Supersymmetry

PreSUSY 10

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For details: “A Supersymmetry Primer”, `hep-ph/9709356v5` (revised December 2008).

Good reasons to believe that the next discoveries beyond the presently known Standard Model will involve **supersymmetry (SUSY)**:

- A possible cold dark matter particle
- A light Higgs boson, in agreement with indirect constraints
- More generally, easy agreement with precision electroweak constraints
- Unification of gauge couplings
- Mathematical beauty

However, they are all insignificant compared to the one really good reason to suspect that supersymmetry is real:

- **The Hierarchy Problem**

An analogy: Coulomb self-energy correction to the electron's mass

(H. Murayama, `hep-ph/0002232`)

If the electron is really point-like, the classical electrostatic contribution to its energy is infinite.

Model the electron as a solid sphere of uniform charge density and radius R:

$$\Delta E_{\rm Coulomb} = \frac{3e^2}{20\pi\epsilon_0 R}$$

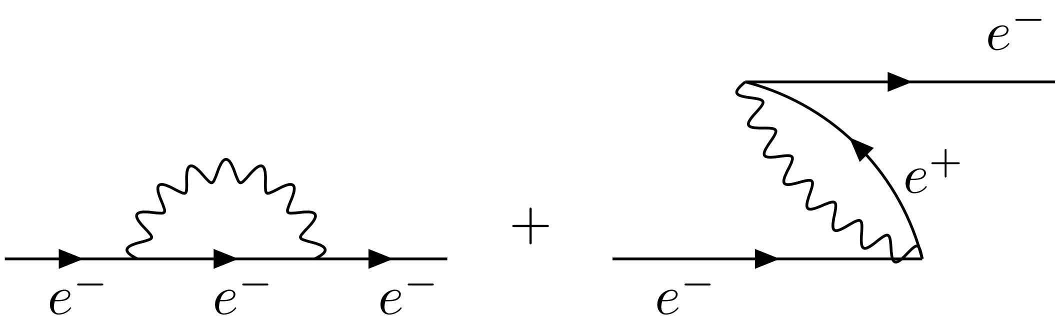
This implies a correction $\Delta m_e = \Delta E_{\rm Coulomb}/c^2$ to the electron mass:

$$m_{e,\rm physical} \;=\; m_{e,\rm bare} + (1\,{\rm MeV}/c^2)\,\left(\frac{0.86\times 10^{-15}\,{\rm meters}}{R}\right).$$

A divergence arises if we try to take $R \longrightarrow 0$. Naively, we might expect $R \gtrsim 10^{-17}$ meters, to avoid having to tune the bare electron mass to better than 1%, for example:

$$0.511\,{\rm MeV}/c^2 \;=\; -100.000\,{\rm MeV}/c^2 \;+\; 100.511\,{\rm MeV}/c^2.$$

However, there is another important quantum mechanical contribution:



The virtual positron effect cancels most of the Coulomb contribution, leaving:

$$m_{e,\text{physical}} = m_{e,\text{bare}} \left[1 + \frac{3\alpha}{4\pi} \ln \left(\frac{\hbar/m_e c}{R} \right) + \ldots \right]$$

with $\hbar/m_e c = 3.9 \times 10^{-13}$ meters. Even if R is as small as the Planck length 1.6×10^{-35} meters, where quantum gravity effects become dominant, this is only a 9% correction.

The existence of a "partner" particle for the electron, the positron, is responsible for eliminating the dangerously huge contribution to its mass.

The “reason” for the positron’s existence can be understood from a **symmetry**, namely the Poincaré invariance of quantum electrodynamics.

If we did not yet know about relativity or the positron, we would have three options:

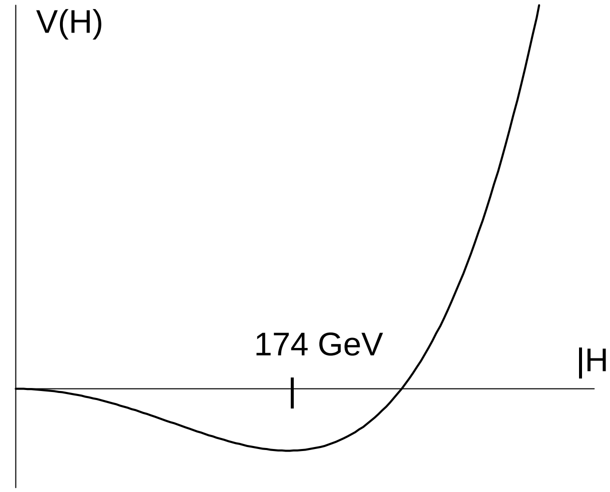
- Assume that the electron is not point-like, and has structure at a measurable size $R \gtrsim 10^{-17}$ meters.
- Assume that the electron is (nearly?) point-like, and there is a mysterious fine-tuning between the bare mass and the Coulomb correction to it.
- Predict that the electron’s symmetry “partner”, the positron, must exist.

Today we know that the last option is the correct explanation.

The Hierarchy Problem

Potential for H, the complex scalar field that is the electrically neutral part of the Standard Model Higgs field, is:

$$V(H) = m_H^2 |H|^2 + \frac{\lambda}{2} |H|^4$$



For electroweak symmetry breaking to give the experimental m_Z, we need:

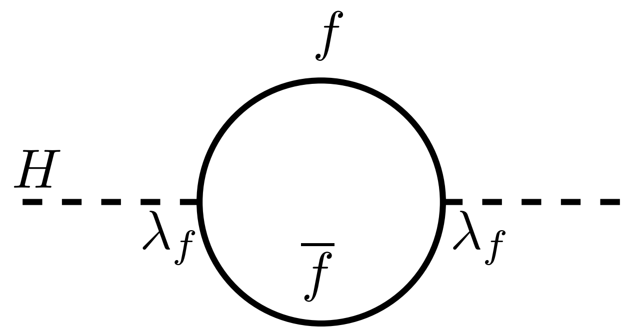
$$\langle H \rangle = \sqrt{-m_H^2/\lambda} \approx 174 \text{ GeV}$$

The requirement of unitarity in the scattering of Higgs bosons and longitudinal W bosons tells us that λ is not much larger than 1. Therefore,

$$-(\text{few hundred GeV})^2 \lesssim m_H^2 < 0.$$

However, this appears fine-tuned (in other words, incredibly and mysteriously lucky!) when we consider the likely size of quantum corrections to m_H^2.

Contributions to m_H^2 from a Dirac fermion loop:



The correction to the Higgs squared mass parameter from this loop diagram is:

$$\Delta m_H^2 = \frac{\lambda_f^2}{16\pi^2}\left[-2M_{\rm UV}^2 + 6m_f^2 \ln\left(M_{\rm UV}/m_f\right) + \ldots\right]$$

where λ_f is the coupling of the fermion to the Higgs field H.

$M_{\rm UV}$ should be interpreted as the ultraviolet cutoff scale(s) at which new physics enters to cut off the loop integrations.

So m_H^2 is sensitive to the **largest** mass scales in the theory.

For example, maybe String Theory is responsible for modifying the high energy behavior of physics, making the theory finite. Compared to field theory, string theory modifies the Feynman integrations over Euclidean momenta:

$$\int d^4p\,[\ldots] \;\rightarrow\; \int d^4p\, e^{-p^2/M_{\rm string}^2}\,[\ldots]$$

Using this, one obtains from each Dirac fermion one-loop diagram:

$$\Delta m_H^2 \;\sim\; -\frac{\lambda_f^2}{8\pi^2} M_{\rm string}^2 + \ldots$$

A typical guess is that $M_{\rm string}$ is comparable to $M_{\rm Planck} \approx 2.4 \times 10^{18}$ GeV.

These huge corrections make it difficult to explain how m_H^2 could be so small.

The Hierarchy Problem

Why should:

$$\frac{|m_H^2|}{M_{\rm Planck}^2} \lesssim 10^{-32}$$

if individual radiative corrections Δm_H^2 are of order $M_{\rm Planck}^2$ or $M_{\rm string}^2$, multiplied by loop factors?

The problem is present even if String Theory is wrong and some other unspecified effects modify physics at $M_{\rm Planck}$, or any other very large mass scale, to make the loop integrals converge.

An incredible coincidence seems to be required to make the corrections to the Higgs squared mass cancel to give a much smaller number.

The systematic cancellation of loop corrections to the Higgs mass squared requires the type of conspiracy that is better known to physicists as a **symmetry**.

Fermion loops and boson loops gave contributions with opposite signs:

$$\Delta m_H^2 = -\frac{\lambda_f^2}{16\pi^2}(2M_{\rm UV}^2) + \ldots \qquad \text{(Dirac fermion)}$$

$$\Delta m_H^2 = +\frac{\lambda_S}{16\pi^2}M_{\rm UV}^2 + \ldots \qquad \text{(complex scalar)}$$

SUPERSYMMETRY, a symmetry between fermions and bosons, makes the cancellation not only possible, but automatic.

There are two complex scalars for every Dirac fermion, and $\lambda_S = \lambda_f^2$.

Supersymmetry

A SUSY transformation turns a boson state into a fermion state, and vice versa. So the operator Q that generates such transformations acts, schematically, like:

$$Q|\text{Boson}\rangle = |\text{Fermion}\rangle; \qquad\qquad Q|\text{Fermion}\rangle = |\text{Boson}\rangle$$

This means that Q must be an anticommuting spinor. This is an intrinsically complex object, so $Q^\dagger$ is also a distinct symmetry generator:

$$Q^\dagger|\text{Boson}\rangle = |\text{Fermion}\rangle; \qquad\qquad Q^\dagger|\text{Fermion}\rangle = |\text{Boson}\rangle$$

The possible forms for such theories are highly restricted by the Haag-Lopuszanski-Sohnius extension of the Coleman-Mandula Theorem. In a 4-dimensional theory with chiral fermions (like the Standard Model) and non-trivial scattering, then Q carries spin-$1/2$ with L helicity, and $Q^\dagger$ has spin-$1/2$ with R helicity, and they must satisfy. . .

The Supersymmetry Algebra

$$\begin{aligned}\{Q, Q^\dagger\} &= P^\mu \\ \{Q, Q\} &= \{Q^\dagger, Q^\dagger\} = 0 \\ [P^\mu, Q] &= [P^\mu, Q^\dagger] = 0 \\ [T^a, Q] &= [T^a, Q^\dagger] = 0\end{aligned}$$

Here $P^\mu = (H, \vec{\mathbf{P}})$ is the generator of spacetime translations, and T^a are the gauge generators. (This is schematic, with spinor indices suppressed for now. We will restore them later.)

The single-particle states of the theory fall into irreducible representations of this algebra, called **supermultiplets**. Fermion and boson members of a given supermultiplet are **superpartners** of each other. By definition, if $|\Omega\rangle$ and $|\Omega'\rangle$ are superpartners, then $|\Omega'\rangle$ is equal to some combination of $Q, Q^\dagger$ acting on $|\Omega\rangle$.

Therefore, since P^2 and T^a commute with $Q, Q^\dagger$, all members of a given supermultiplet must have the same (mass)^2 and gauge quantum numbers.

Types of supermultiplets

Chiral (or "Scalar" or "Matter" or "Wess-Zumino") supermultiplet:

1 two-component Weyl fermion, helicity $\pm\frac{1}{2}$. ($n_F = 2$)

2 real spin-0 scalars = 1 complex scalar. ($n_B = 2$)

The Standard Model quarks, leptons and Higgs bosons must fit into these.

Gauge (or "Vector") supermultiplet:

1 two-component Weyl fermion gaugino, helicity $\pm\frac{1}{2}$. ($n_F = 2$)

1 real spin-1 massless gauge vector boson. ($n_B = 2$)

The Standard Model $\gamma, Z, W^{\pm}, g$ must fit into these.

Gravitational supermultiplet:

1 two-component Weyl fermion gravitino, helicity $\pm\frac{3}{2}$. ($n_F = 2$)

1 real spin-2 massless graviton. ($n_B = 2$)

How do the Standard Model quarks and leptons fit in?

Each quark or charged lepton is 1 Dirac = 2 Weyl fermions

Electron: $\Psi_e = \begin{pmatrix} e_L \\ e_R \end{pmatrix}$ $\begin{matrix} \longleftarrow \text{two-component Weyl LH fermion} \\ \longleftarrow \text{two-component Weyl RH fermion} \end{matrix}$

Each of e_L and e_R is part of a chiral supermultiplet, so each has a complex, spin-0 superpartner, called $\widetilde{e}_L$ and $\widetilde{e}_R$ respectively. They are called the "left-handed selectron" and "right-handed selectron", although they carry no spin.

The conjugate of a right-handed Weyl spinor is a left-handed Weyl spinor. Define two-component left-handed Weyl fields: $e \equiv e_L$ and $\bar{e} \equiv e_R^\dagger$. So, there are two left-handed chiral supermultiplets for the electron:

$$(e,\ \widetilde{e}_L) \quad \text{and} \quad (\bar{e},\ \widetilde{e}_R^*).$$

The other charged leptons and quarks are similar. We do not need ν_R in the Standard Model, so there is only one neutrino chiral supermultiplet for each family:

$$(\nu_e,\ \widetilde{\nu}_e).$$

Chiral supermultiplets of the Minimal Supersymmetric Standard Model (MSSM):

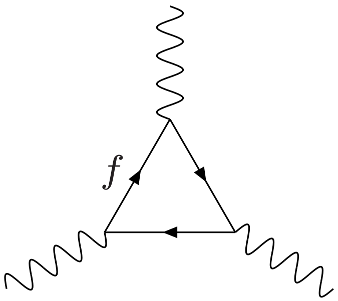
Names		spin 0	spin 1/2	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
squarks, quarks	Q	$(\widetilde{u}_L\ \ \widetilde{d}_L)$	$(u_L\ \ d_L)$	$(\,\mathbf{3},\ \mathbf{2}\,,\ \frac{1}{6})$
($\times 3$ families)	$\bar{u}$	$\widetilde{u}_R^*$	$u_R^\dagger$	$(\,\overline{\mathbf{3}},\ \mathbf{1},\ -\frac{2}{3})$
	$\bar{d}$	$\widetilde{d}_R^*$	$d_R^\dagger$	$(\,\overline{\mathbf{3}},\ \mathbf{1},\ \frac{1}{3})$
sleptons, leptons	L	$(\widetilde{\nu}\ \ \widetilde{e}_L)$	$(\nu\ \ e_L)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$
($\times 3$ families)	$\bar{e}$	$\widetilde{e}_R^*$	$e_R^\dagger$	$(\,\mathbf{1},\ \mathbf{1},\ 1)$
Higgs, higgsinos	H_u	$(H_u^+\ \ H_u^0)$	$(\widetilde{H}_u^+\ \ \widetilde{H}_u^0)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ +\frac{1}{2})$
	H_d	$(H_d^0\ \ H_d^-)$	$(\widetilde{H}_d^0\ \ \widetilde{H}_d^-)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$

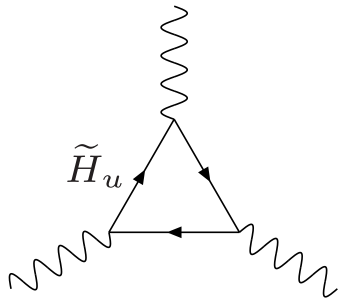
The superpartners of the Standard Model particles are written with a $\widetilde{\ }$. The scalar names are obtained by putting an "s" in front, so they are generically called **squarks** and **sleptons**, short for "scalar quark" and "scalar lepton".

