Harmony of Scattering Amplitudes: From QCD to *N* = 8 Supergravity

SUSY 2010, Bonn

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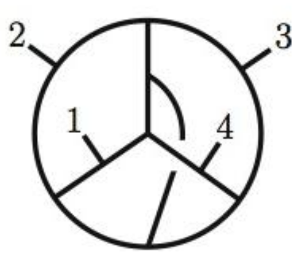
Zvi Bern, UCLA

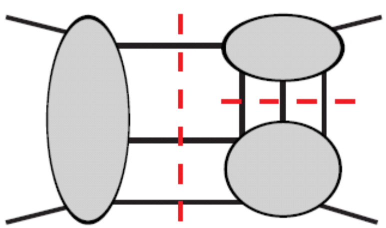
ZB, L. Dixon, R. Roiban, hep-th/0611086
ZB, J.J. Carrasco, L. Dixon, H. Johansson, D. Kosower and R. Roiban , hep-th/0702112
ZB, J. J. Carrasco, L. Dixon, H. Johansson, and R. Roiban , arXiv:0808.4112 arXiv:0905.2326 arXiv:1008.3327
ZB, J.J.M. Carrasco, H. Ita, H. Johansson, R. Roiban, arXiv:0903.5348
ZB, J.J.M. Carrasco and H. Johansson, arXiv:1004.0476
ZB, T. Dennen, Y.-t. Huang, M. Kiermaier, arXiv:1004.0693

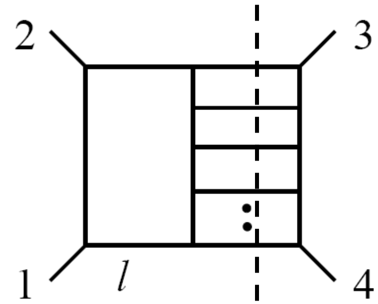
Outline

Will outline some new developments in understanding multiloop scattering amplitudes with a focus on *N* = 8 supergravity and its UV properties.

1. **Modern unitarity method for loop amplitudes.**
2. **NLO QCD and susy phenomenology**
3. **A hidden structure in gauge and gravity theories**
   — **a duality between color and kinematics**
   — **gravity as a double copy of gauge theory**
4. **Reexamination of compatibility of quantum mechanics and general relativity.**

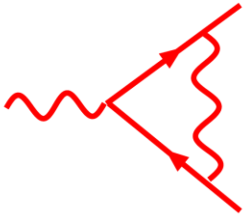






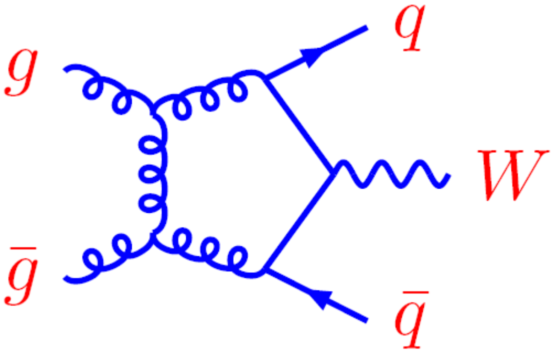
State-of-the-Art Feynman Diagram Calculations

In 1948 Schwinger computed anomalous magnetic moment of the electron.



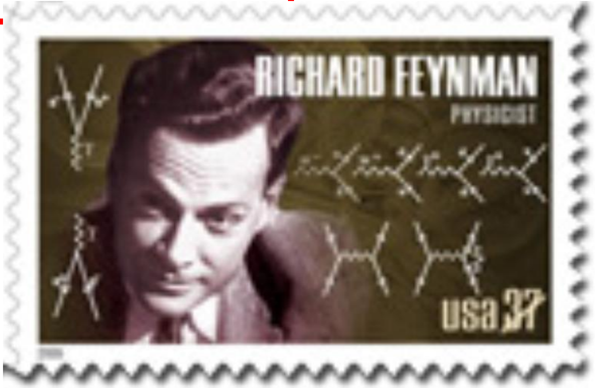
In 2009 typical 1-loop modern example:

$$gg \to W + 2 \text{ jets}$$



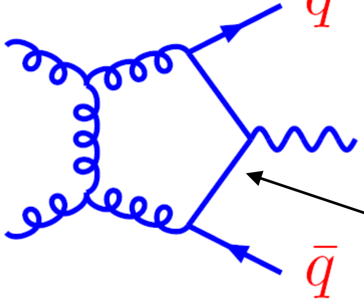
60 years later at 1 loop only 2 (and sometimes 3) legs more than Schwinger!

Why are Feynman diagrams difficult for high-loop or high-multiplicity processes?



- Vertices and propagators involve unphysical gauge-dependent off-shell states. An important origin of the complexity.

$$\int \frac{d^3\vec{p}\, dE}{(2\pi)^4}$$



$$E^2 - \vec{p}^2 \neq m^2$$

Individual Feynman diagrams unphysical

Einstein's relation between momentum and energy violated in the loops. Unphysical states! Not gauge invariant.

- **All steps should be in terms of gauge invariant on-shell physical states. On-shell formalism. Need to rewrite quantum field theory!**

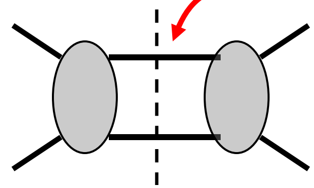
ZB, Dixon, Dunbar, Kosower

Bern, Dixon, Dunbar and Kosower

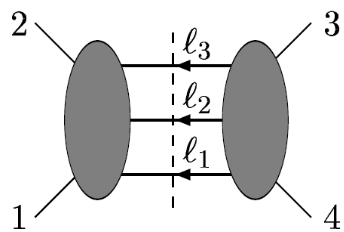
Unitarity Method: Rewrite of QFT

on-shell

Two-particle cut:

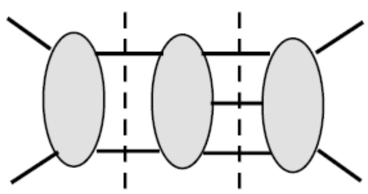


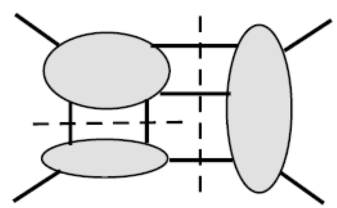
Three-particle cut:



Systematic assembly of complete amplitudes from cuts for any number of particles or loops.

Generalized unitarity as a practical tool:



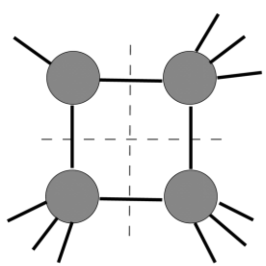


Different cuts merged to give an expression with correct cuts in all channels.

