{dN_\nu}{dE_\nu} \right)^{\mathrm{Inj}}$$

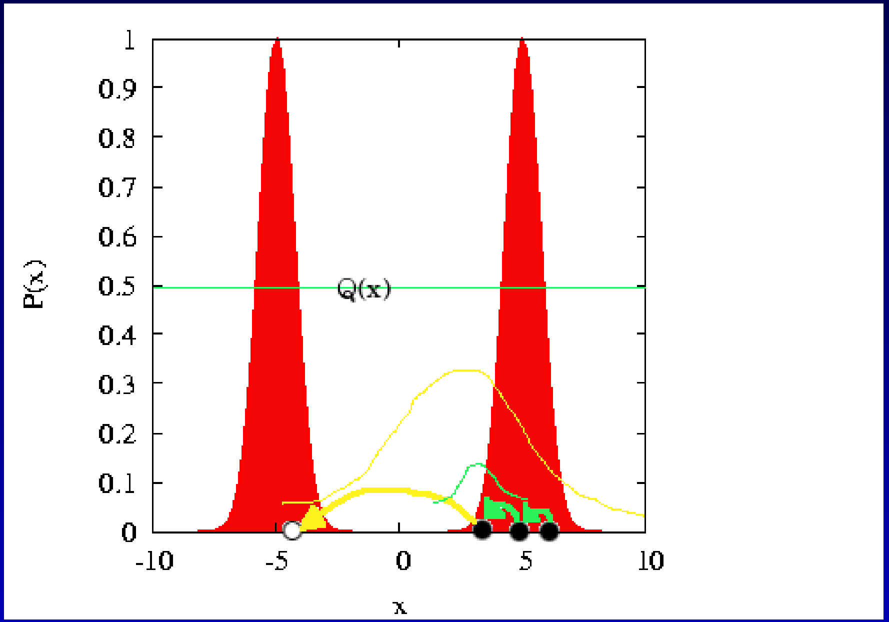
$$N_{\mathrm{ev}} \approx \int \int \frac{dN_{\nu_\mu}}{dE_{\nu_\mu}} \frac{d\sigma_\nu}{dy} R_\mu((1-y)\, E_\nu)\, A_{\mathrm{eff}}\, dE_{\nu_\mu}\, dy$$

Naturalness priors



Potential Problem

Often, people use a flat $Q(x)$. The trouble with this *"blind drunk"* sampling is the following situation:



Either large or small proposal widths σ lead to low efficiencies of sampling. Our proposal is to determine a $Q(x)$ closer to $P(x)$ *semi-automatically*.

Bank Sampling

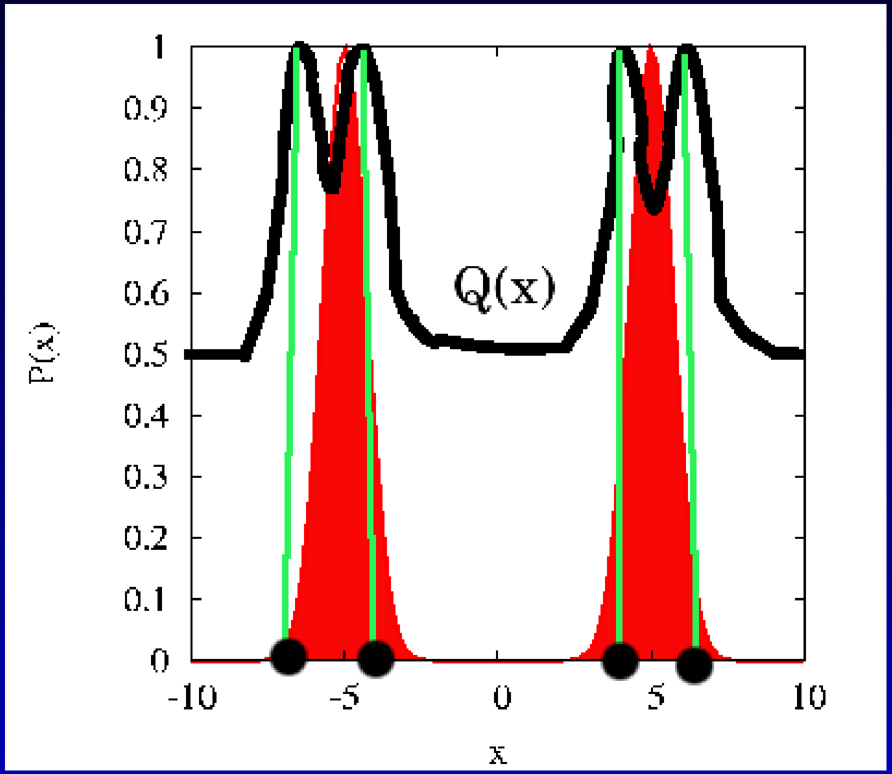


Figure 1: Bank points determined from previous runs: want to have at least one point in each maximum. *Knowledgeable drunk*

Proposal Distribution

$$Q_{bank}(\mathbf{x};\mathbf{x}^{(t)}) = (1-\lambda)K(\mathbf{x};\mathbf{x}^{(t)}) + \lambda \sum_{i=1}^{N} w_i K(\mathbf{x};\mathbf{y}^{(i)})$$

w_i are a set of N weights: $\sum_{i=1}^{N} w_i = 1$, $0 < \lambda < 1$, while K is the proposal distribution.