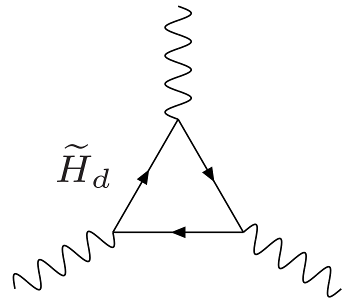
The Standard Model Higgs boson requires two different chiral supermultiplets, H_u and H_d. The fermionic partners of the Higgs scalar fields are called **higgsinos**. There are two charged and two neutral Weyl fermion higgsino degrees of freedom.

Why do we need two Higgs supermultiplets? Two reasons:

1) Anomaly Cancellation







$$\sum_{\text{SM fermions}} Y_f^3 = 0 \qquad + \; 2\left(\frac{1}{2}\right)^3 \qquad + \; 2\left(-\frac{1}{2}\right)^3 \quad = \quad 0$$

This anomaly cancellation occurs if and only if **both** $\widetilde{H}_u$ and $\widetilde{H}_d$ higgsinos are present. Otherwise, the electroweak gauge symmetry would not be allowed!

2) Quark and Lepton masses

Only the H_u Higgs scalar can give masses to charge $+2/3$ quarks (top).

Only the H_d Higgs scalar can give masses to charge $-1/3$ quarks (bottom) and the charged leptons. We will show this later.

The vector bosons of the Standard Model live in gauge supermultiplets:

Names	spin 1/2	spin 1	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	$(\ \mathbf{8},\ \mathbf{1}\ ,\ 0)$
winos, W bosons	$\widetilde{W}^{\pm}\quad \widetilde{W}^{0}$	$W^{\pm}\quad W^{0}$	$(\ \mathbf{1},\ \mathbf{3}\ ,\ 0)$
bino, B boson	$\widetilde{B}^{0}$	B^{0}	$(\ \mathbf{1},\ \mathbf{1}\ ,\ 0)$

The spin-$1/2$ **gauginos** transform as the adjoint representation of the gauge group. Each gaugino carries a $\widetilde{\ }$. The color-octet superpartner of the gluon is called the **gluino**. The $SU(2)_L$ gauginos are called **winos**, and the $U(1)_Y$ gaugino is called the **bino**.

However, the winos and the bino are not mass eigenstate particles; they mix with each other and with the higgsinos of the same charge.

Recall that if supersymmetry were an exact symmetry, then superpartners would have to be exactly degenerate with each other. For example,

$$m_{\tilde{e}_L} = m_{\tilde{e}_R} = m_e = 0.511 \text{ MeV}$$

$$m_{\tilde{u}_L} = m_{\tilde{u}_R} = m_u$$

$$m_{\tilde{g}} = m_{\text{gluon}} = 0 + \text{QCD-scale effects}$$

etc.

But new particles with these properties have been ruled out long ago, so:

Supersymmetry must be broken in the vacuum state chosen by Nature.

Supersymmetry is thought to be spontaneously broken and therefore hidden, the same way that the full electroweak symmetry $SU(2)_L \times U(1)_Y$ is hidden from very low-energy experiments.

For a clue as to the nature of SUSY breaking, return to our motivation in the Hierarchy Problem. The Higgs mass parameter gets corrections from each chiral supermultiplet:

$$\Delta m_H^2 = \frac{1}{16\pi^2}(\lambda_S - \lambda_F^2)M_{\rm UV}^2 + \ldots$$

The corresponding formula for Higgsinos has no term proportional to $M_{\rm UV}^2$; fermion masses always diverge at worst like $\ln(M_{\rm UV})$. Therefore, if supersymmetry were exact and unbroken, it must be that:

$$\lambda_S = \lambda_F^2,$$

in other words, the dimensionless (scalar)^4 couplings are the squares of the (scalar)-(fermion)-(antifermion) couplings.

If we want SUSY to be a solution to the hierarchy problem, we must demand that this is still true even after SUSY is broken:

The breaking of supersymmetry must be "soft". This means that it does not change the dimensionless terms in the Lagrangian.

The effective Lagrangian of the MSSM is therefore:

$$\mathcal{L} = \mathcal{L}_{\rm SUSY} + \mathcal{L}_{\rm soft}$$

- $\mathcal{L}_{\rm SUSY}$ contains all of the gauge, Yukawa, and dimensionless scalar couplings, and preserves exact supersymmetry
- $\mathcal{L}_{\rm soft}$ violates supersymmetry, and contains only mass terms and couplings with *positive* mass dimension.

If $m_{\rm soft}$ is the largest mass scale in $\mathcal{L}_{\rm soft}$, then by dimensional analysis,

$$\Delta m_H^2 = m_{\rm soft}^2 \left[\frac{\lambda}{16\pi^2} \ln(M_{\rm UV}/m_{\rm soft}) + \ldots \right],$$

where λ stands for dimensionless couplings. This is because Δm_H^2 must vanish in the limit $m_{\rm soft} \rightarrow 0$, in which SUSY is restored. Therefore, we expect that $m_{\rm soft}$ should not be much larger than roughly 1000 GeV.

This is the best reason to be optimistic that SUSY will be discovered at the Fermilab Tevatron or the CERN Large Hadron Collider.

Without further justification, soft SUSY breaking might seem like a rather arbitrary requirement. Fortunately, it arises naturally from the spontaneous breaking of theories with exact SUSY.

Is there any good reason why the superpartners of the Standard Model particles should be heavy enough to have avoided discovery so far? Yes!

- All of the particles in the MSSM that have been discovered as of 1995 (quarks, leptons, gauge bosons) would be exactly massless if the electroweak symmetry were not broken. So their masses are expected to be at most of order $v = 174$ GeV, the electroweak breaking scale. **They are required to be light.**

- All of the particles in the MSSM that have **not** yet been discovered as of 2010 (squarks, sleptons, gauginos, Higgsinos, Higgs scalars) can get a mass even without electroweak symmetry breaking. **They are not required to be light.**

The simplest SUSY model: a free chiral supermultiplet

The minimum particle content for a SUSY theory is a complex scalar ϕ and its superpartner fermion ψ. We must at least have kinetic terms for each, so:

$$S = \int d^4x \, (\mathcal{L}_{\rm scalar} + \mathcal{L}_{\rm fermion})$$

$$\mathcal{L}_{\rm scalar} = -\partial^\mu \phi^* \partial_\mu \phi \qquad\qquad \mathcal{L}_{\rm fermion} = -i\psi^\dagger \overline{\sigma}^\mu \partial_\mu \psi$$

A SUSY transformation should turn ϕ into ψ, so try:

$$\delta\phi = \epsilon\psi; \qquad\qquad \delta\phi^* = \epsilon^\dagger \psi^\dagger$$

where $\epsilon =$ infinitesimal, anticommuting, constant spinor, with dimension [mass]$^{-1/2}$, that parameterizes the SUSY transformation. Then we find:

$$\delta\mathcal{L}_{\rm scalar} = -\epsilon\partial^\mu\psi\partial_\mu\phi^* - \epsilon^\dagger\partial^\mu\psi^\dagger\partial_\mu\phi.$$

We would like for this to be canceled by an appropriate SUSY transformation of the fermion field. . .

To have any chance, $\delta\psi$ should be linear in $\epsilon^\dagger$ and in ϕ, and must contain one spacetime derivative. There is only one possibility, up to a multiplicative constant:

$$\delta\psi_\alpha = i(\sigma^\mu\epsilon^\dagger)_\alpha\partial_\mu\phi; \qquad\qquad \delta\psi^\dagger_{\dot\alpha} = -i(\epsilon\sigma^\mu)_{\dot\alpha}\partial_\mu\phi^*$$

With this guess, one finds:

$$\delta\mathcal{L}_{\text{fermion}} \;=\; -\delta\mathcal{L}_{\text{scalar}} + \big(\text{total derivative}\big)$$

so the action S is indeed invariant under the SUSY transformation, justifying the guess of the multiplicative factor. This is called the free Wess-Zumino model.

Furthermore, if we take the commutator of two SUSY transformations:

$$\delta_{\epsilon_2}(\delta_{\epsilon_1}\phi) - \delta_{\epsilon_1}(\delta_{\epsilon_2}\phi) \;=\; i(\epsilon_1\sigma^\mu\epsilon_2 - \epsilon_2\sigma^\mu\epsilon_1)\partial_\mu\phi$$

Since ∂_μ corresponds to the spacetime 4-momentum P_μ, this has exactly the form demanded by the SUSY algebra discussed earlier. (More on this soon.)

The fact that two SUSY transformations give back another symmetry (namely a spacetime translation) means that the SUSY algebra "closes".

If we do the same check for the fermion ψ:

$$\begin{aligned}\delta_{\epsilon_2}(\delta_{\epsilon_1}\psi_\alpha) - \delta_{\epsilon_1}(\delta_{\epsilon_2}\psi_\alpha) \quad &= \quad i(\epsilon_1\sigma^\mu\epsilon_2 - \epsilon_2\sigma^\mu\epsilon_1)\partial_\mu\psi_\alpha \\ &\quad -i\epsilon_{1\alpha}(\epsilon_2^\dagger\overline{\sigma}^\mu\partial_\mu\psi) + i\epsilon_{2\alpha}(\epsilon_1^\dagger\overline{\sigma}^\mu\partial_\mu\psi)\end{aligned}$$

The first line is expected, but the second line only vanishes on-shell (when the classical equation of motion $\overline{\sigma}^\mu\partial_\mu\psi = 0$ is satisfied). This seems like a problem, since we want SUSY to be a valid symmetry of the quantum theory (off-shell)!

To show that there is no problem, we introduce another bosonic spin-0 field, F, called an auxiliary field. Its Lagrangian density is:

$$\mathcal{L}_{\text{aux}} = F^*F$$

Note that F has no kinetic term, and has dimensions $[\text{mass}]^2$, unlike an ordinary scalar field. It has the not-very-exciting equations of motion $F = F^* = 0$.

The auxiliary field F does not affect the dynamics, classically or in the quantum theory. But it does appear in modified SUSY transformation laws:

$$\begin{aligned} \delta\phi &= \epsilon\psi \\ \delta\psi_\alpha &= i(\sigma^\mu\epsilon^\dagger)_\alpha\partial_\mu\phi + \epsilon_\alpha F \\ \delta F &= i\epsilon^\dagger\overline{\sigma}^\mu\partial_\mu\psi \end{aligned}$$

Now the total Lagrangian

$$\mathcal{L} = -\partial^\mu\phi^*\partial_\mu\phi - i\psi^\dagger\overline{\sigma}^\mu\partial_\mu\psi + F^*F$$

is still invariant, and also one can now check:

$$\delta_{\epsilon_2}(\delta_{\epsilon_1}X) - \delta_{\epsilon_1}(\delta_{\epsilon_2}X) \;=\; i(\epsilon_1\sigma^\mu\epsilon_2 - \epsilon_2\sigma^\mu\epsilon_1)\partial_\mu X$$

for each of $X = \phi, \phi^*, \psi, \psi^\dagger, F, F^*$, without using equations of motion.

So in the “modified” theory, SUSY does close off-shell as well as on-shell.

The auxiliary field F is really just a book-keeping device to make this simple. We can see why it is needed by considering the number of degrees of freedom on-shell (classically) and off-shell (quantum mechanically):

	ϕ	ψ	F
on-shell ($n_B = n_F = 2$)	2	2	0
off-shell ($n_B = n_F = 4$)	2	4	2

(Going on-shell eliminates half of the propagating degrees of freedom of the fermion, because the Lagrangian density is linear in time derivatives, so that the fermionic canonical momenta are not independent phase-space variables.)

The auxiliary field also plays an important role when we add interactions to the theory, and in gaining a simple understanding of SUSY breaking.

Noether's Theorem tells us that for every symmetry, there is a conserved current, and SUSY is not an exception. The **supercurrent** J^μ_α is an anti-commuting 4-vector that also carries a spinor index.

By the usual Noether procedure, one finds for the supercurrent (and its conjugate $J^\dagger$), in terms of the variations of the fields δX for $X = (\phi, \phi^*, \psi, \psi^\dagger, F, F^*)$:

$$\epsilon J^\mu + \epsilon^\dagger J^{\dagger\mu} \quad \equiv \quad \sum_X \delta X \, \frac{\delta \mathcal{L}}{\delta(\partial_\mu X)} - K^\mu,$$

where K^μ satisfies $\delta\mathcal{L} = \partial_\mu K^\mu$. One finds:

$$J^\mu_\alpha = (\sigma^\nu \overline{\sigma}^\mu \psi)_\alpha \, \partial_\nu \phi^*; \qquad\qquad J^{\dagger\mu}_{\dot\alpha} = (\psi^\dagger \overline{\sigma}^\mu \sigma^\nu)_{\dot\alpha} \, \partial_\nu \phi.$$

The supercurrent and its hermitian conjugate are separately conserved:

$$\partial_\mu J^\mu_\alpha = 0; \qquad\qquad \partial_\mu J^{\dagger\mu}_{\dot\alpha} = 0,$$

as can be verified by use of the equations of motion.