Bern, Dixon and Kosower
Britto, Cachazo and Feng; Forde;
Ossala, Pittau, Papadopolous, and many others

complex momenta to solve cuts

Britto, Cachazo and Feng

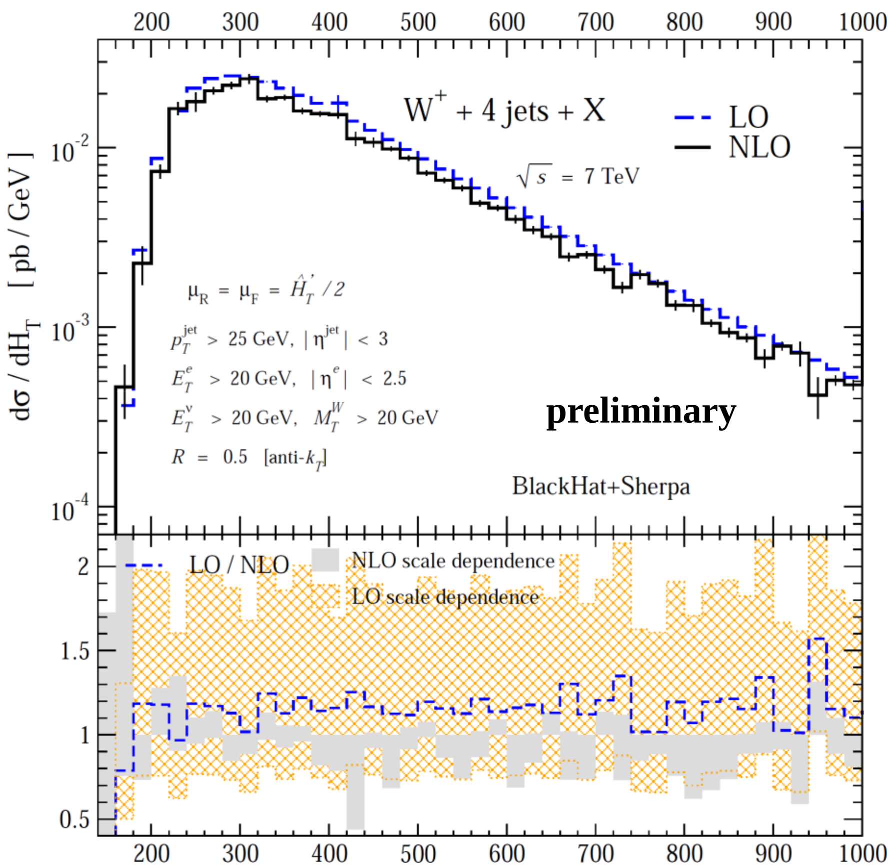


Unitarity method now a standard tool for NLO QCD

Applications of new ideas to collider phenomenology

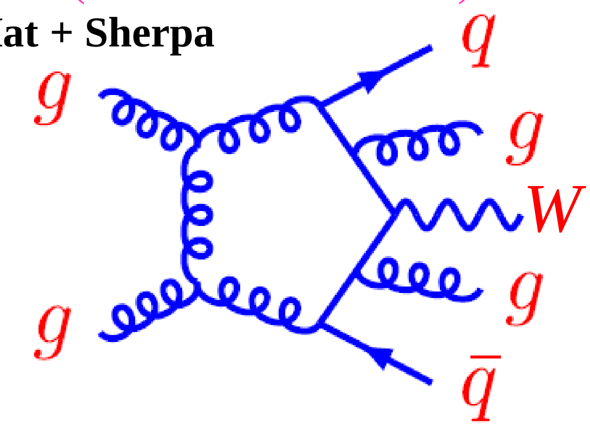
Berger, ZB, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre (BlackHat collaboration)

W+4 jets H_T distribution



H_T [GeV] –total transverse energy

BlackHat + Sherpa



NLO QCD provides the *best* available theoretical predictions. Leptonic decays of W and Z's give missing energy.

- **On-shell methods really work.**
- **2 legs beyond Feynman diagrams.**

Such calculations are very helpful in experimental searches for susy and other new physics

The Structure of (Supersymmetric) Gauge and Gravity Scattering Amplitudes

Gravity vs Gauge Theory

Consider the gravity Lagrangian

$$L_{\text{gravity}} = \frac{2}{\kappa^2}\sqrt{-g}\,R$$

flat metric

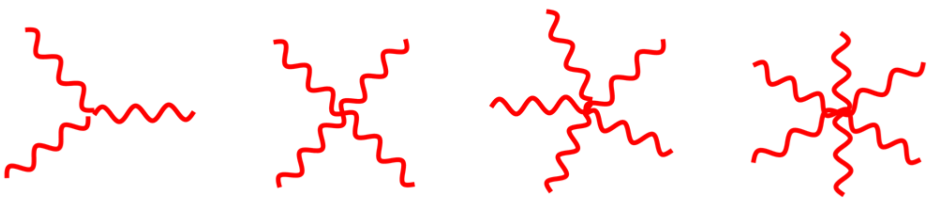
graviton field

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

metric

$$\kappa^2 = 32\pi G_{\text{Newton}}$$

Infinite number of complicated interactions



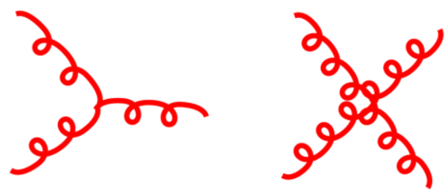
$+\cdots$

terrible mess

Non-renormalizable

Compare to Yang-Mills Lagrangian on which QCD is based

$$L_{\text{YM}} = \frac{1}{g^2}F^2$$

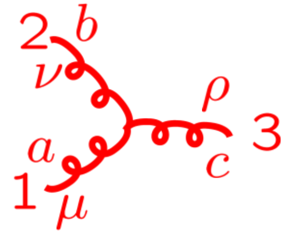


Only three and four point interactions

Gravity seems so much more complicated than gauge theory.

Three Vertices

Standard Feynman diagram approach.



Three-gluon vertex:

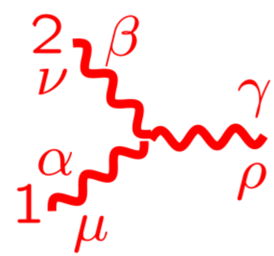
$$V_{3\,\mu\nu\sigma}^{abc} = -gf^{abc}(\eta_{\mu\nu}(k_1-k_2)_\rho+\eta_{\nu\rho}(k_1-k_2)_\mu+\eta_{\rho\mu}(k_1-k_2)_\nu)$$

$$k_i^2 = E_i^2 - \vec{k}_i^2 \neq 0$$

Three-graviton vertex:

$$G_{3\mu\alpha,\nu\beta,\sigma\gamma}(k_1,k_2,k_3) =$$

$$\begin{aligned}
&\mathrm{sym}[-\tfrac{1}{2}P_3(k_1\cdot k_2\eta_{\mu\alpha}\eta_{\nu\beta}\eta_{\sigma\gamma}) - \tfrac{1}{2}P_6(k_{1\nu}k_{1\beta}\eta_{\mu\alpha}\eta_{\sigma\gamma}) + \tfrac{1}{2}P_3(k_1\cdot k_2\eta_{\mu\nu}\eta_{\alpha\beta}\eta_{\sigma\gamma})\\
&\quad + P_6(k_1\cdot k_2\eta_{\mu\alpha}\eta_{\nu\sigma}\eta_{\beta\gamma}) + 2P_3(k_{1\nu}k_{1\gamma}\eta_{\mu\alpha}\eta_{\beta\sigma}) - P_3(k_{1\beta}k_{2\mu}\eta_{\alpha\nu}\eta_{\sigma\gamma})\\
&\quad + P_3(k_{1\sigma}k_{2\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + P_6(k_{1\sigma}k_{1\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + 2P_6(k_{1\nu}k_{2\gamma}\eta_{\beta\mu}\eta_{\alpha\sigma})\\
&\quad + 2P_3(k_{1\nu}k_{2\mu}\eta_{\beta\sigma}\eta_{\gamma\alpha}) - 2P_3(k_1\cdot k_2\eta_{\alpha\nu}\eta_{\beta\sigma}\eta_{\gamma\mu})]
\end{aligned}$$



About 100 terms in three vertex

Naïve conclusion: Gravity is a nasty mess.