With probability $(1-\lambda)$ propose a local Metropolis step of the usual kind, i.e. “close” to the last point in the chain. With probability λ, teleport to the vicinity of one of the number of “banked” points, chosen with weight w_i from within the bank.

Collider Check

Need corroboration with *direct detection*.

If we can pin particle physics down, a comparison between the predicted relic density and that observed is a test of the cosmological assumptions used in the prediction.[a]

Thus, if it doesn't fit, you change the cosmology until it does.

---

[a] BCA, G. Belanger, F. Boudjema, A. Pukhov, JHEP 0412 (2004) 020.; M. Nojiri, D. Tovey, JHEP 0603 (2006) 063

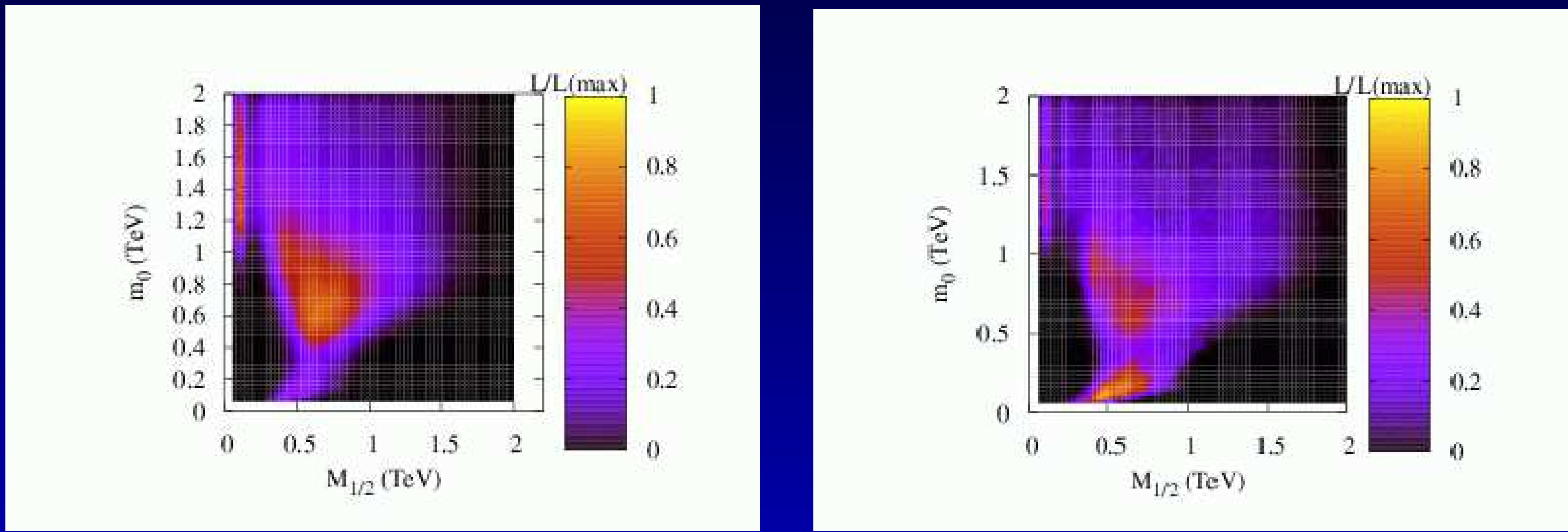
CMSSM Regions

After WMAP+LEP2, bulk region diminished. Need specific mechanism to reduce overabundance:

- $\tilde{\tau}$ coannihilation: small m_0, $m_{\tilde{\tau}_1} \approx m_{\chi_1^0}$. Boltzmann factor $\exp(-\Delta M/T_f)$ controls ratio of species. $\tilde{\tau}_1 \chi_1^0 \to \tau\gamma$, $\tilde{\tau}_1 \tilde{\tau}_1 \to \tau\bar{\tau}$.
- Higgs Funnel: $\chi_1^0 \chi_1^0 \to A \to b\bar{b}/\tau\bar{\tau}$ at large $\tan\beta$. Also via[a] h at large m_0 small $M_{1/2}$.
- Focus region: Higgsino LSP at large m_0: $\chi_1^0 \chi_1^0 \to WW/ZZ/Zh/t\bar{t}$.
- $\tilde{t}$ coannihilation: high $-A_0$, $m_{\tilde{t}_1} \approx m_{\chi_1^0}$. $\tilde{t}_1 \chi_1^0 \to gt$, $\tilde{t}\tilde{t} \to tt$