From the conserved supercurrents one can construct the conserved charges:

$$Q_\alpha = \sqrt{2} \int d^3x \, J^0_\alpha; \qquad\qquad Q^\dagger_{\dot\alpha} = \sqrt{2} \int d^3x \, J^{\dagger 0}_{\dot\alpha},$$

As quantum mechanical operators, they satisfy:

$$\left[\epsilon Q + \epsilon^\dagger Q^\dagger, X\right] = -i\sqrt{2}\, \delta X$$

for any field X. Let us also introduce the 4-momentum operator $P^\mu = (H, \vec{P})$, which satisfies:

$$[P_\mu, X] = i\partial_\mu X.$$

Now by using the canonical commutation relations of the fields, one finds:

$$\begin{aligned} \left[\epsilon_2 Q + \epsilon_2^\dagger Q^\dagger,\, \epsilon_1 Q + \epsilon_1^\dagger Q^\dagger\right] &= 2(\epsilon_2 \sigma_\mu \epsilon_1^\dagger - \epsilon_1 \sigma_\mu \epsilon_2^\dagger)\, P^\mu \\ \left[\epsilon Q + \epsilon^\dagger Q^\dagger,\, P\right] &= 0 \end{aligned}$$

This implies. . .

The SUSY Algebra

$$\{Q_\alpha, Q^\dagger_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha} P_\mu,$$
$$\{Q_\alpha, Q_\beta\} = \{Q^\dagger_{\dot\alpha}, Q^\dagger_{\dot\beta}\} = 0$$
$$[Q_\alpha, P^\mu] = [Q^\dagger_{\dot\alpha}, P^\mu] = 0$$

This time in non-schematic form, with the spinor indices and the factors of 2 in their proper places.

(The commutators turned into anti-commutators in the first two, when we extracted the anti-commutating spinors ϵ_1, ϵ_2.)

Masses and Interactions for Chiral Supermultiplets

The Lagrangian describing a collection of free, massless, chiral supermultiplets is

$$\mathcal{L} = -\partial^\mu \phi^{*i} \partial_\mu \phi_i - i\psi^{\dagger i} \overline{\sigma}^\mu \partial_\mu \psi_i + F^{*i} F_i.$$

Question: How do we make mass terms and interactions for these fields, while still preserving supersymmetry invariance?

Answer: choose a **superpotential**,

$$W = \tfrac{1}{2} M^{ij} \phi_i \phi_j + \tfrac{1}{6} y^{ijk} \phi_i \phi_j \phi_k.$$

It cannot depend on ϕ^{*i}, only the ϕ_i. It must be an analytic function of the scalar fields treated as complex variables.

The superpotential W contains masses M^{ij} and couplings y^{ijk}, which must be symmetric under interchange of i, j, k.

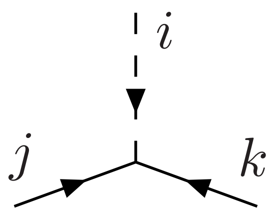
Supersymmetry is very restrictive; you cannot just do anything you want!

The superpotential $W = M^{ij}\phi_i\phi_j + y^{ijk}\phi_i\phi_j\phi_k$ determines all non-gauge masses and interactions.

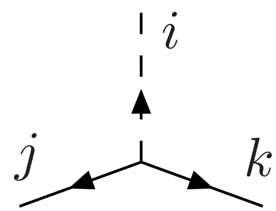
Both scalars and fermions have squared mass matrix $M_{ik}M^{kj}$.

The interaction Feynman rules for the chiral supermultiplets are:

Yukawa interactions:

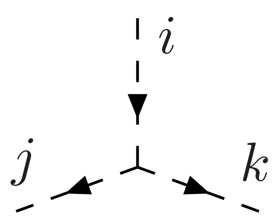


$-iy^{ijk}$

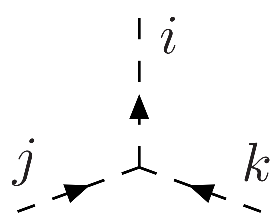


$-iy_{ijk}$

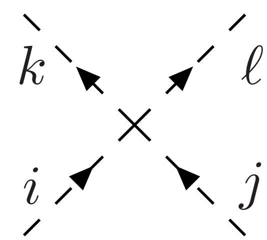
Scalar interactions:



$-iM^{in}y_{njk}$



$-iM_{in}y^{njk}$



$-iy^{ijn}y_{k\ell n}$

Supersymmetric Gauge Theories

A gauge or vector supermultiplet contains physical fields:

- a gauge boson A_μ^a
- a gaugino λ_α^a.

The index a runs over the gauge group generators $[1, 2, \ldots, 8$ for $SU(3)_C$; $1, 2, 3$ for $SU(2)_L$; 1 for $U(1)_Y]$.

Suppose the gauge coupling constant is g and the structure constants of the group are f^{abc}. The Lagrangian for the gauge supermultiplet is:

$$\mathcal{L} = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a - i\lambda^{\dagger a} \overline{\sigma}^\mu \nabla_\mu \lambda^a + \frac{1}{2} D^a D^a$$

where D^a is a real spin-0 auxiliary field with no kinetic term, and

$$\nabla_\mu \lambda^a \equiv \partial_\mu \lambda^a - g f^{abc} A_\mu^b \lambda^c$$

The auxiliary field D^a is again needed so that the SUSY algebra closes on-shell. Counting fermion and boson degrees of freedom on-shell and off-shell:

	A_μ	λ	D
on-shell $(n_B = n_F = 2)$	2	2	0
off-shell $(n_B = n_F = 4)$	3	4	1

To make a gauge-invariant supersymmetric Lagrangian involving both gauge and chiral supermultiplets, one must turn the ordinary derivatives into covariant ones:

$$\partial_\mu \phi_i \quad \rightarrow \quad \nabla_\mu \phi_i = \partial_\mu \phi_i + igA^a_\mu (T^a \phi)_i$$
$$\partial_\mu \psi_i \quad \rightarrow \quad \nabla_\mu \psi_i = \partial_\mu \psi_i + igA^a_\mu (T^a \psi)_i$$

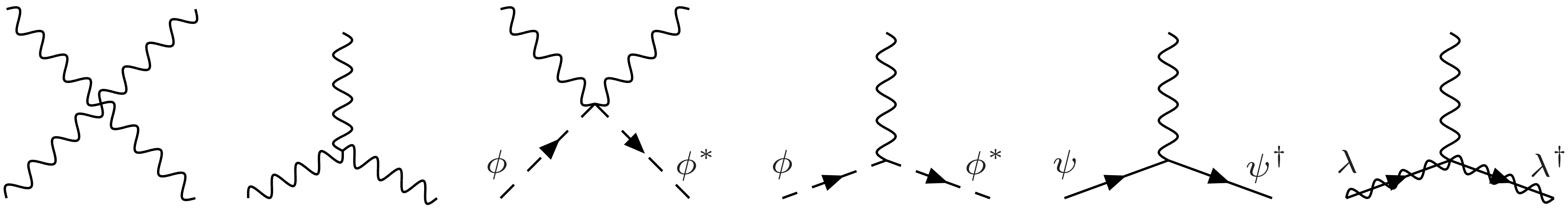
One must also add three new terms to the Lagrangian:

$$\mathcal{L} \;=\; \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{chiral}} - \sqrt{2} g(\phi^* T^a \psi)\lambda^a - \sqrt{2} g \lambda^{\dagger a}(\psi^\dagger T^a \phi)$$
$$+g(\phi^* T^a \phi) D^a .$$

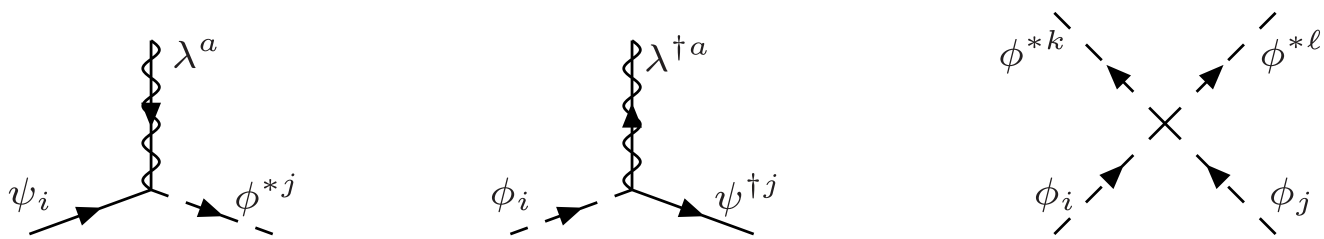
You can check (after some algebra) that this full Lagrangian is now invariant under both SUSY transformations and gauge transformations.

Supersymmetric gauge interactions

The following interactions are dictated by ordinary gauge invariance alone:



SUSY also predicts interactions that have gauge coupling strength, but are not gauge interactions in the usual sense:



$-i\sqrt{2}g_a(T^a)_i{}^j$ $\quad$ $-i\sqrt{2}g_a(T^a)_i{}^j$ $\quad$ $-ig_a^2(T_i^{ak}T_j^{a\ell}+T_i^{a\ell}T_j^{ak})$

These interactions are entirely determined by supersymmetry and the gauge group. Experimental measurements of the magnitudes of these couplings will provide an important test that we really have SUSY.

Soft SUSY-breaking Lagrangians

It has been shown that the quadratic sensitivity to $M_{\rm UV}$ is still absent in SUSY theories with these SUSY-breaking terms added in:

$$\begin{aligned} \mathcal{L}_{\rm soft} &= -\tfrac{1}{2}\left(M_a\,\lambda^a\lambda^a + \text{c.c.}\right) - (m^2)^i_j\phi^{*j}\phi_i \\ &\quad -\Big(\tfrac{1}{2}b^{ij}\phi_i\phi_j + \tfrac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \text{c.c.}\Big), \end{aligned}$$

They consist of:

- gaugino masses M_a,
- scalar (mass)^2 terms $(m^2)^j_i$ and b^{ij},
- (scalar)^3 couplings a^{ijk}

How to build a SUSY Model:

- Choose a gauge symmetry group.

  (In the MSSM, this is already done: $SU(3)_C \times SU(2)_L \times U(1)_Y$.)

- Choose a superpotential W; must be invariant under the gauge symmetry.

  (In the MSSM, this is almost already done: Yukawa couplings are dictated by the observed fermion masses.)

- Choose a soft SUSY-breaking Lagrangian, or else choose a method for spontaneous SUSY breakdown.

  (This is where almost all of the unknowns and arbitrariness in the MSSM are.)

Let us do this for the MSSM now, and then explore the consequences.

The Superpotential for the Minimal SUSY Standard Model:

$$W_{\rm MSSM} = \tilde{\bar{u}}\mathbf{y_u}\tilde{Q}H_u - \tilde{\bar{d}}\mathbf{y_d}\tilde{Q}H_d - \tilde{\bar{e}}\mathbf{y_e}\tilde{L}H_d + \mu H_u H_d$$

The objects H_u, H_d, $\tilde{Q}$, $\tilde{L}$, $\tilde{\bar{u}}$, $\tilde{\bar{d}}$, $\tilde{\bar{e}}$ appearing here are the scalar fields appearing in the left-handed chiral supermultiplets. Recall that $\bar{u}$, $\bar{d}$, $\bar{e}$ are the conjugates of the right-handed parts of the quark and lepton fields.

The dimensionless Yukawa couplings $\mathbf{y_u}$, $\mathbf{y_d}$ and $\mathbf{y_e}$ are 3×3 matrices in family space. Up to a normalization, and higher-order quantum corrections, they are the same as in the Standard Model.

We need both H_u and H_d, because terms like $\tilde{\bar{u}}\mathbf{y_u}\tilde{Q}H_d^*$ and $\tilde{\bar{d}}\mathbf{y_d}\tilde{Q}H_u^*$ are not analytic, and so not allowed in the superpotential.

In the approximation that only the t, b, τ Yukawa couplings are included:

$$\mathbf{y_u} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix}; \qquad \mathbf{y_d} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix}; \qquad \mathbf{y_e} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

the superpotential becomes

$$\begin{aligned} W_{\rm MSSM} \;\approx\; & y_t(\bar{t}tH_u^0 - \bar{t}bH_u^+) - y_b(\bar{b}tH_d^- - \bar{b}bH_d^0) \\ & -y_\tau(\bar{\tau}\nu_\tau H_d^- - \bar{\tau}\tau H_d^0) + \mu(H_u^+ H_d^- - H_u^0 H_d^0) \end{aligned}$$

Here the ˜ are omitted to reduce clutter, and $Q_3 = (t\, b)$; $L_3 = (\nu_\tau\, \tau)$; $H_u = (H_u^+\, H_u^0)$; $H_d = (H_d^0\, H_d^-)$ $\bar{u}_3 = \bar{t}$; $\bar{d}_3 = \bar{b}$; $\bar{e}_3 = \bar{\tau}$.

The minus signs are arranged so that if the neutral Higgs scalars get positive VEVs $\langle H_u^0 \rangle = v_u$ and $\langle H_d^0 \rangle = v_d$, and the Yukawa couplings are defined positive, then the fermion masses are also positive:

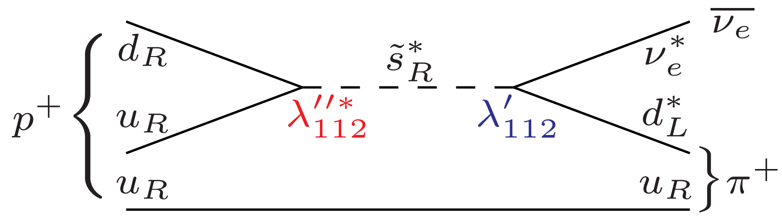
$$m_t = y_t v_u; \qquad m_b = y_b v_d; \qquad m_\tau = y_\tau v_d.$$

Actually, the most general possible superpotential would also include:

$$W_{\Delta L=1} = \frac{1}{2}\lambda_{ijk} L_i L_j \bar{e}_k + \lambda'_{ijk} L_i Q_j \bar{d}_k + \mu'_i L_i H_u$$
$$W_{\Delta B=1} = \frac{1}{2}\lambda''_{ijk} \bar{u}_i \bar{d}_j \bar{d}_k$$

These violate lepton number ($\Delta L = 1$) or baryon number ($\Delta B = 1$).

If both types of couplings were present, and of order 1, then the proton would decay in a tiny fraction of a second through diagrams like this:



Many other proton decay modes, and other experimental limits on B and L violation, give strong constraints on these terms in the superpotential.

One cannot simply require B and L conservation, since they are already known to be violated by non-perturbative electroweak effects. Instead, in the MSSM, one postulates a new discrete symmetry called **Matter Parity**, also known as **R-parity**.