Definitely not a good approach.

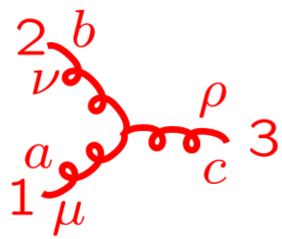
Simplicity of Gravity Amplitudes

People were looking at gravity the wrong way. On-shell viewpoint much more powerful.

On-shell **three vertices contains all information:**

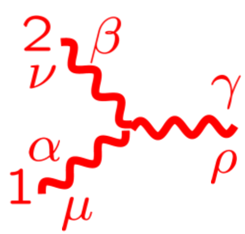
$$k_i^2 = 0$$

gauge theory:



$$-gf^{abc}(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic})$$

gravity:



$$i\kappa(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic}) \times (\eta_{\alpha\beta}(k_1 - k_2)_\gamma + \text{cyclic})$$

double copy of Yang-Mills vertex.

- **Using modern on-shell methods, any gravity scattering amplitude constructible solely from *on-shell* 3 vertex.**
- **Higher-point vertices irrelevant! On-shell recursion for trees, unitarity method for loops.**

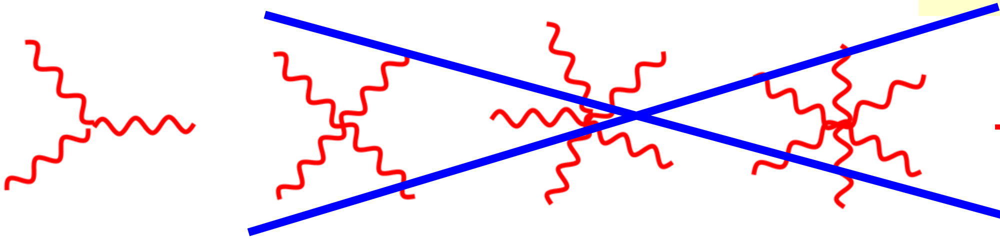
Gravity vs Gauge Theory

Consider the gravity Lagrangian

$$L_{\text{gravity}} = \frac{2}{\kappa^2}\sqrt{-g}\,R \qquad \underbrace{g_{\mu\nu}}_{\text{metric}} = \underbrace{\eta_{\mu\nu}}_{\text{flat metric}} + \kappa \underbrace{h_{\mu\nu}}_{\text{graviton field}}$$

$$\kappa^2 = 32\pi G_{\text{Newton}}$$

Infinite number of irrelevant interactions!

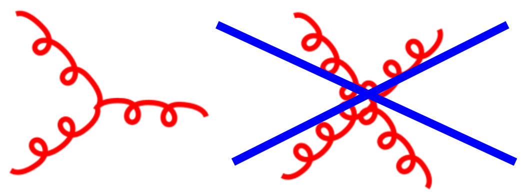


$+ \cdots$

Simple relation to gauge theory

Compare to Yang-Mills Lagrangian

$$L_{\text{YM}} = \frac{1}{g^2}F^2$$



Only three-point interactions

Gravity seems ~~so much~~ no more complicated than gauge theory.

Duality Between Color and Kinematics

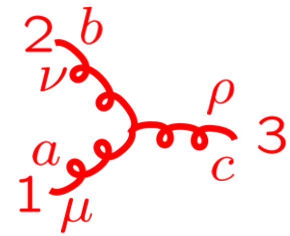
ZB, Carrasco, Johansson

coupling constant

color factor

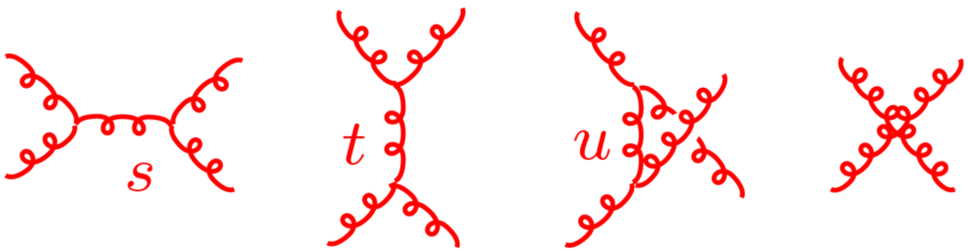
momentum dependent kinematic factor

$$-gf^{abc}(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic})$$



Color factors based on a Lie algebra: $[T^a, T^b] = if^{abc}T^c$

Jacobi Identity $f^{a_1 a_2 b} f^{b a_4 a_3} + f^{a_4 a_2 b} f^{b a_3 a_1} + f^{a_4 a_1 b} f^{b a_2 a_3} = 0$



Use 1 = *s/s* = *t/t* = *u/u*
to assign 4-point diagram to others.

$$\mathcal{A}_4^{\text{tree}} = g^2\left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\right)$$

$s = (k_1 + k_2)^2$

$t = (k_1 + k_4)^2$

$u = (k_1 + k_3)^2$

Color factors satisfy Jacobi identity:

Numerator factors satisfy similar identity:

$$c_u = c_s - c_t$$

$$n_u = n_s - n_t$$

Color and kinematics satisfy similar identities

Duality Between Color and Kinematics

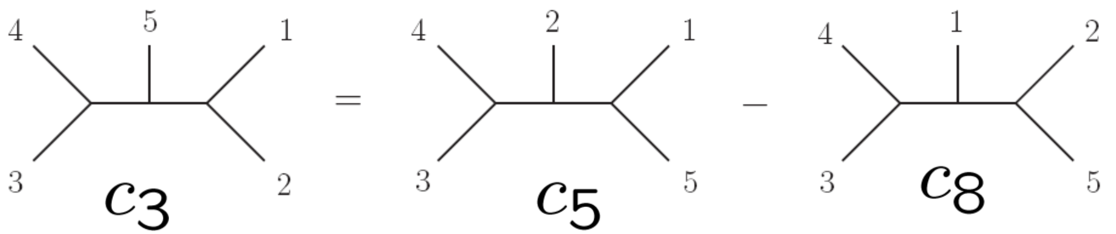
Consider five-point amplitude:

color factor

kinematic numerator factor

$$\mathcal{A}_5^{\text{tree}} = \sum_{i=1}^{15} \frac{c_i\, n_i}{D_i}$$

Feynman propagators



$$c_3 \equiv f^{a_3 a_4 b} f^{b a_5 c} f^{c a_1 a_2}\,, \qquad c_5 \equiv f^{a_3 a_4 b} f^{b a_2 c} f^{c a_1 a_5}\,, \qquad c_8 \equiv f^{a_3 a_4 b} f^{b a_1 c} f^{c a_2 a_5}$$

$$\boxed{c_3 - c_5 + c_8 = 0 \quad \Leftrightarrow \quad n_3 - n_5 + n_8 = 0}$$

Claim: We can *always* find a rearrangement where color and kinematics satisfy the *same* Jacobi constraint equations.