[a] Datta, Djouadi, Drees, hep-ph/0504090

Comparison

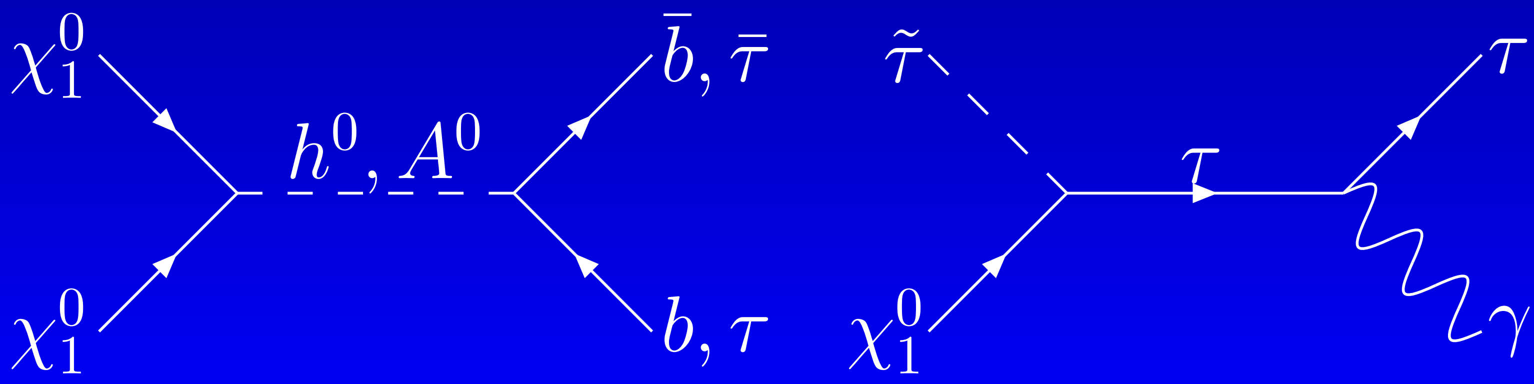


- LHS: allowing non thermal-χ_1^0 contribution
- RHS: only χ_1^0 dark matter
- *(flat priors)*

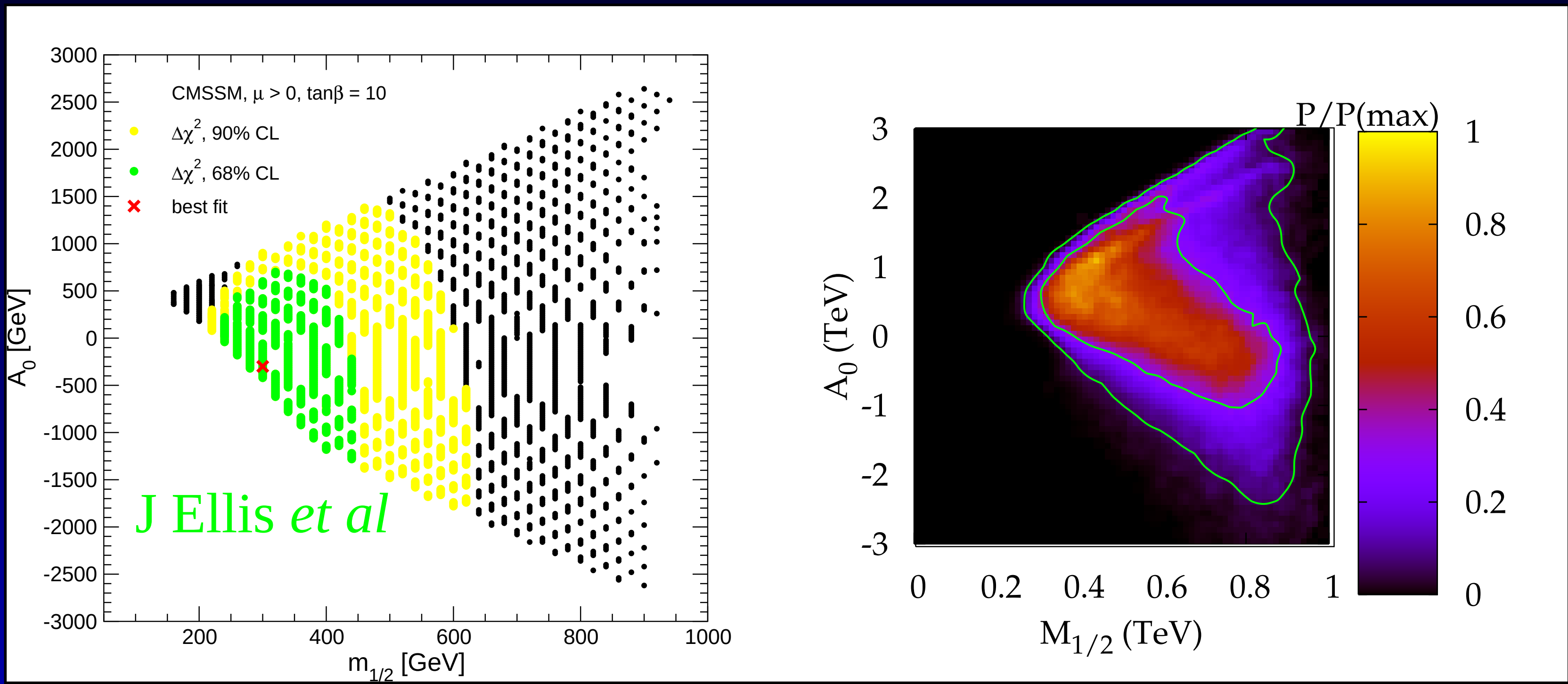
Annihilation Mechanism

Define stau co-annihilation when $m_{\tilde{\tau}}$ is within 10% of $m_{\chi_1^0}$ and Higgs pole when $m_{h,A}$ is within 10% of $2m_{\chi_1^0}$.

mechanism	flat prior	natural prior
h^0−pole	0.025	0.07
A^0−pole	0.41	0.14
$\tilde{\tau}$−co-annihilation	0.26	0.18
rest	0.31	0.61