Matter parity is a multiplicatively conserved quantum number defined as:

$$P_M = (-1)^{3(B-L)}$$

for each particle in the theory. All quark and lepton supermultiplets carry $P_M = -1$, and the Higgs and gauge supermultiplets carry $P_M = +1$. This eliminates all of the dangerous $\Delta L = 1$ and $\Delta B = 1$ terms from the superpotential, saving the proton.

R-parity is defined for each particle with spin S by:

$$P_R = (-1)^{3(B-L)+2S}$$

This is **exactly equivalent** to matter parity, because the product of $(-1)^{2S}$ is always $+1$ for any interaction vertex that conserves angular momentum.

However, particle within the same supermultiplet do not carry the same R-parity. All of the known Standard Model particles and the Higgs scalar bosons carry $P_R = +1$, while all of the squarks and sleptons and higgsinos and gauginos carry $P_R = -1$.

Consequences of R-parity

The particles with odd R-parity ($P_R = -1$) are the “supersymmetric particles” or “sparticles”.

Every interaction vertex in the theory must contain an even number of $P_R = -1$ sparticles. Three important consequences:

- The lightest sparticle with $P_R = -1$, called the “Lightest Supersymmetric Particle” or LSP, must be absolutely stable. If the LSP is electrically neutral, it interacts only weakly with ordinary matter, and so can make an attractive candidate for the non-baryonic dark matter required by cosmology.
- In collider experiments, sparticles can only be produced in even numbers (usually two-at-a-time).
- Each sparticle other than the LSP must eventually decay into a state that contains an odd number of LSPs (usually just one). The LSP escapes the detector, with a missing momentum signature.

The Lightest SUSY Particle as Cold Dark Matter

Recent results in experimental cosmology suggest the existence of cold dark matter with a density:

$$\Omega_{\text{CDM}} h^2 = 0.11 \pm 0.02 \qquad \text{(WMAP)}$$

where h = Hubble constant in units of 100 km/(sec Mpc).

A stable particle which freezes out of thermal equilibrium will have $\Omega h^2 = 0.11$ today if its thermal-averaged annihilation cross-section is, roughly:

$$\langle \sigma v \rangle = 1 \text{ pb}$$

As a crude estimate, a weakly interacting particle that annihilates with a characteristic mass scale M will have

$$\langle \sigma v \rangle \sim \frac{\alpha^2}{M^2} \sim 1 \text{ pb} \left(\frac{150 \text{ GeV}}{M} \right)^2$$

So, a stable, weakly interacting particle with mass of order 100 GeV is a likely candidate. In particular, a neutralino LSP ($\tilde{N}_1$) may do it, if R-parity is conserved.

The Soft SUSY-breaking Lagrangian for the MSSM

$$
\begin{aligned}
\mathcal{L}_{\rm soft}^{\rm MSSM} \quad = \quad & -\tfrac{1}{2}\left(M_3\widetilde{g}\widetilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B}\right) + {\rm c.c.} \\
& -\left(\widetilde{\overline{u}}\,\mathbf{a_u}\,\widetilde{Q}H_u - \widetilde{\overline{d}}\,\mathbf{a_d}\,\widetilde{Q}H_d - \widetilde{\overline{e}}\,\mathbf{a_e}\,\widetilde{L}H_d\right) + {\rm c.c.} \\
& -\widetilde{Q}^\dagger\,\mathbf{m^2_{\tilde{Q}}}\,\widetilde{Q} - \widetilde{L}^\dagger\,\mathbf{m^2_{\tilde{L}}}\,\widetilde{L} - \widetilde{\overline{u}}\,\mathbf{m^2_{\tilde{\overline{u}}}}\,\widetilde{\overline{u}}^\dagger - \widetilde{\overline{d}}\,\mathbf{m^2_{\tilde{\overline{d}}}}\,\widetilde{\overline{d}}^\dagger - \widetilde{\overline{e}}\,\mathbf{m^2_{\tilde{\overline{e}}}}\,\widetilde{\overline{e}}^\dagger \\
& - m^2_{H_u}H_u^*H_u - m^2_{H_d}H_d^*H_d - (bH_uH_d + {\rm c.c.})\,.
\end{aligned}
$$

The first line gives masses to the MSSM gauginos (gluino $\widetilde{g}$, winos $\widetilde{W}$, bino $\widetilde{B}$).

The second line consists of (scalar)^3 interactions.

The third line is (mass)^2 terms for the squarks and sleptons.

The last line is Higgs (mass)^2 terms.

If SUSY is to solve the Hierarchy Problem, we expect:

$$
\begin{aligned}
& M_1,\, M_2,\, M_3,\, \mathbf{a_u},\, \mathbf{a_d},\, \mathbf{a_e} \,\sim\, m_{\rm soft}; \\
& \mathbf{m^2_{\tilde{Q}}},\, \mathbf{m^2_{\tilde{L}}},\, \mathbf{m^2_{\tilde{\overline{u}}}},\, \mathbf{m^2_{\tilde{\overline{d}}}},\, \mathbf{m^2_{\tilde{\overline{e}}}},\, m^2_{H_u},\, m^2_{H_d},\, b \,\sim\, m^2_{\rm soft}
\end{aligned}
$$

where $m_{\rm soft} \,\lesssim\, 1$ TeV.

The soft SUSY-breaking Lagrangian of the MSSM contains 105 new parameters not found in the Standard Model.

Most of what we do not already know about SUSY is expressed by the question: "How is supersymmetry broken?"

Many proposals have been made.

The question can be answered experimentally by discovering the pattern of Higgs and squark and slepton and gaugino masses, because they are the main terms in the SUSY-breaking Lagrangian.

Electroweak symmetry breaking and the Higgs bosons

In SUSY, there are two complex Higgs scalar doublets, $(H_u^+,\ H_u^0)$ and $(H_d^0,\ H_d^-)$, rather than one in the Standard Model.

The Higgs VEVs can be parameterized:

$$v_u = \langle H_u^0 \rangle, \qquad v_d = \langle H_d^0 \rangle, \qquad \text{where}$$
$$v_u^2 + v_d^2 = v^2 = 2m_Z^2/(g^2 + g'^2) \approx (174\,\text{GeV})^2$$
$$\tan\beta = v_u/v_d.$$

The quark and lepton masses are related to these VEVs and the superpotential Yukawa couplings by:

$$y_t = \frac{m_t}{v\sin\beta}, \qquad y_b = \frac{m_b}{v\cos\beta}, \qquad y_\tau = \frac{m_\tau}{v\cos\beta}, \qquad \text{etc.}$$

If we want the Yukawa couplings to avoid getting non-perturbatively large up to very high scales, we need:

$$1.5 \lesssim \tan\beta \lesssim 55$$

Define mass-eigenstate Higgs bosons: h^0, H^0, A^0, G^0, H^+, G^+ by:

$$\begin{pmatrix} H_u^0 \\ H_d^0 \end{pmatrix} = \begin{pmatrix} v_u \\ v_d \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h^0 \\ H^0 \end{pmatrix} + \frac{i}{\sqrt{2}} \begin{pmatrix} \sin\beta & \cos\beta \\ -\cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}$$

$$\begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix} = \begin{pmatrix} \sin\beta & \cos\beta \\ -\cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} G^+ \\ H^+ \end{pmatrix}$$

Now, expand the potential to second order in these fields to obtain the masses:

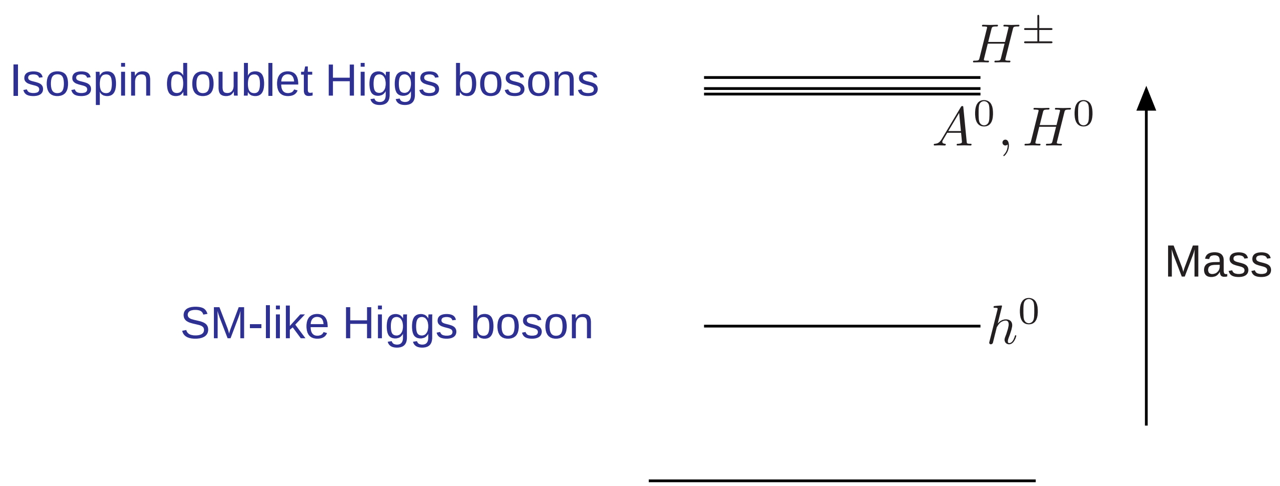
$$\begin{aligned} m_{A^0}^2 &= 2b/\sin 2\beta \\ m_{h^0,H^0}^2 &= \tfrac{1}{2}\Big(m_{A^0}^2 + m_Z^2 \mp \sqrt{(m_{A^0}^2 + m_Z^2)^2 - 4m_Z^2 m_{A^0}^2 \cos^2 2\beta}\Big), \\ m_{H^\pm}^2 &= m_{A^0}^2 + m_W^2 \end{aligned}$$

The Goldstone bosons have $m_{G^0} = m_{G^\pm} = 0$; they are absorbed by the Z, $W^\pm$ bosons to give them masses, just as in the Standard Model.

The decoupling limit for the Higgs bosons

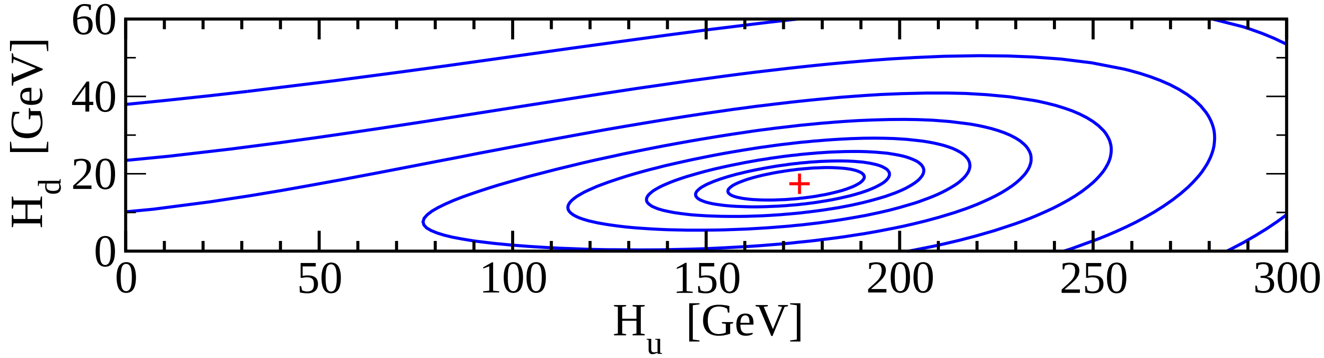
If $m_{A^0} \gg m_Z$, then:

- h^0 has the same couplings as would a Standard Model Higgs boson of the same mass
- $\alpha \approx \beta - \pi/2$
- $A^0, H^0, H^\pm$ form an isospin doublet, and are much heavier than h^0
- h^0 mass is maximized



Many models of SUSY breaking approximate this decoupling limit.

Typical contour map of the Higgs potential in SUSY:



The Standard Model-like Higgs boson h^0 corresponds to oscillations along the shallow direction with $(H_u^0 - v_u, H_d^0 - v_d) \propto (\cos\alpha, -\sin\alpha)$. At tree-level,

$$m_{h^0} < m_Z.$$

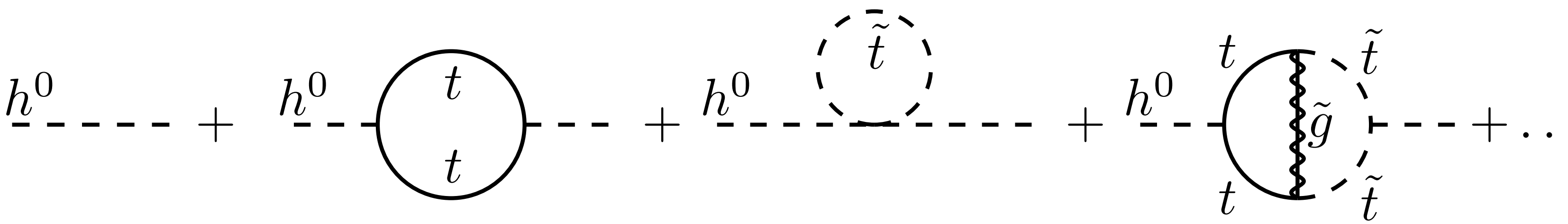
This has been ruled out by LEP2. However, taking into account loop effects, m_{h^0} is considerably larger. Assuming that all superpartners are lighter than 1000 GeV, and that perturbation theory is valid to very high energies, one finds:

$$m_{h^0} \lesssim 130 \text{ GeV}$$

in the MSSM. By adding more supermultiplets, or not requiring that the theory stays perturbative, one can get up to 200 GeV.

Radiative corrections to the Higgs mass in SUSY:

$$m_{h^0}^2 = m_Z^2 \cos^2(2\beta) + \frac{3}{4\pi^2} y_t^2 m_t^2 \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) + \ldots$$



At tree-level:	m_Z^2	pure electroweak
At one-loop:	$y_t^2 m_t^2$	top Yukawa comes in
At two-loop:	$\alpha_S y_t^2 m_t^2$	SUSYQCD comes in
At three-loop:	$\alpha_S^2 y_t^2 m_t^2$	

Even the three-loop corrections can add 1 GeV or so to m_{h^0}.

This is larger than the experimental uncertainty expected at the LHC.