- **Color and kinematics satisfy same equations!**
- **Nontrivial constraints on amplitudes.**

There is now a string-theory understanding.

Bjerrum-Bohr, Damgaard, Vanhove; Stieberger; Mafra; Tye and Zhang

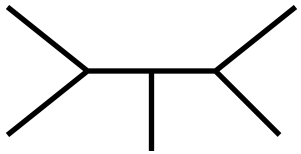
Higher-Point Gravity and Gauge Theory

ZB, Carrasco, Johansson

gauge theory: $$\frac{1}{g^{n-2}}\mathcal{A}_n^{\text{tree}}(1,2,3,\ldots,n) = \sum_i \frac{n_i\, c_i}{\prod_{\alpha_i} p^2_{\alpha_i}}$$

sum over diagrams with only 3 vertices

gravity: $$-i\left(\frac{2}{\kappa}\right)^{(n-2)}\mathcal{M}_n^{\text{tree}}(1,2,\ldots,n) = \sum_i \frac{n_i\, \tilde{n}_i}{\prod_{\alpha_i} p^2_{\alpha_i}}$$



Holds if the n_i satisfy the duality. $\tilde{n}_i$ is from 2nd gauge theory

Gravity numerators are a double-copy of gauge-theory ones!

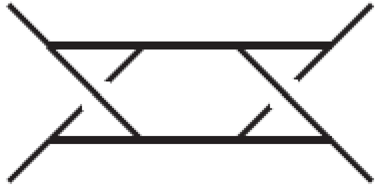
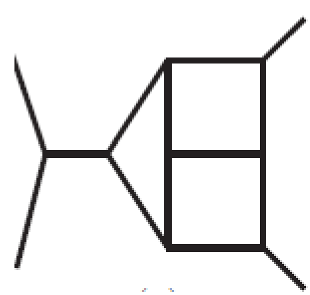
Proved using on-shell recursion relations that if duality holds, gravity numerators are 2 copies of gauge-theory ones.

ZB, Dennen, Huang, Kiermaier

Cries out for a unified description of the sort given by string theory!

Loop-Level Generalization

ZB, Carrasco, Johansson (2010)

sum is over diagrams

kinematic numerator

color factor

$$\frac{(-i)^L}{g^{n-2+2L}}\mathcal{A}_n^{\rm loop} = \sum_j \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_j} \frac{n_j c_j}{\prod_{\alpha_j} p^2_{\alpha_j}}$$

gauge theory

propagators

$$\frac{(-i)^{L+1}}{(\kappa/2)^{n-2+2L}}\mathcal{M}_n^{\rm loop} = \sum_j \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_j} \frac{n_j \tilde{n}_j}{\prod_{\alpha_j} p^2_{\alpha_j}}$$

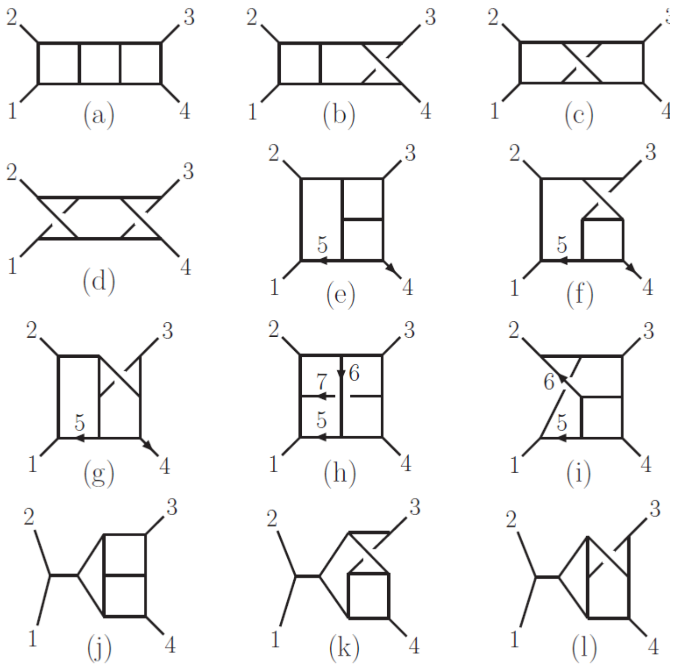
gravity

symmetry factor

- Loop-level conjecture is identical to tree-level one except for symmetry factors and loop integration.
- Gravity double copy works if numerator satisfies duality.
- Does *not* work for Feynman diagrams.

Explicit Three-Loop Check

ZB, Carrasco, Johansson (2010)



$$c_i = c_j - c_k \;\Rightarrow\; n_i = n_j - n_k$$

For *N*=4 sYM we have the abililty to go to high loop orders. Go to 3 loops. (1 & 2 loops work.)

Similar to earlier form with found with Dixon and Roiban, except now duality exposed.

Integral $I^{(x)}$	$\mathcal{N}=4$ Super-Yang-Mills ($\sqrt{\mathcal{N}=8 \text{ supergravity}}$) numerator
(a)–(d)	s^2
(e)–(g)	$\left(s\left(-\tau_{35}+\tau_{45}+t\right) - t\left(\tau_{25}+\tau_{45}\right) + u\left(\tau_{25}+\tau_{35}\right) - s^2 \right)/3$
(h)	$\left(s\left(2\tau_{15}-\tau_{16}+2\tau_{26}-\tau_{27}+2\tau_{35}+\tau_{36}+\tau_{37}-u\right) \right.$ $\left. +t\left(\tau_{16}+\tau_{26}-\tau_{37}+2\tau_{36}-2\tau_{15}-2\tau_{27}-2\tau_{35}-3\tau_{17}\right)+s^2 \right)/3$
(i)	$\left(s\left(-\tau_{25}-\tau_{26}-\tau_{35}+\tau_{36}+\tau_{45}+2t\right) \right.$ $\left. +t\left(\tau_{26}+\tau_{35}+2\tau_{36}+2\tau_{45}+3\tau_{46}\right)+u\,\tau_{25}+s^2 \right)/3$
(j)-(l)	$s(t-u)/3$

$$\tau_{ij} = 2k_i \cdot l_j$$

- **Duality works!**
- **Double copy works!**

Lagrangians

ZB, Dennen, Huang, Kiermaier

$$L_{\mathrm{YM}} = \frac{1}{g^2} F^2 \qquad L_{\mathrm{gravity}} = \frac{2}{\kappa^2} \sqrt{-g}\, R$$

How can one take two copies of the gauge-theory Lagrangian to give a gravity Lagrangian?