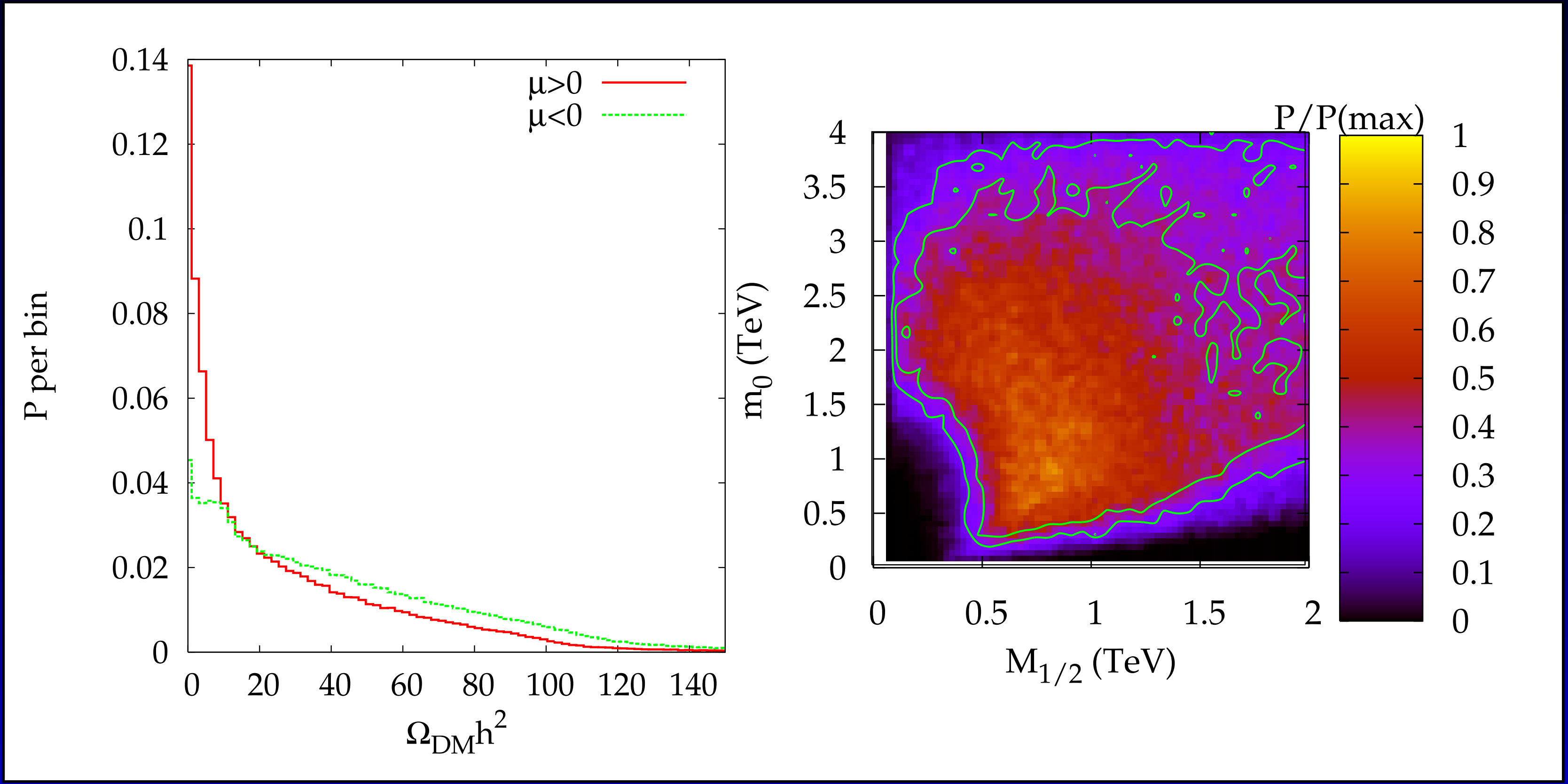


Comparison



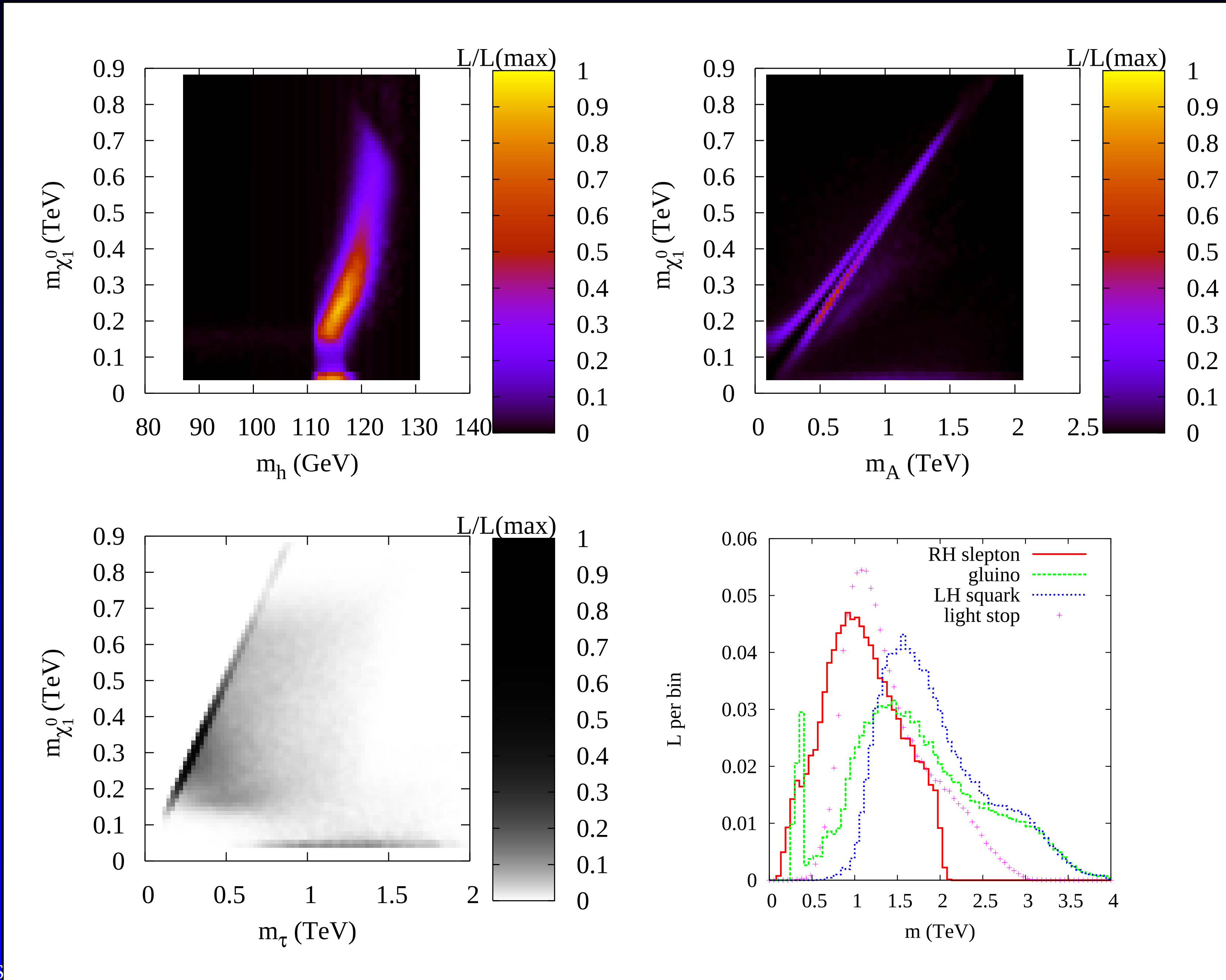
- Fix $\tan\beta = 10$ and all SM inputs
- Restrict $m_0, M_{1/2} < 1$ TeV.
- *Same* fits!

No Dark Matter Fits



Huge χ^2 from the dark matter relic density.

Sanity Check



LHC vs LC in SUSY Measurement

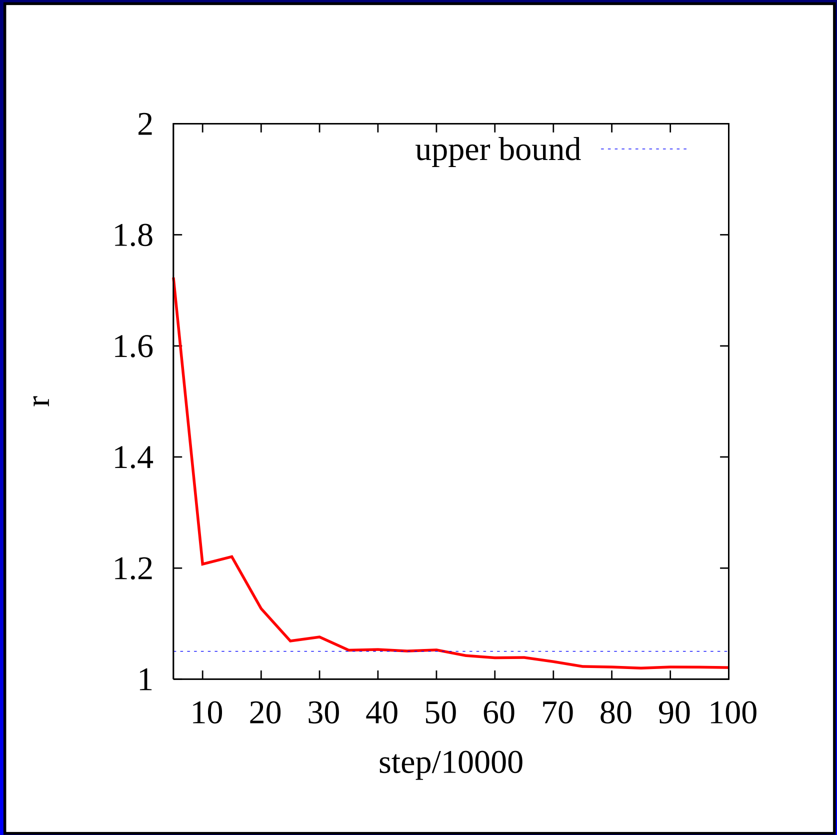
- LHC (start date 2007) produces strongly interacting particles up to a few TeV. Precision measurements of mass *differences* possible if the decay chains exist: possibly per mille for leptons, several percent for jets.
- ILC has several energy options: 500-1000 GeV, CLIC up to 3 TeV. Linear colliders produce less strong particles but much easier to make precision measurements of masses/couplings.

Q: What energy for LC?
Q: What do we get from LHC[a]?