Supersymmetry

PreSUSY 10

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Covered in Lecture 1: The hierarchy problem as motivation; the SUSY algebra; types of supermultiplets; soft SUSY breaking; the Wess-Zumino model; auxiliary fields; supersymmetric masses and interactions; SUSY gauge theories; the MSSM superpotential; R-parity and matter parity; soft breaking in the MSSM; SUSY Higgs sector.

Covered in Lecture 2: Neutralino, chargino, gluino, squark, and slepton masses in the MSSM; hints of an Organizing Principle for SUSY breaking; the Flavor-Preserving MSSM; Origins of SUSY breaking; the mSUGRA parameter space; superpartner decays; Tevatron and LHC signals.

Neutralinos

The neutral higgsinos ($\tilde{H}_u^0$, $\tilde{H}_d^0$) and the neutral gauginos ($\tilde{B}$, $\tilde{W}^0$) mix with each other after electroweak symmetry breaking to form four **neutralino** fermions. In the gauge eigenstate basis $\psi_i^0 = (\tilde{B},\ \tilde{W}^0,\ \tilde{H}_d^0,\ \tilde{H}_u^0)$ for $i = 1, 2, 3, 4$, the neutralino mass terms in the Lagrangian are

$$\mathcal{L}_{\text{neutralino mass}} = -\tfrac{1}{2}(\psi^0)^T \mathbf{M}_{\tilde{N}} \psi^0$$

$$\mathbf{M}_{\tilde{N}} = \begin{pmatrix} M_1 & 0 & -g' v_d/\sqrt{2} & g' v_u/\sqrt{2} \\ 0 & M_2 & g v_d/\sqrt{2} & -g v_u/\sqrt{2} \\ -g' v_d/\sqrt{2} & g v_d/\sqrt{2} & 0 & -\mu \\ g' v_u/\sqrt{2} & -g v_u/\sqrt{2} & -\mu & 0 \end{pmatrix}$$

The diagonal terms are just the gaugino masses in the soft SUSY-breaking Lagrangian. The $-\mu$ entries can be traced back to the superpotential. The off-diagonal terms come from the gaugino-Higgs-Higgsino interactions, and are always less than m_Z.

The physical neutralino mass eigenstates $\tilde{N}_i$ (another popular notation is $\tilde{\chi}_i^0$) are obtained by diagonalizing the mass matrix with a unitary matrix.

$$\tilde{N}_i \quad = \quad \mathbf{N}_{ij}\psi_j^0,$$

where

$$\text{diag}(m_{\tilde{N}_1},\, m_{\tilde{N}_2},\, m_{\tilde{N}_3},\, m_{\tilde{N}_4}) = \mathbf{N}^*\mathbf{M}\mathbf{N}^{-1},$$

with $m_{\tilde{N}_1} < m_{\tilde{N}_2} < m_{\tilde{N}_3} < m_{\tilde{N}_4}$.

In many models of SUSY breaking, one finds:

$$M_1 \approx 0.5 M_2 < |\mu| \qquad \text{and} \qquad m_Z \ll |\mu|$$

where the "0.5" is really $\frac{5}{3}\tan^2\theta_W$. In that case, the lightest neutralino state $\tilde{N}_1$ is mostly bino, with mass nearly equal to M_1.

The lightest neutralino fermion, $\tilde{N}_1$, is a likely candidate for the cold dark matter that seems to be required by cosmology.

Charginos

Similarly, the charged higgsinos H_u^+, H_d^- and the charged winos W^+, W^- mix to form **chargino** fermion mass eigenstates.

$$\mathcal{L}_{\text{chargino mass}} = -\tfrac{1}{2}(\psi^\pm)^T \mathbf{M}_{\widetilde{C}} \psi^\pm + \text{c.c.}$$

where, in 2×2 block form,

$$\mathbf{M}_{\widetilde{C}} \quad = \quad \begin{pmatrix} \mathbf{0} & \mathbf{X}^T \\ \mathbf{X} & \mathbf{0} \end{pmatrix} \qquad \text{with} \qquad \mathbf{X} = \begin{pmatrix} M_2 & gv_u \\ gv_d & \mu \end{pmatrix}$$

The mass eigenstates $\widetilde{C}^\pm_{1,2}$ (many other sources use $\tilde{\chi}^\pm_{1,2}$) are related to the gauge eigenstates by two unitary 2×2 matrices $\mathbf{U}$ and $\mathbf{V}$ according to

$$\begin{pmatrix} \widetilde{C}_1^+ \\ \widetilde{C}_2^+ \end{pmatrix} = \mathbf{V} \begin{pmatrix} \widetilde{W}^+ \\ \widetilde{H}_u^+ \end{pmatrix}; \qquad \begin{pmatrix} \widetilde{C}_1^- \\ \widetilde{C}_2^- \end{pmatrix} = \mathbf{U} \begin{pmatrix} \widetilde{W}^- \\ \widetilde{H}_d^- \end{pmatrix}.$$

Note that the mixing matrix for the positively charged left-handed fermions is different from that for the negatively charged left-handed fermions.

The chargino mixing matrices are chosen so that

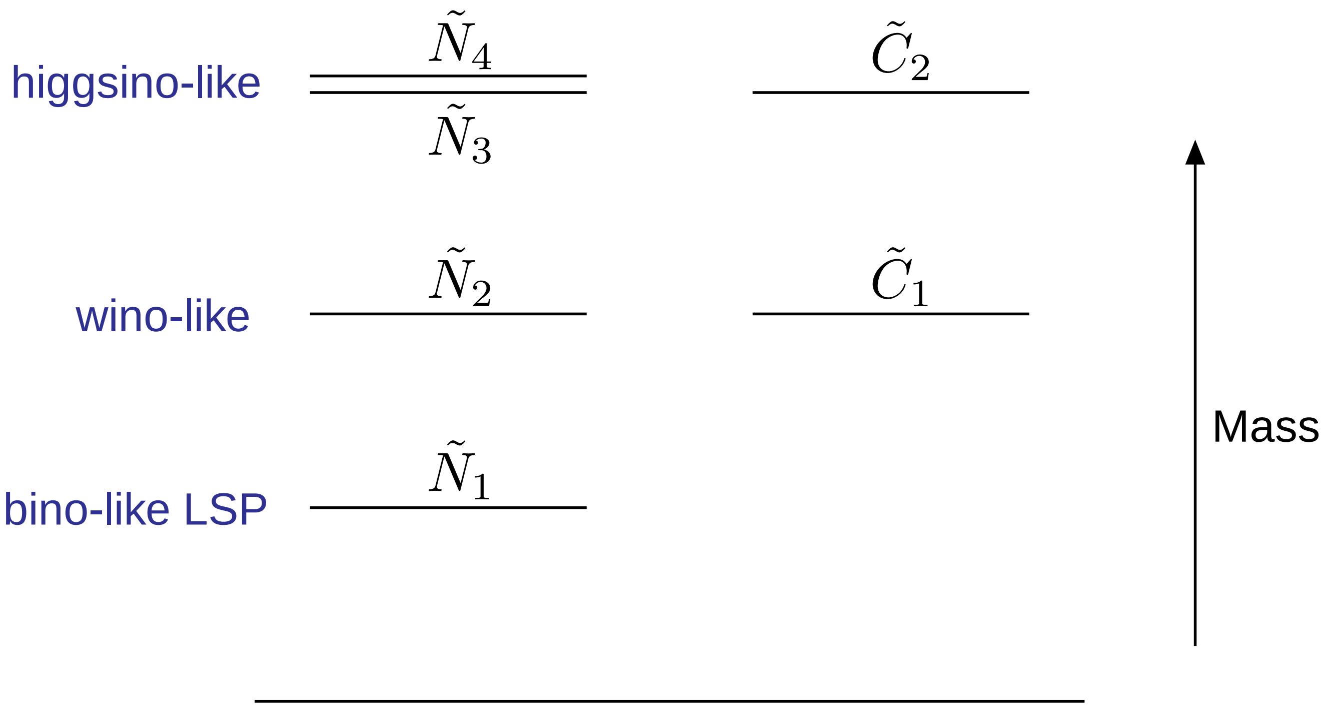
$$\mathbf{U}^*\mathbf{X}\mathbf{V}^{-1} = \begin{pmatrix} m_{\widetilde{C}_1} & 0 \\ 0 & m_{\widetilde{C}_2} \end{pmatrix},$$

with positive real entries $m_{\widetilde{C}_i}$. In this case, one can solve for them in simple closed form:

$$\begin{aligned} m^2_{\widetilde{C}_1}, m^2_{\widetilde{C}_2} \quad = \quad & \frac{1}{2}\Big[|M_2|^2 + |\mu|^2 + 2m^2_W \\ & \mp\sqrt{(|M_2|^2 + |\mu|^2 + 2m^2_W)^2 - 4|\mu M_2 - m^2_W \sin 2\beta|^2}\Big]. \end{aligned}$$

In many models of SUSY breaking, one finds that $M_2 \ll |\mu|$, so the lighter chargino is mostly wino with mass close to M_2, and the heavier is mostly higgsino with mass close to $|\mu|$.

A typical mass hierarchy for the neutralinos and charginos, assuming $m_Z \ll |\mu|$ and $M_1 \approx 0.5 M_2 < |\mu|$.



Although this is a very popular scenario, it is NOT guaranteed.

The Gluino

The gluino is an $SU(3)_C$ color octet fermion, so it does not have the right quantum numbers to mix with any other state. Therefore, at tree-level, its mass is the same as the corresponding parameter in the soft SUSY-breaking Lagrangian:

$$M_{\tilde{g}} = M_3$$

However, the quantum corrections to this are quite large (again, because this is a color octet!). If one calculates the one-loop pole mass of the gluino, one finds:

$$M_{\tilde{g}} = M_3(Q)\left(1 + \frac{\alpha_s}{4\pi}\left[15 + 6\ln(Q/M_3) + \sum A_{\tilde{q}}\right]\right)$$

where Q is the renormalization scale, the sum is over all 12 squark multiplets, and

$$A_{\tilde{q}} = \int_0^1 dx\, x \ln\left[x m_{\tilde{q}}^2/M_3^2 + (1-x)m_q^2/M_3^2 - x(1-x) - i\epsilon\right].$$

This correction can be of order 5% to 25%, depending on the squark masses. It tends to **increase** the gluino mass, compared to the tree-level prediction.

Squarks and Sleptons

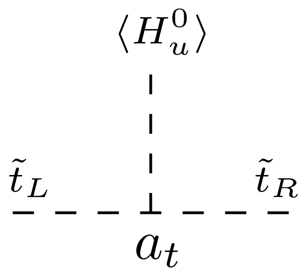
To treat these in complete generality, we would have to take into account arbitrary mixing. So the mass eigenstates would be obtained by diagonalizing:

- a 6×6 (mass)^2 matrix for up-type squarks ($\widetilde{u}_L$, $\widetilde{c}_L$, $\widetilde{t}_L$, $\widetilde{u}_R$, $\widetilde{c}_R$, $\widetilde{t}_R$),
- a 6×6 (mass)^2 matrix for down-type squarks ($\widetilde{d}_L$, $\widetilde{s}_L$, $\widetilde{b}_L$, $\widetilde{d}_R$, $\widetilde{s}_R$, $\widetilde{b}_R$),
- a 6×6 (mass)^2 matrix for charged sleptons ($\widetilde{e}_L$, $\widetilde{\mu}_L$, $\widetilde{\tau}_L$, $\widetilde{e}_R$, $\widetilde{\mu}_R$, $\widetilde{\tau}_R$),
- a 3×3 (mass)^2 matrix for sneutrinos ($\widetilde{\nu}_e$, $\widetilde{\nu}_\mu$, $\widetilde{\nu}_\tau$)

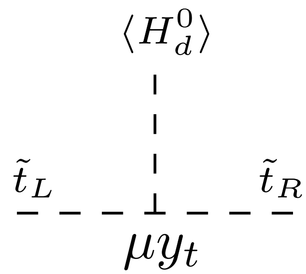
Fortunately, in viable models, most of these mixing angles are very small.

The first- and second-family squarks and sleptons have negligible Yukawa couplings, so they end up in 7 very nearly degenerate, unmixed pairs $(\widetilde{e}_R, \widetilde{\mu}_R)$, $(\widetilde{\nu}_e, \widetilde{\nu}_\mu)$, $(\widetilde{e}_L, \widetilde{\mu}_L)$, $(\widetilde{u}_R, \widetilde{c}_R)$, $(\widetilde{d}_R, \widetilde{s}_R)$, $(\widetilde{u}_L, \widetilde{c}_L)$, $(\widetilde{d}_L, \widetilde{s}_L)$.

For the third-family squarks and sleptons, there are additional effects proportional the large Yukawa (y_t, y_b, y_τ) and soft (a_t, a_b, a_τ) couplings. For the top quark, we have corrections with the diagrammatic representations:



and



The first diagram comes directly from the soft SUSY-breaking Lagrangian, and the others from the F-term contribution to the scalar potential. So, in the $(\tilde{t}_L, \tilde{t}_R)$ basis, the top squark (mass)^2 matrix is:

$$\begin{pmatrix} m^2_{\tilde{Q}_3} + m_t^2 + \Delta_{\tilde{t}_L} & a_t^* v_u - \mu y_t v_d \\ a_t v_u - \mu^* y_t v_d & m^2_{\tilde{\bar{u}}_3} + m_t^2 + \Delta_{\tilde{t}_R} \end{pmatrix}$$

Therefore, the top-squark system has a significant mixing, with the off-diagonal entries “repelling” the two (mass)^2 eigenvalues.

Diagonalizing the top squark (mass)^2 matrix, one finds mass eigenstates:

$$\begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix} = \begin{pmatrix} c_{\tilde{t}} & -s^*_{\tilde{t}} \\ s_{\tilde{t}} & c_{\tilde{t}} \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}$$

where $m^2_{\tilde{t}_1} < m^2_{\tilde{t}_2}$ by convention, and $|c_{\tilde{t}}|^2 + |s_{\tilde{t}}|^2 = 1$.