Add zero to the YM Lagrangian in a special way:

$$\mathcal{L}'_5 = -\tfrac{1}{2} g^3 \left(f^{a_1 a_2 b} f^{b a_3 c} + f^{a_2 a_3 b} f^{b a_1 c} + f^{a_3 a_1 b} f^{b a_2 c} \right) f^{c a_4 a_5} \\ \times \partial_{[\mu} A^{a_1}_{\nu]} A^{a_2}_{\rho} A^{a_3 \mu} \frac{1}{\Box} (A^{a_4 \nu} A^{a_5 \rho}) \quad = 0$$

Through five points:

- **Feynman diagrams satisfy the color-kinematic duality.**
- **Introduce auxiliary field to convert contact interactions into three-point interactions.**
- **Take two copies: you get gravity!**

$$A^{\mu} \tilde{A}^{\nu} \rightarrow h^{\mu\nu}$$

At each order need to add more and more vanishing terms.

Lagrangians

One can continue this process but things get more complicated:

- **At six points (vanishing) Lagrangian correction has ~100 terms.**
- **Beyond six points it has not been constructed.**

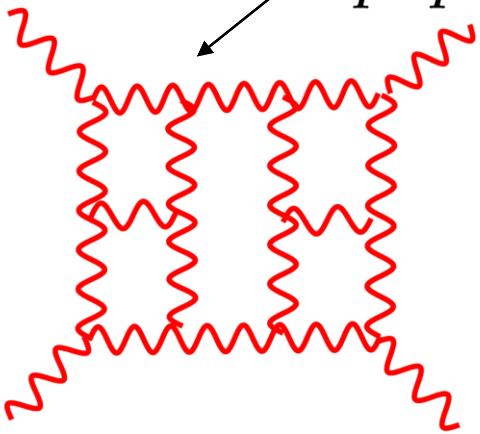
Nevertheless, double-copy structure suggests that *all* classical solutions in gravity theories are convolutions of gauge theory solutions when appropriate variables are used.

$$g_{\mu\nu}(x) \sim \int dy \, A_\mu(x-y)\tilde{A}_\nu(y)$$

UV Properties of Gravity

Power Counting at High-Loop Orders

$$\kappa = \sqrt{32\pi G_N} \quad \longleftarrow \text{ Dimensionful coupling}$$



Gravity: $$\int \prod_{i=1}^{L} \frac{dp_i^D}{(2\pi)^D} \frac{(\kappa\, p_j^\mu p_j^\nu)\cdots}{\text{propagators}}$$

Gauge theory: $$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{(g\, p_j^\nu)\cdots}{\text{propagators}}$$

Extra powers of loop momenta in numerator means integrals are badly behaved in the UV.

Non-renormalizable by power counting.

Reasons to focus on *N* = 8 supergravity:

- **With more susy expect better UV properties.**
- **High symmetry implies technical simplicity.**

Opinions from the 80's

Unfortunately, in the absence of further mechanisms for cancellation, the analogous $N = 8$ $D = 4$ supergravity theory would seem set to diverge at the three-loop order.

Howe, Stelle (1984)

It is therefore very likely that all supergravity theories will diverge at three loops in four dimensions. … The final word on these issues may have to await further explicit calculations.

Marcus, Sagnotti (1985)

The idea that *all* supergravity theories diverge (at three loops) has been widely accepted for over 25 years

R^4 **is expected counterterm**

Novel $N = 8$ Supergravity UV Cancellations

Have constructed a case that correct UV finiteness condition is:

$$\boxed{D < \frac{6}{L} + 4} \qquad (L > 1)$$

UV finite in $D = 4$
Same as $N = 4$ sYM!

D : dimension
L : loop order

Three pillars to our case:

- **Demonstration of *all*-loop order UV cancellations from "no-triangle property".** ZB, Dixon, Roiban
- **Identification of tree-level cancellations responsible for improved UV behavior.** ZB, Carrasco, Ita, Johansson, Forde
- **Explicit 3,4 loop calculations.** ZB, Carrasco, Dixon, Johansson, Kosower, Roiban

Key claim: The most important cancellations are generic to gravity theories. Supersymmetry helps make the theory finite, but is *not* the key ingredient for finiteness.

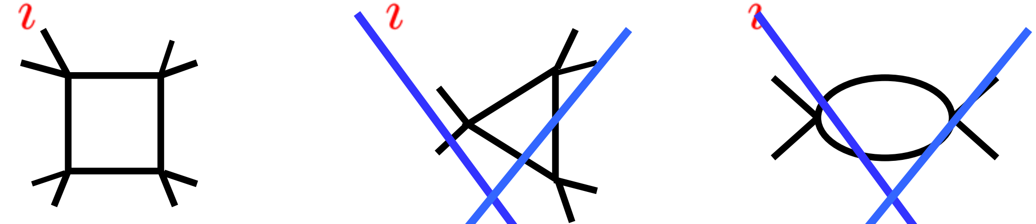
N = 8 Supergravity No-Triangle Property

ZB, Dixon, Perelstein, Rozowsky; ZB, Bjerrum-Bohr and Dunbar; Bjerrum-Bohr, Dunbar, Ita, Perkins, Risager; Proofs by Bjerrum-Bohr and Vanhove; Arkani-Hamed, Cachazo and Kaplan.

One-loop *D* = 4 theorem: Any one loop amplitude is a linear combination of scalar box, triangle and bubble integrals with rational coefficients:

Brown, Feynman; Passarino and Veltman, etc

$$A_n^{\text{1-loop}} = \sum_i d_i I_4^{(i)} + \sum_i c_i I_3^{(i)} + \sum_i b_i I_2^{(i)}$$

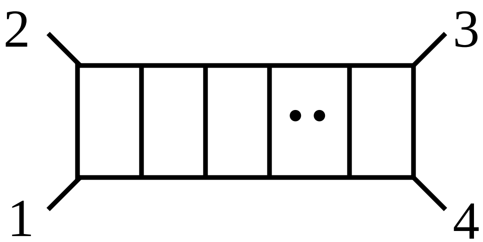


$$\int \frac{d^4p}{(p^2)^4} \qquad \cancel{\int \frac{d^4p}{(p^2)^3}} \qquad \cancel{\int \frac{d^4p}{(p^2)^2}}$$

- **In *N* = 4 Yang-Mills *only box* integrals appear. No triangle integrals and no bubble integrals.**
- **The "no-triangle property" is the statement that same holds in *N* = 8 supergravity. Non-trivial constraint on analytic form of amplitudes.**

$N = 8$ L-Loop UV Cancellations

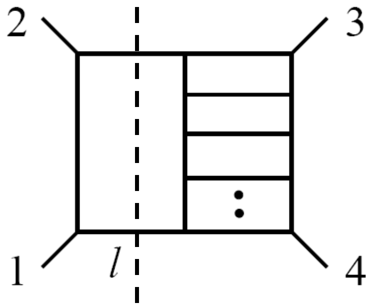
ZB, Dixon, Roiban



$$[(k_1 + k_2)^2]^{2(L-2)}$$

numerator factor

From 2 particle cut:

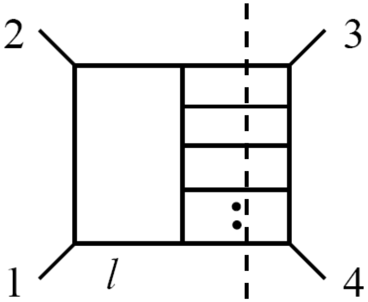


1 in $N = 4$ YM

$$[(l + k_4)^2]^{2(L-2)}$$

numerator factor

L-particle cut



- **Numerator violates one-loop "no-triangle" property.**
- **Too many powers of loop momentum in one-loop subamplitude.**
- **After cancellations behavior is same as in $N = 4$ Yang-Mills!**

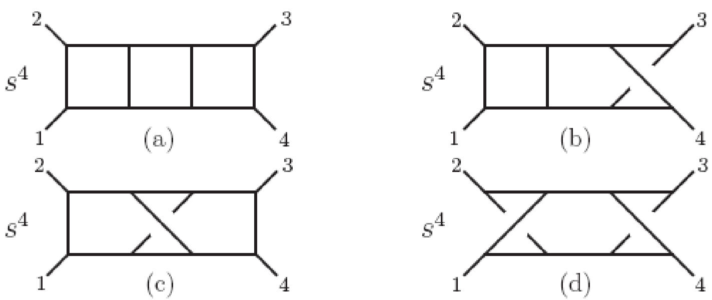
- **UV cancellation exist to *all* loop orders! (not a proof of finiteness)**
- **These *all-loop* cancellations *not* explained by any known supersymmetry arguments.**
- **Existence of these cancellations drive our calculations!**

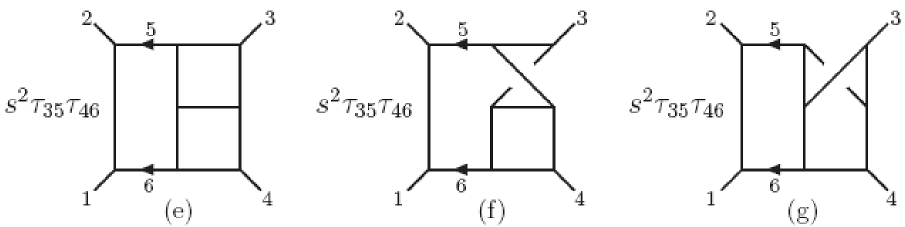
Complete Three-Loop $N = 8$ Supergravity Result

ZB, Carrasco, Dixon, Johansson, Kosower, Roiban; hep-th/0702112
ZB, Carrasco, Dixon, Johansson, Roiban arXiv:0808.4112 [hep-th]

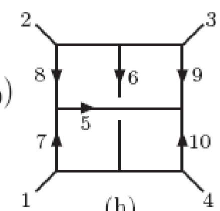
$$M_4^{(3)} = \left(\frac{\kappa}{2}\right)^8 stu M_4^{\rm tree} \sum_{S_3} \left[I^{(a)} + I^{(b)} + \frac{1}{2} I^{(c)} + \frac{1}{4} I^{(d)} + 2 I^{(e)} + 2 I^{(f)} + 4 I^{(g)} + \frac{1}{2} I^{(h)} + 2 I^{(i)} \right]$$

$$\tau_{ij} = 2 k_i \cdot k_j$$





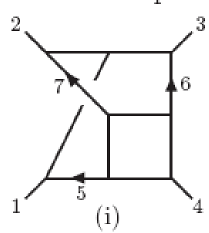
$$(s(\tau_{26} + \tau_{36}) + t(\tau_{15} + \tau_{25}) + st)^2$$
$$+ (s^2(\tau_{26} + \tau_{36}) - t^2(\tau_{15} + \tau_{25}))(\tau_{17} + \tau_{28} + \tau_{39} + \tau_{4,10})$$
$$+ s^2(\tau_{17}\tau_{28} + \tau_{39}\tau_{4,10}) + t^2(\tau_{28}\tau_{39} + \tau_{17}\tau_{4,10})$$
$$+ u^2(\tau_{17}\tau_{39} + \tau_{28}\tau_{4,10})$$



Three loops is not only UV finite it is "superfinite"— cancellations beyond those needed for finiteness in $D = 4$.

Finite for $D < 6$

$$(s\tau_{45} - t\tau_{46})^2 - \tau_{27}(s^2\tau_{45} + t^2\tau_{46})$$
$$- \tau_{15}(s^2\tau_{47} + u^2\tau_{46}) - \tau_{36}(t^2\tau_{47} + u^2\tau_{45})$$
$$+ l_5^2 s^2 t + l_6^2 s t^2 - \frac{1}{3} l_7^2 stu$$



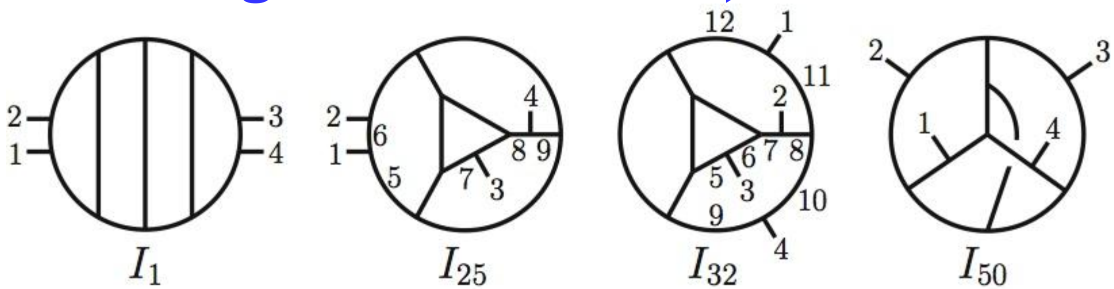
$$\mathrm{UVpole}_{D=6-2\epsilon} = \frac{1}{\epsilon} \frac{5\zeta_3}{(4\pi)^9} \left(\frac{\kappa}{2}\right)^8 (stu)^2 M_4^{\rm tree}$$

Identical power count as $N = 4$ super-Yang-Mills

Four-Loop Amplitude Construction

ZB, Carrasco, Dixon, Johansson, Roiban

Get 50 distinct diagrams or integrals (ones with two- or three-point subdiagrams not needed).



Journal submission has mathematica files with all 50 diagrams

$$M_4^{4\text{-loop}} = \left(\frac{\kappa}{2}\right)^{10} stu M_4^{\text{tree}} \sum_{S_4} \sum_{i=1}^{50} c_i I_i$$

Integral

symmetry factor

leg perms

John Joseph shaved!
UV finite for $D < 5.5$
It is very finite!