[a] LHC/ILC Working Group Report: `hep-ph/0410364`

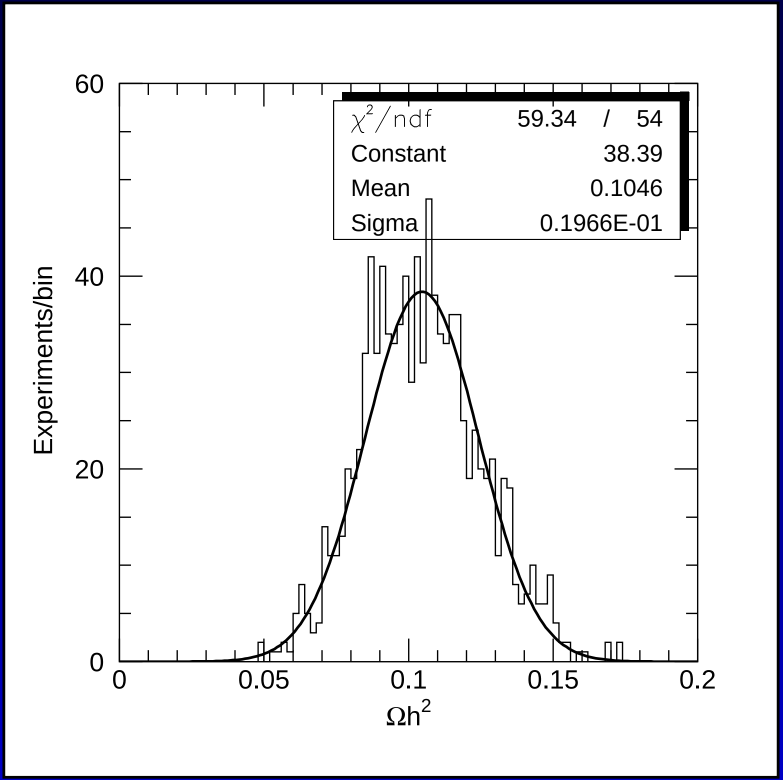
Convergence

We run 9$\times$1 000 000 points. By comparing the 9 independent chains with random starting points, we can provide a statistical measure of convergence: an upper bound r on the excepted variance decrease for infinite statistics.



Predicting Ωh^2

Not much left that's allowed but edge measurements allow reasonable Ωh^2 error[a] for 300 fb^{-1}.



$\mathcal{Q}$: What about other bits of parameter space?

[a] M Nojiri, G Polesello, D Tovey, JHEP 0603 (2006) 063, `hep-ph/0512204`.

Bulk Region

M Nojiri, G Polesello, D Tovey, JHEP 0603 (2006) 063, `hep-ph/0512204`. for 300 fb^{-1}. SPA point $m_0 = 70$ GeV, $m_{1/2} = 250$ GeV, $A_0 = -300$ GeV, $\tan\beta = 10$, $\mu > 0$: $\Omega h^2 = 0.108$. Put in m_{ll}^{max}, m_{llq}^{max}, m_{lq}^{low}, m_{lq}^{high}, m_{llq}^{min}, $m_{l_L} - m_{\chi_1^0}$, $m_{ll}^{max}(\chi_4^0)$, $m_{\tau\tau}^{max}$, m_h.

$\tilde\chi_1^0 \tilde\chi_1^0 \to \ell^+ \ell^-$	40%
$\tilde\chi_1^0 \tilde\chi_1^0 \to \tau^+ \tau^-$	28%
$\tilde\chi_1^0 \tilde\chi_1^0 \to \nu \bar\nu$	3%
$\tilde\chi_1^0 \tilde\tau_1 \to Z \tau$	4%
$\tilde\chi_1^0 \tilde\tau_1 \to A \tau$	18%
$\tilde\tau_1 \tilde\tau_1 \to \tau \tau$	2%