In a completely analogous way, there is a non-trivial mixing for the bottom squark and tau slepton states:

$$\begin{pmatrix} \tilde{b}_1 \\ \tilde{b}_2 \end{pmatrix} = \begin{pmatrix} c_{\tilde{b}} & -s^*_{\tilde{b}} \\ s_{\tilde{b}} & c_{\tilde{b}} \end{pmatrix} \begin{pmatrix} \tilde{b}_L \\ \tilde{b}_R \end{pmatrix};$$

$$\begin{pmatrix} \tilde{\tau}_1 \\ \tilde{\tau}_2 \end{pmatrix} = \begin{pmatrix} c_{\tilde{\tau}} & -s^*_{\tilde{\tau}} \\ s_{\tilde{\tau}} & c_{\tilde{\tau}} \end{pmatrix} \begin{pmatrix} \tilde{\tau}_L \\ \tilde{\tau}_R \end{pmatrix}$$

The same sort of mixing occurs for the first- and second-family squarks and sleptons, but is considered negligible because the Yukawa couplings are small, and in most viable models the relevant a-terms are also.

The undiscovered particles in the MSSM:

Names	Spin	P_R	Mass Eigenstates	Gauge Eigenstates
Higgs bosons	0	$+1$	$h^0 \; H^0 \; A^0 \; H^\pm$	$H_u^0 \; H_d^0 \; H_u^+ \; H_d^-$
squarks	0	-1	$\widetilde{u}_L \; \widetilde{u}_R \; \widetilde{d}_L \; \widetilde{d}_R$	" "
			$\widetilde{s}_L \; \widetilde{s}_R \; \widetilde{c}_L \; \widetilde{c}_R$	" "
			$\widetilde{t}_1 \; \widetilde{t}_2 \; \widetilde{b}_1 \; \widetilde{b}_2$	$\widetilde{t}_L \; \widetilde{t}_R \; \widetilde{b}_L \; \widetilde{b}_R$
sleptons	0	-1	$\widetilde{e}_L \; \widetilde{e}_R \; \widetilde{\nu}_e$	" "
			$\widetilde{\mu}_L \; \widetilde{\mu}_R \; \widetilde{\nu}_\mu$	" "
			$\widetilde{\tau}_1 \; \widetilde{\tau}_2 \; \widetilde{\nu}_\tau$	$\widetilde{\tau}_L \; \widetilde{\tau}_R \; \widetilde{\nu}_\tau$
neutralinos	$1/2$	-1	$\widetilde{N}_1 \; \widetilde{N}_2 \; \widetilde{N}_3 \; \widetilde{N}_4$	$\widetilde{B}^0 \; \widetilde{W}^0 \; \widetilde{H}_u^0 \; \widetilde{H}_d^0$
charginos	$1/2$	-1	$\widetilde{C}_1^\pm \; \widetilde{C}_2^\pm$	$\widetilde{W}^\pm \; \widetilde{H}_u^+ \; \widetilde{H}_d^-$
gluino	$1/2$	-1	$\widetilde{g}$	" "

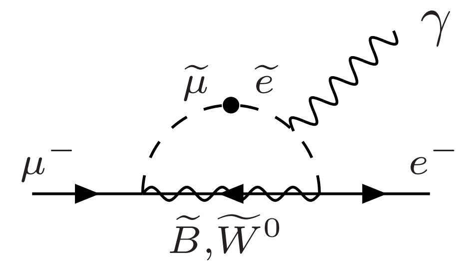
There are 105 new parameters associated with SUSY breaking in the MSSM.

How are we supposed to make any meaningful predictions in the face of this uncertainty?

Fortunately, we already know that the MSSM soft terms cannot be arbitrary, because of experimental constraints on flavor violation.

Hints of an Organizing Principle

For example, if there is a smuon-selectron mixing (mass)^2 term $\mathcal{L} = -m^2_{\tilde{\mu}^*_L \tilde{e}_L} \tilde{\mu}^*_L \tilde{e}_L$, and $\tilde{M} =$ Max$[m_{\tilde{e}_L}, m_{\tilde{e}_R}, M_2]$, then by calculating this one-loop diagram, one finds the decay width:



$\mu^- \to e^- \gamma$

$$\Gamma(\mu^- \to e^- \gamma) = 5 \times 10^{-15}\ \text{eV} \left(\frac{m^2_{\tilde{\mu}^*_L \tilde{e}_L}}{\tilde{M}^2} \right)^2 \left(\frac{100\ \text{GeV}}{\tilde{M}} \right)^4$$

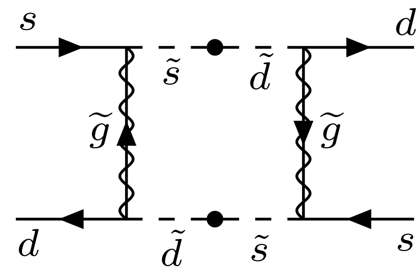
For comparison, the experimental limit is (from MEGA at LAMPF):

$$\Gamma(\mu^- \to e^- \gamma) < 3.6 \times 10^{-21}\ \text{eV}.$$

So the amount of smuon-selectron mixing in the soft Lagrangian is limited by:

$$\left(\frac{m^2_{\tilde{\mu}^*_L \tilde{e}_L}}{\tilde{M}^2} \right) < 10^{-3} \left(\frac{\tilde{M}}{100\ \text{GeV}} \right)^2$$

Another example: $K^0 \leftrightarrow \overline{K^0}$ mixing:



$K^0 \leftrightarrow \overline{K^0}$

This constrains the flavor-violating SUSY breaking terms:

$\mathcal{L} = -m^2_{\tilde{d}^*_L \tilde{s}_L} \tilde{d}^*_L \tilde{s}_L - m^2_{\tilde{d}_R \tilde{s}^*_R} \tilde{d}_R \tilde{s}^*_R.$

Comparing this diagram with the observed Δm_{K^0} gives:

$$\frac{\mathrm{Re}[m^2_{\tilde{d}^*_L \tilde{s}_L} m^2_{\tilde{d}_R \tilde{s}^*_R}]^{1/2}}{\tilde{M}^2} \lesssim 0.001 \left(\frac{\tilde{M}}{500 \text{ GeV}} \right)$$

where $\tilde{M}$ is the dominant squark or gluino mass.

The experimental values of ϵ and ϵ'/ϵ in the effective Hamiltonian for the $K^0, \overline{K^0}$ system also give strong constraints on the amount of $\tilde{d}_L, \tilde{s}_L$ and $\tilde{d}_R, \tilde{s}_R$ mixing and CP violation in the soft terms.

Similarly:

The $D^0, \overline{D^0}$ system constrains $\tilde{u}_L, \tilde{c}_L$ and $\tilde{u}_R, \tilde{c}_R$ soft SUSY-breaking mixing.

The $B_d^0, \overline{B_d^0}$ system constrains $\tilde{d}_L, \tilde{b}_L$ and $\tilde{d}_R, \tilde{b}_R$ soft SUSY-breaking mixing.

The soft-SUSY breaking masses must be either VERY heavy, or nearly flavor-blind, to avoid flavor-changing violating experimental limits.

The Flavor-Preserving Minimal Supersymmetric Standard Model

Take an idealized limit in which in which the squark and slepton (mass)^2 matrices are flavor-blind, each proportional to the 3×3 identity matrix in family space:

$$\mathbf{m^2_{\tilde{Q}}} = m^2_{\tilde{Q}}\mathbf{1}; \quad \mathbf{m^2_{\bar{\tilde{u}}}} = m^2_{\tilde{\bar{u}}}\mathbf{1}; \quad \mathbf{m^2_{\bar{\tilde{d}}}} = m^2_{\tilde{\bar{d}}}\mathbf{1}; \quad \mathbf{m^2_{\tilde{L}}} = m^2_{\tilde{L}}\mathbf{1}; \quad \mathbf{m^2_{\bar{\tilde{e}}}} = m^2_{\tilde{\bar{e}}}\mathbf{1}.$$

Then all squark and slepton mixing angles are rendered trivial, because squarks and sleptons with the same electroweak quantum numbers will be degenerate in mass and can be rotated into each other at will. Also assume:

$$\mathbf{a_u} = A_{u0}\,\mathbf{y_u}; \qquad \mathbf{a_d} = A_{d0}\,\mathbf{y_d}; \qquad \mathbf{a_e} = A_{e0}\,\mathbf{y_e},$$

and no new CP-violating phases:

$$M_1,\ M_2,\ M_3,\ A_{u0},\ A_{d0},\ A_{e0} = \text{real}$$

The Higgs mass parameters $m^2_{H_u}$ and $m^2_{H_d}$ are real, and μ and b can be chosen real by convention.

The Flavor-Preserving Minimal Supersymmetric Standard Model (continued)

The new parameters, besides those already found in the Standard Model, are:

- M_1, M_2, M_3 (3 real gaugino masses)
- $m^2_{\tilde{Q}}$, $m^2_{\tilde{\bar{u}}}$, $m^2_{\tilde{\bar{d}}}$, $m^2_{\tilde{L}}$, $m^2_{\tilde{\bar{e}}}$ (5 squark and slepton mass^2 parameters)
- A_{u0}, A_{d0}, A_{e0} (3 real scalar^3 couplings)
- $m^2_{H_u}$, $m^2_{H_d}$, b, μ (4 real parameters)

So there are 15 real parameters in this model.

The parameters μ and $b \equiv B\mu$ are often traded for the known Higgs VEV $v = 174$ GeV, $\tan\beta$, and sign(μ).

Most viable SUSY breaking models are special cases of this.

However, these are Lagrangian parameters that run with the renormalization scale, Q. Therefore, one must also choose an "input scale" Q_0 where the flavor-independence holds.

What is the input scale Q_0 ?

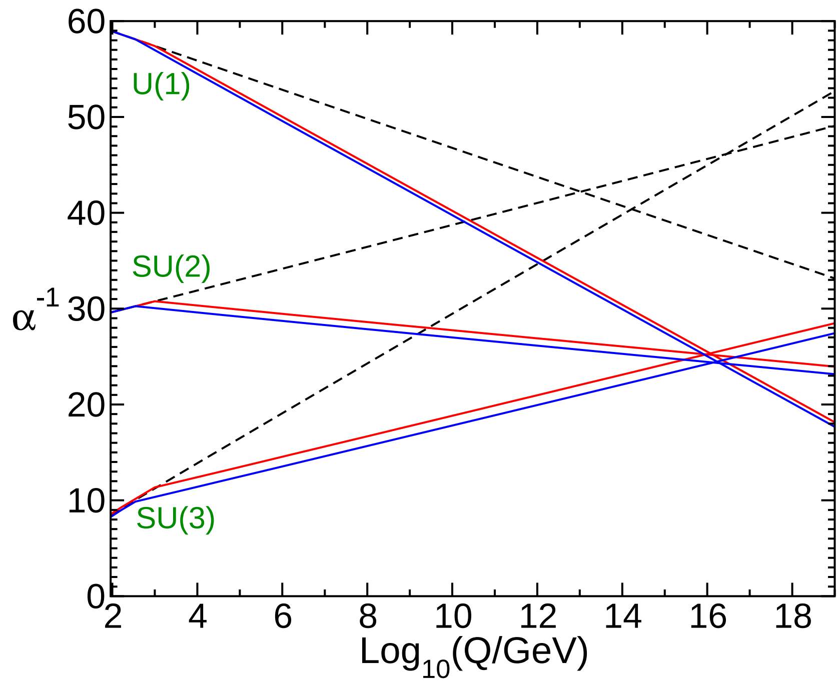
Perhaps:

- $Q_0 = M_{\rm Planck}$, or
- $Q_0 = M_{\rm string}$, or
- $Q_0 = M_{\rm GUT}$, or
- Q_0 is some other scale associated with the type of SUSY breaking.

In any case, one can pick the SUSY-breaking parameters at Q_0 as boundary conditions, then run them down to the weak scale using their renormalization group (RG) equations. Flavor violation will remain small, because the Yukawa couplings of the first two families are small.

At the weak scale, use the renormalized parameters to predict physical masses, decay rates, cross-sections, dark matter relic density, etc.

A reason to be optimistic that this program can succeed: the SUSY unification of gauge couplings. The measured α_1, α_2, α_3 are run up to high scales using the RG equations of the Standard Model (dashed lines) and the MSSM (solid lines).



At one-loop order, the RG equations are:

$$\frac{d}{d(\ln Q)}\alpha_a^{-1} = -\frac{b_a}{2\pi} \qquad\qquad (a = 1, 2, 3)$$

with $b_a^{\rm SM}=(41/10,-19/6,-7)$ in the Standard Model, and $b_a^{\rm MSSM}=(33/5,1,-3)$ in the MSSM because of the extra particles in the loops. The results for the MSSM are in agreement with unification at $M_{\rm GUT} \approx 2 \times 10^{16}$ GeV.

If this hint is real, we can reasonably hope that a similar extrapolation for the soft SUSY-breaking parameters can also work.

Origins of SUSY breaking

Up to now, we have simply put SUSY breaking into the MSSM explicitly.

To gain deeper understanding, let us consider how SUSY could be spontaneously broken. This means that the Lagrangian is invariant under SUSY transformations, but the ground state is not:

$$Q_\alpha|0\rangle \neq 0, \qquad\qquad Q^\dagger_{\dot\alpha}|0\rangle \neq 0.$$

The SUSY algebra tells us that the Hamiltonian is related to the SUSY charges by:

$$H = P^0 = \tfrac{1}{4}(Q_1 Q_1^\dagger + Q_1^\dagger Q_1 + Q_2 Q_2^\dagger + Q_2^\dagger Q_2).$$

Therefore, if SUSY is unbroken in the ground state, then $H|0\rangle = 0$, so the ground state energy is 0. Conversely, if SUSY is spontaneously broken, then the ground state must have positive energy, since

$$\langle 0|H|0\rangle = \tfrac{1}{4}\Big(\|Q_1^\dagger|0\rangle\|^2 + \|Q_1|0\rangle\|^2 + \|Q_2^\dagger|0\rangle\|^2 + \|Q_2|0\rangle\|^2\Big) > 0$$

To achieve spontaneous SUSY breaking, we need a theory in which the prospective ground state $|0\rangle$ has positive energy.

In SUSY, the potential energy can be written, using the equations of motion, as:

$$V = \sum_i |F_i|^2 + \frac{1}{2} \sum_a D^a D^a,$$

a sum of squares of auxiliary fields. So, for spontaneous SUSY breaking, one must arrange a stable (or quasi-stable) ground state with either $\langle F_i \rangle \neq 0$ or $\langle D^a \rangle \neq 0$, for at least one i or a.

Models of SUSY breaking where

- $\langle F_i \rangle \neq 0$ are called “O’Raifeartaigh models” or “F-term breaking models”
- $\langle D^a \rangle \neq 0$ are called “Fayet-Iliopoulis models” or “D-term breaking models”

F-term breaking is used in (almost) all known realistic models.