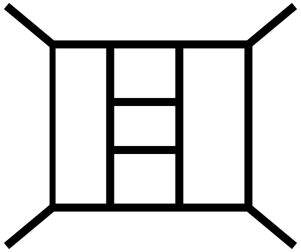
"I'm not shaving until we finish the calculation"
— John Joseph Carrasco

Five Loops is the New Challenge

• **Recent papers argue that susy protection does not extend beyond 7 loops.**

Bossard, Howe, Stelle; Elvang, Freedman, Kiermaier; Green, Russo, Vanhove ; Green and Bjornsson

• **If no other cancellations, this implies a worse behavior at 5 loops than for $\mathcal{N} = 4$ sYM theory. All known potential purely susy explanations exhausted. Testable!**



However, we know that *all-loop* cancellations exist *not* explained by any known susy explanation.

Summary

- **Unitarity method has widespread applications in phenomenology and theoretical studies of gravity and gauge theories.**

- **A new duality conjectured between color and kinematics.**

- **Conjecture that Gravity ~ (gauge theory) x (gauge theory) for diagram numerators to all loop orders when duality is manifest. Three-loop confirmation.**

- ***N* = 8 supergravity has ultraviolet cancellations with no known supersymmetry explanation.**

- **At four points three and four loops, *established* that cancellations are complete and *N* = 8 supergravity has same UV power counting as *N* = 4 super-Yang-Mills theory (which is finite).**

- ***N* = 8 supergravity may well be the first example of a *D* = 4 unitary point-like perturbatively UV finite theory of gravity. Demonstrating this remains a challenge.**

Extra Transparancies

Where is First Potential UV Divergence in *D*= 4 *N* = 8 Sugra?

Various opinions over the years:

3 loops	Conventional superspace power counting	Green, Schwarz, Brink (1982) Howe and Stelle (1989) Marcus and Sagnotti (1985)
5 loops	Partial analysis of unitarity cuts; *If* $\mathcal{N}$ = 6 harmonic superspace exists; algebraic renormalisation argument	Bern, Dixon, Dunbar, Perelstein, Rozowsky (1998) Howe and Stelle (2003,2009)
6 loops	*If* $\mathcal{N}$ = 7 harmonic superspace exists	Howe and Stelle (2003)
7 loops	*If* $\mathcal{N}$ = 8 harmonic superspace exists; lightcone gauge locality arguments; Algebraic renormalization arguments; Field theory pure spinors	Grisaru and Siegel (1982); Howe, Stelle and Bossard (2009) Vanhove; Bjornsson, Green (2010) Kiermaier, Elvang, Freedman(2010) Ramond Kallosh (2010)
8 loops	Explicit identification of potential susy invariant counterterm with full non-linear susy	Kallosh; Howe and Lindström (1981)
9 loops	*Assume* Berkovits' superstring non-renormalization theorems can be carried over to *D*=4 $\mathcal{N}$ = 8 supergravity and extrapolate to 9 loops	Green, Russo, Vanhove (2006) (retracted)

No divergence demonstrated above. Arguments based on lack of susy protection! We will present contrary evidence of all-loop finiteness.

To end debate, we need solid results!

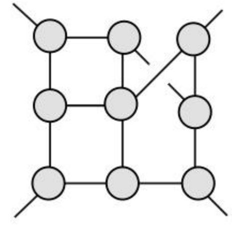
Four-Loop Construction

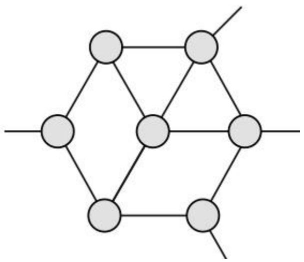
ZB, Carrasco, Dixon, Johansson, Roiban

numerator

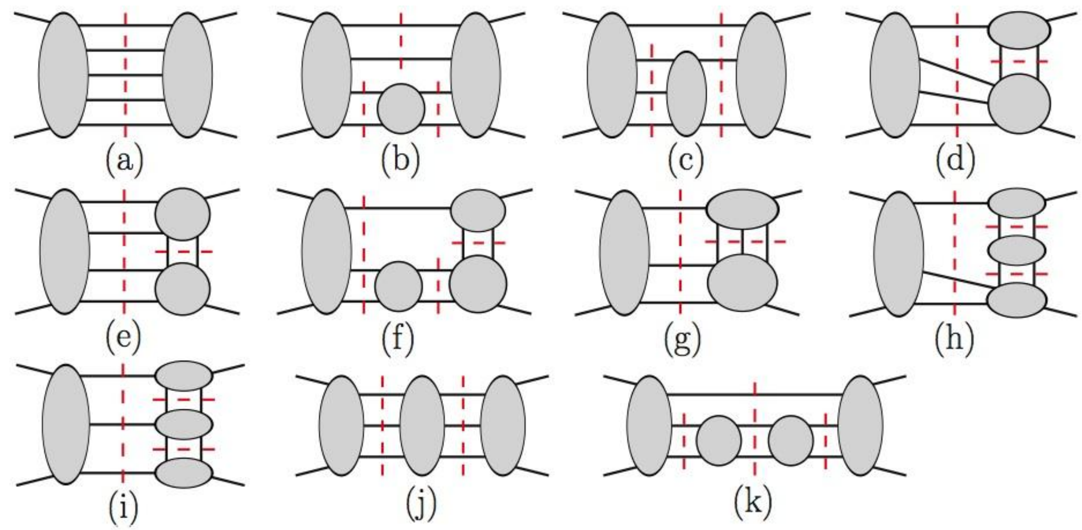
$$I_i = \int d^D l_1 d^D l_2 d^D l_3 d^D l_4 \, \frac{N_i(l_j, k_j)}{l_1^2 l_2^2 l_3^2 l_4^2 l_5^2 l_6^2 l_7^2 l_8^2 l_9^2 l_{10}^2 l_{11}^2 l_{12}^2 l_{13}^2}$$

Determine numerators from 2906 maximal and near maximal cuts





Completeness of expression confirmed using 26 generalized cuts sufficient for obtaining the complete expression