Neutralino mass matrix

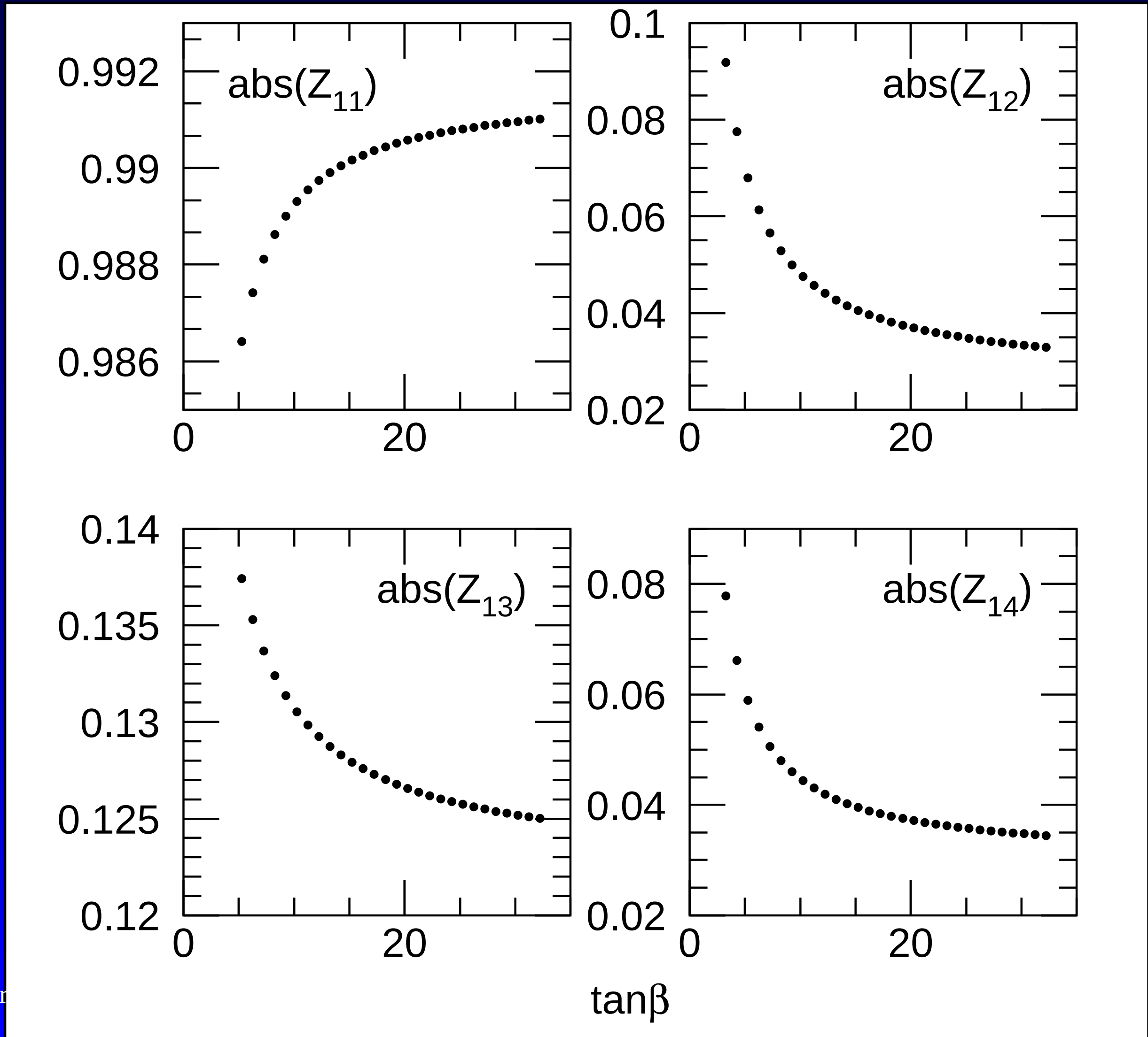
Neutralino masses measured: $\chi^0_{1,2,4}$ but need mixing matrix to determine couplings. Left with $\tan\beta$.

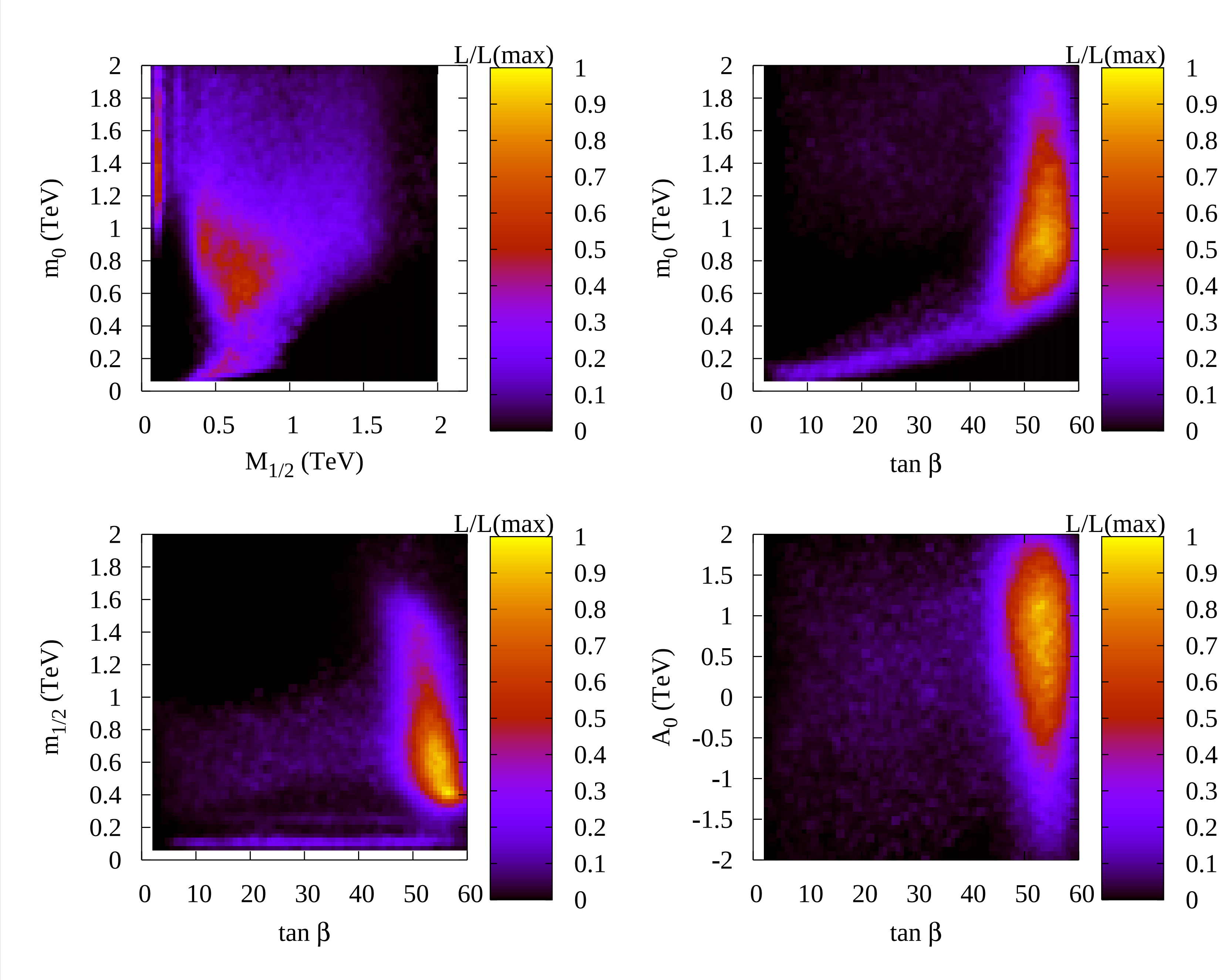
$$\begin{bmatrix} M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W \\ 0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W \\ -m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -\mu \\ m_Z s_\beta s_W & -m_Z s_\beta c_W & -\mu & 0 \end{bmatrix}$$