This can only happen if the chiral supermultiplet is a singlet.

Spontaneous Breaking of SUSY requires us to extend the MSSM

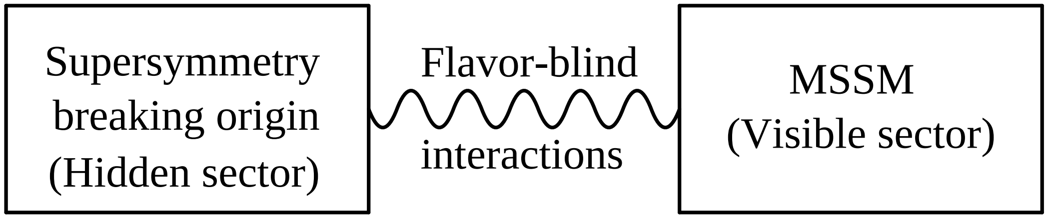
There is no gauge-singlet chiral supermultiplet in the MSSM that could get a non-zero F-term VEV.

Even if there were such an $\langle F \rangle$, there is another general obstacle. Gaugino masses cannot arise in a renormalizable SUSY theory at tree-level. This is because SUSY does not contain any (gaugino)-(gaugino)-(scalar) coupling that could turn into a gaugino mass term when the scalar gets a VEV.

We also have the clue that SUSY breaking must be essentially flavor-blind in order to not conflict with experiment.

This leads to the following general schematic picture of SUSY breaking. . .

The MSSM soft SUSY-breaking terms arise indirectly or radiatively, not from tree-level renormalizable couplings directly to the SUSY-breaking sector.



Spontaneous SUSY breaking occurs in a "hidden sector" of particles with no (or tiny) direct couplings to the "visible sector" chiral supermultiplets of the MSSM. However, the two sectors do share some mediating interactions that transmit SUSY-breaking effects indirectly. As a bonus, if the mediating interactions are flavor-blind, then the soft SUSY-breaking terms of the MSSM will be also.

By dimensional analysis,

$$m_{\rm soft} \sim \frac{\langle F \rangle}{M}$$

where M is a mass scale associated with the physics that mediates between the two sectors.

Planck-scale Mediated SUSY Breaking (also known as "gravity mediation")

The idea: SUSY breaking is transmitted from a hidden sector to the MSSM by the new interactions, including gravity, that enter near the Planck mass scale M_P.

If SUSY is broken in the hidden sector by some VEV $\langle F \rangle$, then the MSSM soft terms should be of order:

$$m_{\rm soft} \sim \frac{\langle F \rangle}{M_P}$$

This follows from dimensional analysis, since $m_{\rm soft}$ must vanish in the limit that SUSY breaking is turned off $(\langle F \rangle \rightarrow 0)$ and in the limit that gravity becomes irrelevant $(M_P \rightarrow \infty)$.

Since we know $m_{\rm soft} \sim$ few hundred GeV, and $M_P \sim 2.4 \times 10^{18}$ GeV:

$$\sqrt{\langle F \rangle} \sim 10^{11} \text{ or } 10^{12} \text{ GeV}$$

Planck-scale Mediated SUSY Breaking (continued)

Write down an effective field theory non-renormalizable Lagrangian that couples F to the MSSM scalar fields ϕ_i and gauginos λ^a:

$$\mathcal{L}_{\text{PMSB}} = -\Big(\frac{f^a}{2M_P}F\lambda^a\lambda^a + \text{c.c.}\Big) - \frac{k_i^j}{M_P^2}FF^*\phi_i\phi^{*j}$$
$$-\Big(\frac{\alpha^{ijk}}{6M_P}F\phi_i\phi_j\phi_k + \frac{\beta^{ij}}{2M_P}F\phi_i\phi_j + \text{c.c.}\Big)$$

This is (part of) a fully supersymmetric Lagrangian that arises in supergravity. When we replace F by its VEV $\langle F\rangle$, we get exactly the MSSM soft SUSY-breaking Lagrangian, with:

- Gaugino masses: $M_a = f^a\langle F\rangle/M_P$
- Scalar squared massed: $(m^2)_i^j = k_i^j|\langle F\rangle|^2/M_P^2$ and $b^{ij} = \beta^{ij}\langle F\rangle/M_P$
- Scalar^3 couplings $a^{ijk} = \alpha^{ijk}\langle F\rangle/M_P$

Unfortunately, it is **not** obvious why these are flavor-blind!

A dramatic simplification occurs if one assumes a “minimal” form for the kinetic terms and gauge interactions in the underlying supergravity theory. (Whether this assumption is reasonable or not remains controversial.)

This means $f^a = f$ for all gauge interactions, $k_i^j = k\delta_i^j$ for all scalar fields, and $\alpha^{ijk} = \alpha y^{ijk}$ and $\beta^{ij} = \beta M^{ij}$. Then all of the MSSM soft terms can be written in terms of just four parameters:

- A common gaugino mass: $m_{1/2} = f\frac{\langle F\rangle}{M_P}$
- A common scalar squared mass: $m_0^2 = k\frac{|\langle F\rangle|^2}{M_P^2}$
- A scalar^3 coupling prefactor: $A_0 = \alpha\frac{\langle F\rangle}{M_P}$
- A scalar mass^2 prefactor $B_0 = \beta\frac{\langle F\rangle}{M_P}$

This simplified parameter space is often called “Minimal Supergravity” or “mSUGRA”.

The "mSUGRA" parameter space

In terms of the four parameters $m_{1/2}$, m_0^2, A_0, and B_0:

$$M_3 = M_2 = M_1 = m_{1/2}$$

$$\mathbf{m^2_{\tilde{Q}}} = \mathbf{m^2_{\tilde{\bar{u}}}} = \mathbf{m^2_{\tilde{\bar{d}}}} = \mathbf{m^2_{\tilde{L}}} = \mathbf{m^2_{\tilde{\bar{e}}}} = m_0^2\,\mathbf{1}$$

$$m^2_{H_u} = m^2_{H_d} = m_0^2$$

$$\mathbf{a_u} = A_0\mathbf{y_u}, \qquad \mathbf{a_d} = A_0\mathbf{y_d}, \qquad \mathbf{a_e} = A_0\mathbf{y_e}$$

$$b = B_0\mu.$$

These values of the soft terms should probably be taken at the renormalization scale $Q_0 = M_P$, and then run down to the weak scale. However, it is traditional to use $Q_0 = M_{\rm GUT}$ instead, only because nobody has any idea how to extrapolate above $M_{\rm GUT}$! Part of the error incurred in doing so can be reabsorbed into the definitions of $m_{1/2}$, m_0^2, A_0, and B_0.

Some particular models can be even more predictive, in principle:

- Dilaton-dominated: $m_0^2 = m_{3/2}^2,\;\; m_{1/2} = -A_0 = \sqrt{3} m_{3/2}$
- Polonyi: $m_0^2 = m_{3/2}^2,\;\; A_0 = (3 - \sqrt{3}) m_{3/2}$
- "No-scale" or "Gaugino mass dominated": $m_{1/2} \gg m_0, A_0$

However, there is no clear theoretical reason why things should be so simple.

The modern viewpoint is to take $m_{1/2}$, m_0^2, A_0, and B_0 as crude, but convenient, parameterizations of our ignorance of SUSY breaking.

It is usual to trade B_0 for the parameter $\tan\beta = v_u/v_d$.
Also, the minimization of the EW potential allows us to eliminate the magnitude (but not the phase) of μ in favor of m_Z.

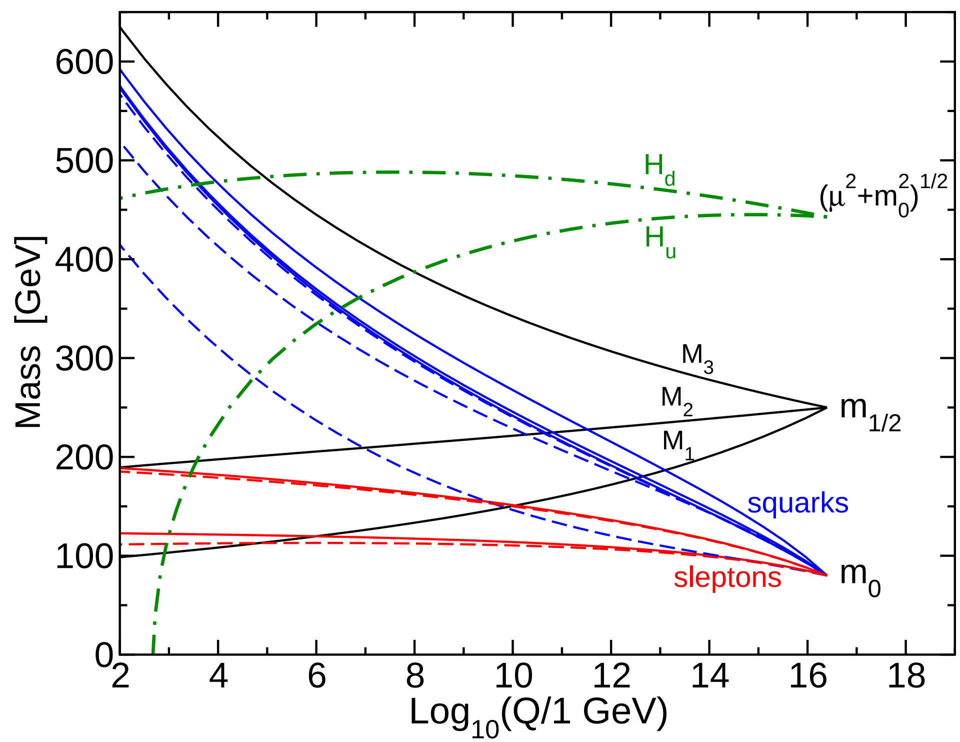
Renormalization Group Running for an mSUGRA model (SPS1A$'$) with $m_{1/2} = 250$ GeV, $m_0 = 70$ GeV, $A_0 = -300$ GeV, $\tan\beta = 10$, and sign(μ) = +1

Gaugino masses M_1, M_2, M_3

Slepton masses (dashed=stau)

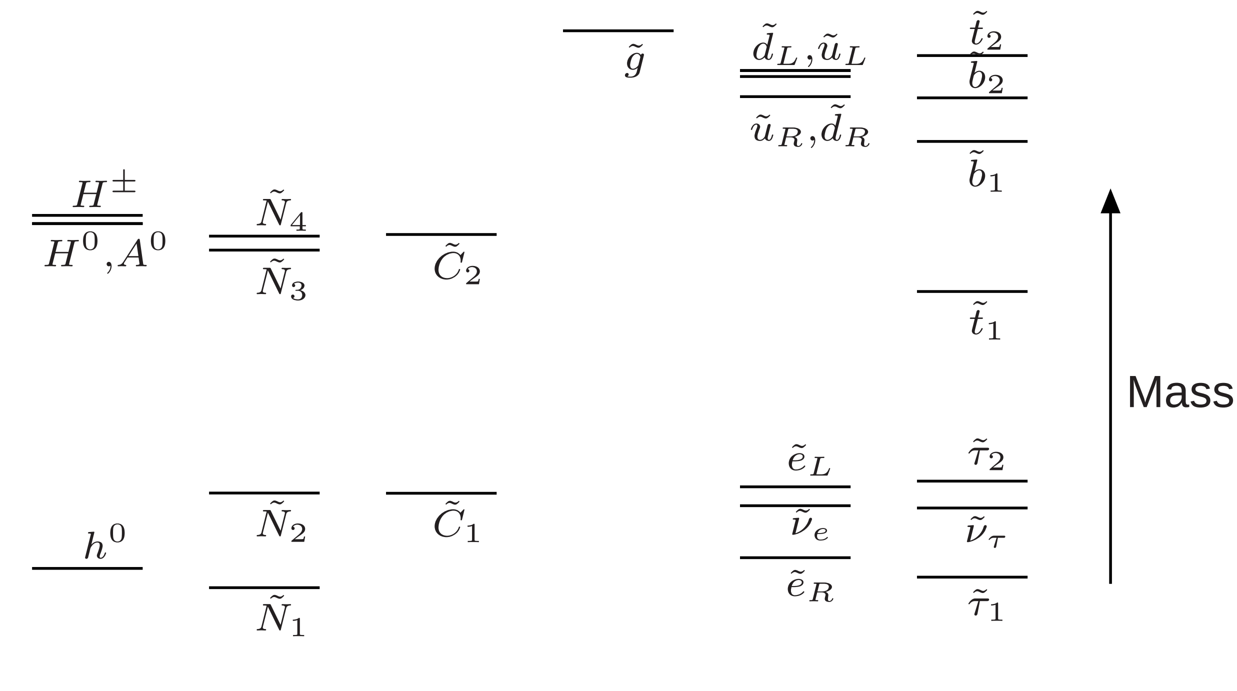
Squark masses (dashed=stop)

Higgs: $(m_{H_u}^2 + \mu^2)^{1/2}$, $(m_{H_d}^2 + \mu^2)^{1/2}$



Electroweak symmetry breaking occurs because $m_{H_u}^2 + \mu^2$ runs negative near the electroweak scale. This is due directly to the large top quark Yukawa coupling.

Here is the resulting sparticle mass spectrum:



This is typical, qualitatively, of mSUGRA models with relatively large $m_{1/2}$.

Notes: The Higgs sector is in the decoupling limit, with h^0 near the LEP2 limit. A neutralino is the LSP. The gluino is the heaviest sparticle. The lightest squark is the top squark. The lightest slepton is the tau slepton.

Computer programs exist that generate the superpartner mass spectrum for you, given a choice of SUSY-breaking model parameters. Some of the publicly available ones:

- `ISASUSY` (Paige, Protopopescu, Baer, Tata)
- `SOFTSUSY` (Allanach)
- `SuSpect` (Djouadi, Kneur, Moultaka)
- `SPheno` (Porod)

These can be interfaced, through the SUSY Les Houches Accords (SLHA) to programs that produce cross-sections, decay rates, and Monte Carlo events: `MadGraph/MadEvent`, `Pythia`, `ISAJET`, `HERWIG`, `WHIZARD` `SUSYGEN`, `SDECAY`, `GRACE`, `CompHEP`, `CalcHEP`, `PROSPINO`, ...

The can also be interfaced to programs that compute the abundance of dark matter: `micrOMEGAs`, `DarkSUSY`, `ISAReD`.