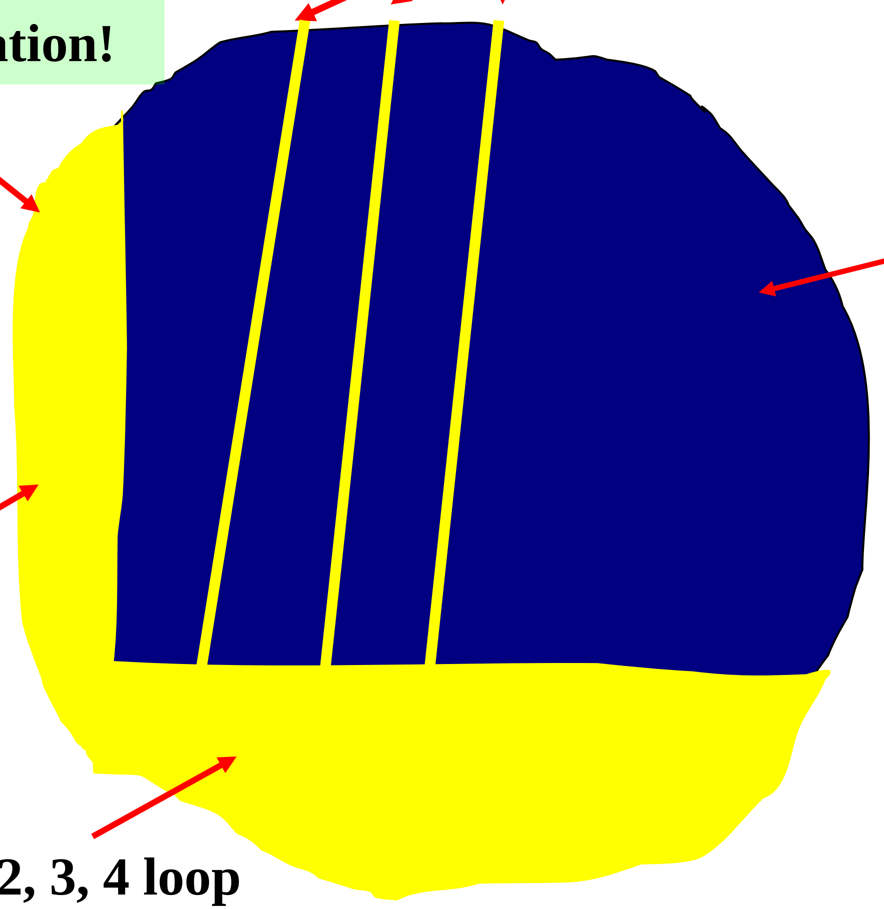
11 most complicated cuts shown

Schematic Illustration of Status

- Same power count as N=4 super-Yang-Mills
- UV behavior unknown

All-loop UV finiteness.
No susy explanation!

from feeding 2, 3 and 4 loop calculations into iterated cuts.



finiteness unproven

loops

No triangle property

explicit 2, 3, 4 loop computations

terms

Through four loops four-point amplitudes of N=8 supergravity are very finite! In at least one non-trivial class of terms this continues to all loop orders.

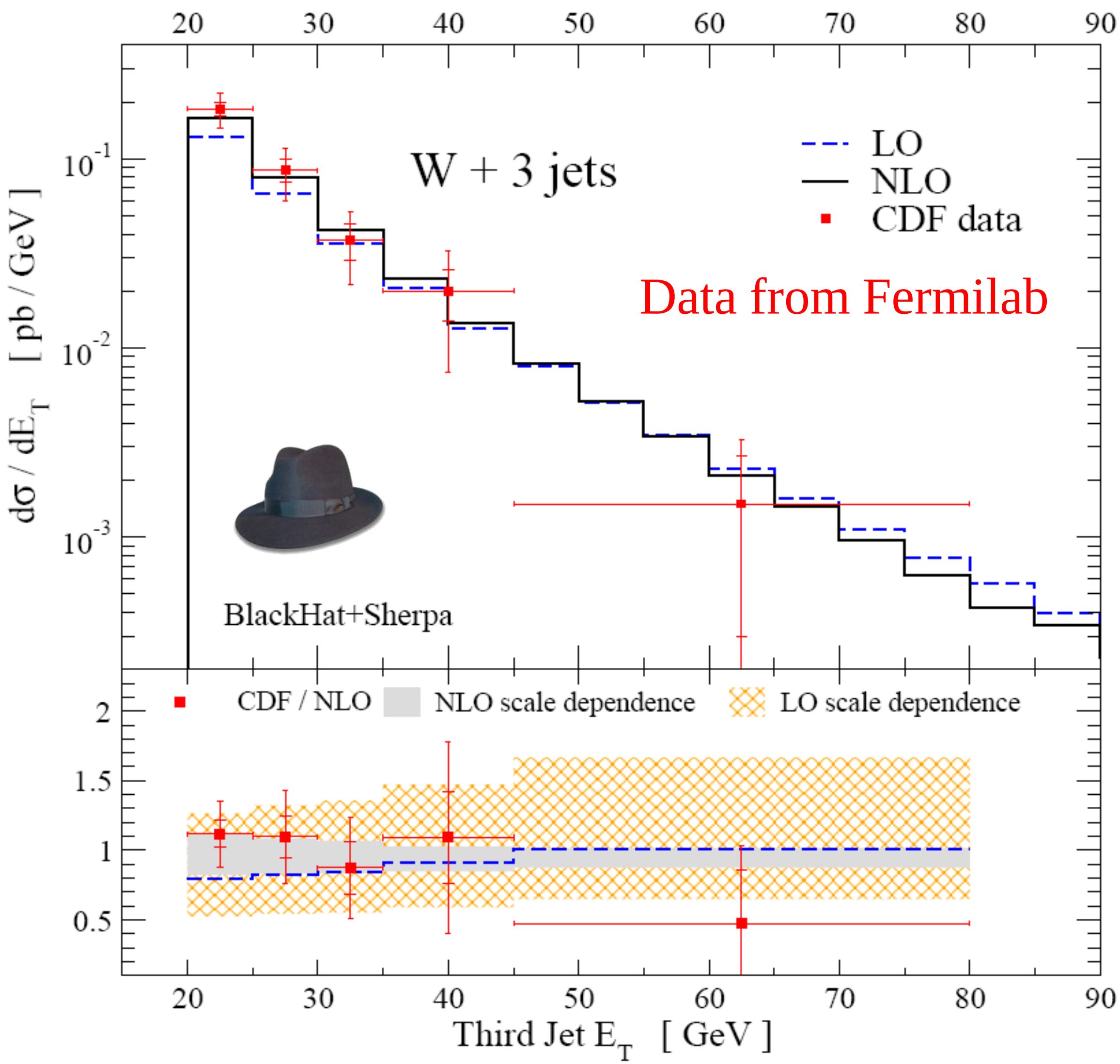
Comments on Consequences of Finiteness

- Suppose $N = 8$ SUGRA is finite to all loop orders. Would this prove that it is a *nonperturbatively* consistent theory of quantum gravity? Of course not!
- At least two reasons to think it needs a nonperturbative completion:
  - — Likely $L!$ or worse growth of the order L coefficients,

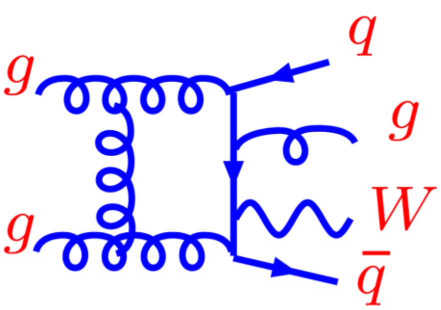
    $$\sim L! \ (s/M_{\mathrm{Pl}}^2)^L$$
  - — Different $E_{7(7)}$ behavior of the perturbative series (invariant!), compared with the $E_{7(7)}$ behavior of the mass spectrum of black holes (non-invariant!)
- Note QED is renormalizable, but its perturbation series has zero radius of convergence in α: $\sim L!\ \alpha^L$. But it has many point-like nonperturbative UV completions —asymptotically free GUTS.

First Useful NLO QCD Calculation of W+3 jets

Berger, ZB, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre (BlackHat collaboration)



BlackHat for one-loop
SHERPA for other parts



Excellent agreement between NLO theory and experiment.

A triumph for on-shell methods!