(2)

Neutralino mass matrix

Neutralino masses measured: $\chi^0_{1,2,4}$ but need mixing matrix to determine couplings. Left with $\tan\beta$.





L/L(max)
m_0 (TeV)
$M_{1/2}$ (TeV)
m_0 (TeV)
tan β
$m_{1/2}$ (TeV)
tan β
A_0 (TeV)
tan β

Uncertainties in Relic Density

Bulk region: $\tilde{B}\tilde{B} \to Z, h \to l\bar{l}$. Coannihilation: $\tilde{\tau}\chi_1^0 \to \tau + X$

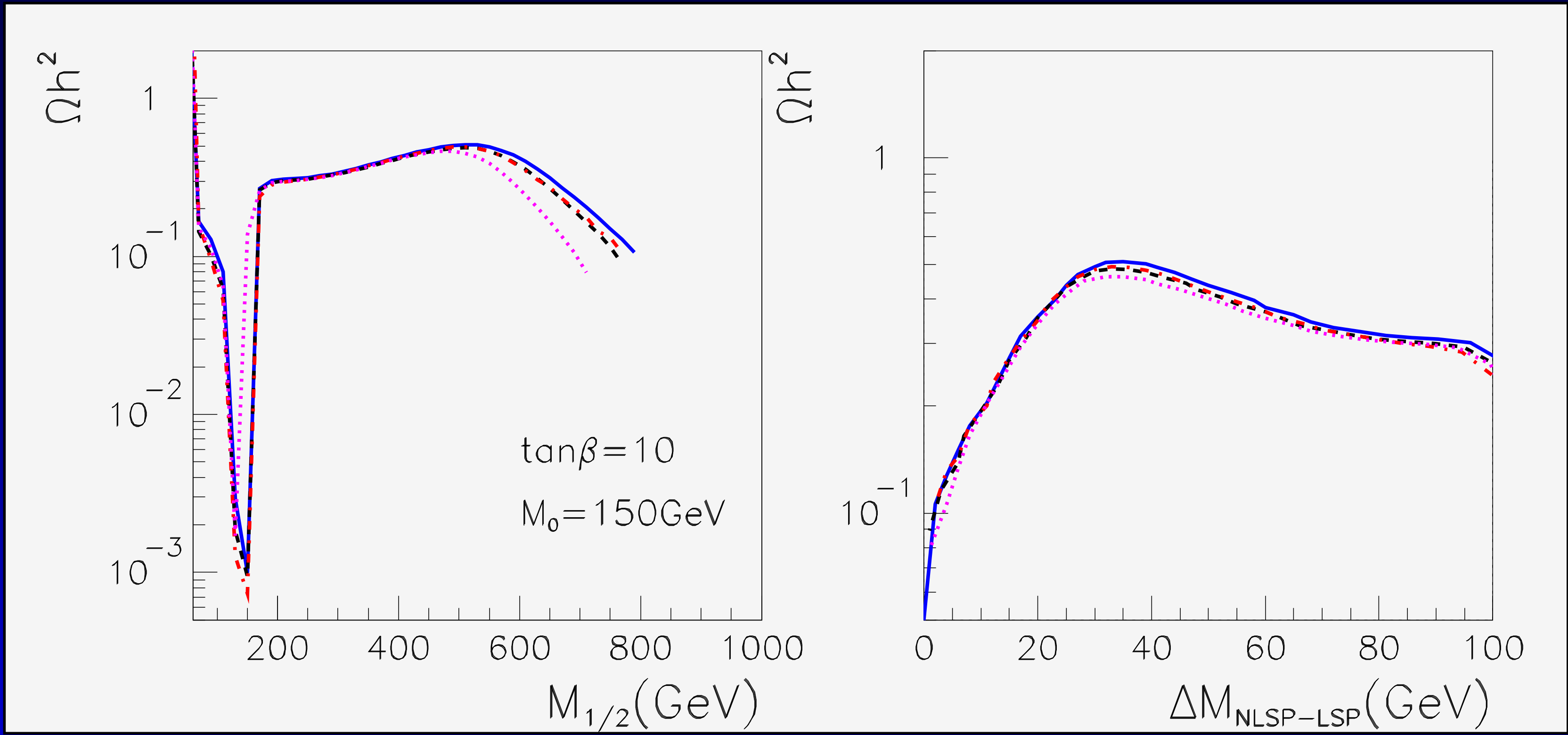


Figure 2: Bulk/coannihilation region. Full: `SoftSusy`, dotted: `SPheno`.