SUSY signatures at colliders

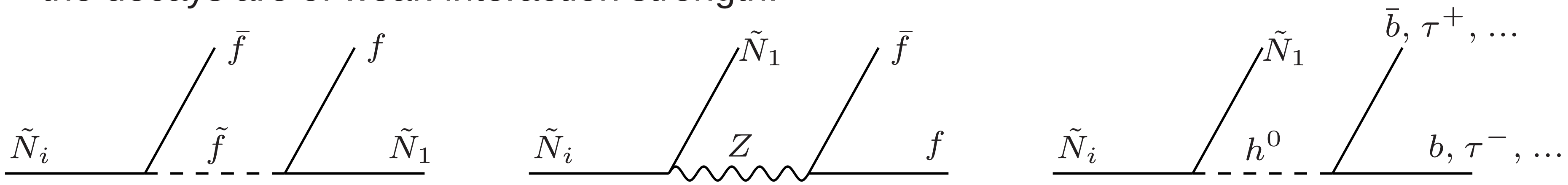
I will concentrate mostly on models with conserved R-parity and a neutralino LSP dark matter candidate $(\tilde{N}_1)$. Recall:

- The most important interactions for producing sparticles are gauge interactions, and interactions related to gauge interactions by SUSY. Their strength is known, up to mixing of sparticles.
- Two sparticles produced in each event, with opposite momenta.
- The LSPs are neutral and extremely weakly interacting, so they carry away energy and momentum.
  - At e^+e^- colliders, the total energy can be accounted for, so one sees missing energy, $\not{E}$.
  - At hadron colliders, the component of the momentum along the beam is unknown on an event-by-event basis, so only the energy component in particles transverse to the beam is observable. So one sees "missing transverse energy", $\not{E}_T$.

Sparticle Decays

1) Neutralino Decays

If R-parity is conserved and $\tilde{N}_1$ is the LSP, then it cannot decay. For the others, the decays are of weak-interaction strength:



In each case, the intermediate boson (squark or slepton $\tilde{f}$, Z boson, or Higgs boson h^0) might be on-shell, if that two-body decay is kinematically allowed.

In general, the visible decays are either:

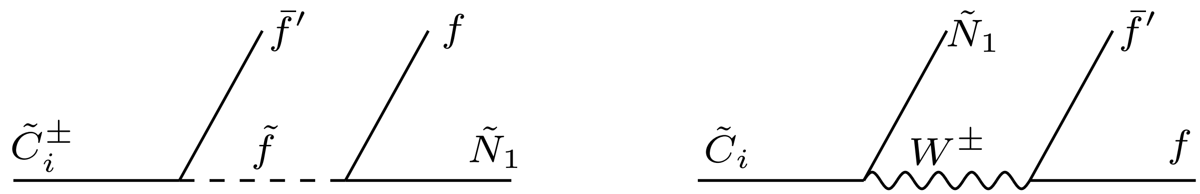
$$\tilde{N}_i \to q\bar{q}\tilde{N}_1 \qquad \text{(seen in detector as } jj + \not{E}\text{)}$$

$$\tilde{N}_i \to \ell^+\ell^-\tilde{N}_1 \qquad \text{(seen in detector as } \ell^+\ell^- + \not{E}\text{)}$$

Some SUSY signals rely on leptons in the final state. This is more likely if sleptons are relatively light. If $\tilde{N}_i \to \tilde{N}_1 h^0$ is kinematically open, then it often dominates. This is called a “spoiler” mode, because leptonic final states are rare.

2) Chargino Decays

Charginos $\tilde{C}_i$ have decays of weak-interaction strength:



In each case, the intermediate boson (squark or slepton $\tilde{f}$, or W boson) might be on-shell, if that two-body decay is kinematically allowed.

In general, the decays are either:

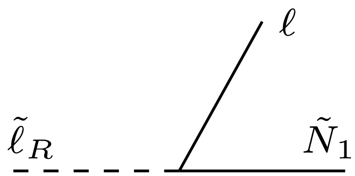
$$\tilde{C}_i^\pm \longrightarrow q\bar{q}'\tilde{N}_1 \qquad \text{(seen in detector as } jj + \not{E}\text{)}$$
$$\tilde{C}_i^\pm \longrightarrow \ell^\pm\nu\tilde{N}_1 \qquad \text{(seen in detector as } \ell^\pm + \not{E}\text{)}$$

Again, leptons in final state are more likely if sleptons are relatively light.

For both neutralinos and charginos, a relatively light, mixed $\tilde{\tau}_1$ can lead to enhanced τ's in the final state. This is especially important for larger $\tan\beta$. Tau identification may be a crucial limiting factor for experimental SUSY.

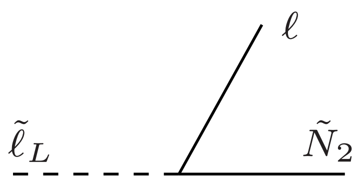
3) Slepton Decays

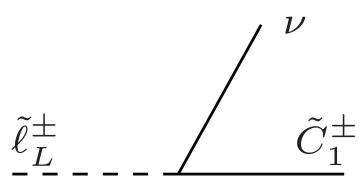
When $\tilde{N}_1$ is the LSP and has a large bino content, the sleptons $\tilde{e}_R$, $\tilde{\mu}_R$ (and often $\tilde{\tau}_1$ and $\tilde{\tau}_2$) prefer the direct two-body decays with strength proportional to g'^2:

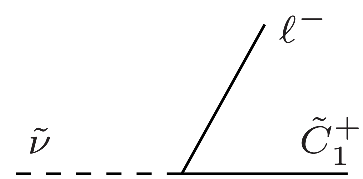


(seen in detector as $\ell^\pm + \not{E}$)

However, the left-handed sleptons $\tilde{e}_L$, $\tilde{\mu}_L$, $\tilde{\nu}$ have no coupling to the bino component of $\tilde{N}_1$, so they often decay preferentially through $\tilde{N}_2$ or $\tilde{C}_1$, which have a large wino content, with strength proportional to g^2:



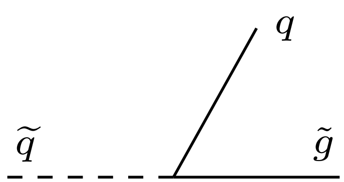




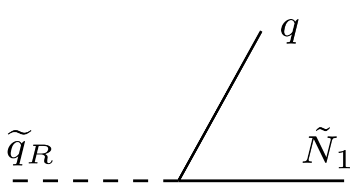
with $\tilde{N}_2$ and $\tilde{C}_1$ decaying as before.

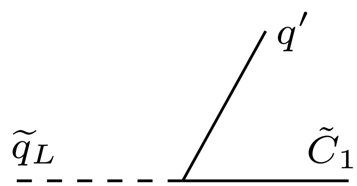
4) Squark Decays

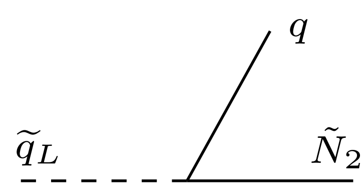
If the decay $\tilde{q} \rightarrow q\tilde{g}$ is kinematically allowed, it will always dominate, because the squark-quark-gluino vertex has QCD strength:



Otherwise, right-handed squarks prefer to decay directly to a bino-like LSP, while left-handed squarks prefer to decay to a wino-like $\tilde{C}_1$ or $\tilde{N}_2$:





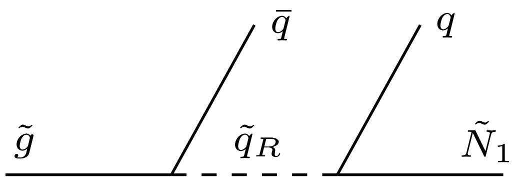


If a top squark is light, then the decays $\tilde{t}_1 \rightarrow t\tilde{g}$ and $\tilde{t}_1 \rightarrow t\tilde{N}_1$ may not be kinematically allowed, and it may decay only into charginos: $\tilde{t}_1 \rightarrow b\tilde{C}_1$. If those decays are also closed, it has $\tilde{t}_1 \rightarrow bW\tilde{N}_1$. If even that is closed, it has only a suppressed flavor-changing decay $\tilde{t}_1 \rightarrow c\tilde{N}_1$ or 4-body decay $\tilde{t}_1 \rightarrow bf\bar{f}'\tilde{N}_1$.

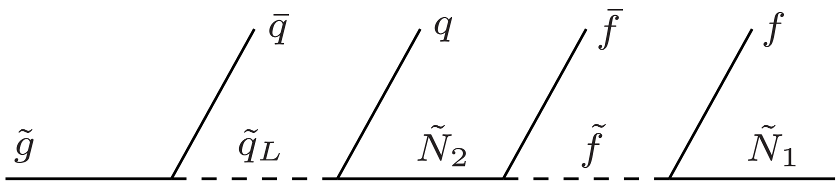
5) Gluino Decays

The gluino can only decay through squarks, either on-shell (if allowed) or virtual.

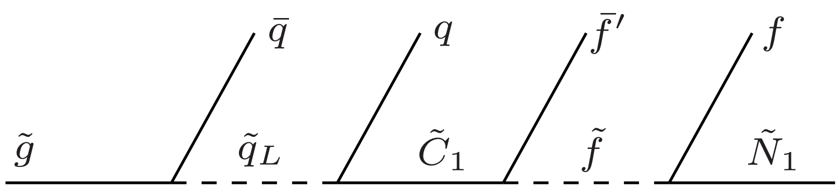
For example:



$jj + \not\!\!E$ or $t\bar{t} + \not\!\!E$



$jjjj + \not\!\!E$ or $t\bar{t}jj + \not\!\!E$ or $jj\ell^+\ell^- + \not\!\!E$

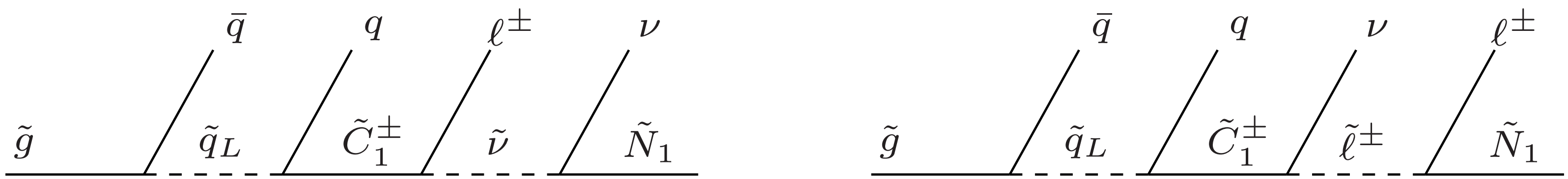


$jjjj + \not\!\!E$ or $t\bar{t}jj + \not\!\!E$ or $jj\ell^\pm + \not\!\!E$

Because $m_{\tilde{t}_1} \ll$ other squark masses, top quarks can appear in these decays.

The possible signatures of gluinos and squarks are typically numerous and complicated because of these and other **cascade decays**.

An important feature of gluino decays with one lepton:



In each case, $\tilde{g} \to jj\ell^\pm + \not{E}$, and the lepton has either charge with equal probability. (The gluino does not "know" about electric charge.)

So, events with at least one gluino, and exactly one charged lepton in the final state from each sparticle that was produced, will have probability 0.5 to have **same-charge leptons**, and probability 0.5 to have opposite-charge leptons.

This is important at hadron colliders, where Standard Model backgrounds with same-charge leptons are much smaller.

$$\text{(SUSY)} \to \ell^+\ell'^+ + \text{ jets } + \not{E}_T$$

SUSY Limits from LEP2 e^+e^- collisions up to $\sqrt{s} = 208$ GeV

The CERN LEP2 collider had the capability of producing all sparticle-antisparticle pairs, except for the gluino:

$$e^+e^- \rightarrow \tilde{\ell}^+\tilde{\ell}^-,\ \tilde{C}_1^+\tilde{C}_1^-,\ \tilde{N}_1\tilde{N}_2,\ \tilde{N}_2\tilde{N}_2,\ \gamma\tilde{N}_1\tilde{N}_1,\ \tilde{q}\tilde{q}^*$$

Exclusions for charged sparticles are typically close to the kinematic limit, except when mass difference are small. For example, at 95% CL:

$$m_{\tilde{C}_1} > 103 \text{ GeV} \qquad (m_{\tilde{C}_1} - m_{\tilde{N}_1} > 3 \text{ GeV or } < 100 \text{ MeV})$$

$$m_{\tilde{C}_1} > 92 \text{ GeV} \qquad (\text{any heavier than } \tilde{N}_1)$$

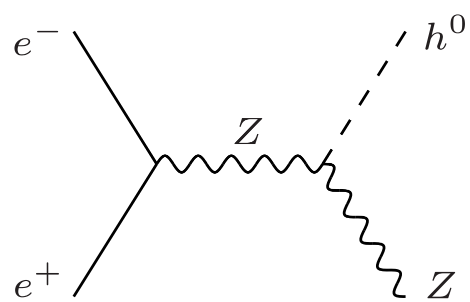
and

$$m_{\tilde{e}_R} > 100 \text{ GeV} \qquad (m_{\tilde{e}_R} - m_{\tilde{N}_1} > 5 \text{ GeV})$$

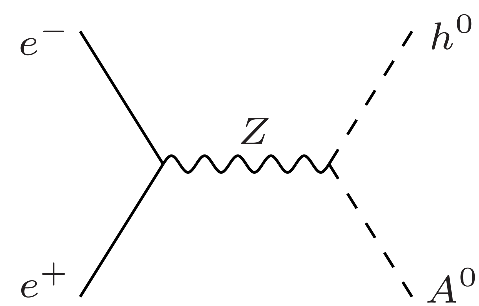
See `http://lepsusy.web.cern.ch/lepsusy/`
for detailed results.

LEP2 Searches for Higgs bosons

The most important constraints on SUSY parameter space come from searches for the MSSM Higgs bosons at LEP2. The relevant processes include:



$$\propto \sin^2(\beta - \alpha)$$



$$\propto \cos^2(\beta - \alpha)$$

The first diagram is the same as for the Standard Model Higgs search in the decoupling limit, where $\sin^2(\beta - \alpha) \approx 1$. Many SUSY models fall into this category, and the LEP2 bound (nearly) applies:

$$m_{h^0} > 114.4\,\mathrm{GeV} \qquad (95\%\ \mathrm{CL})$$

General bounds in SUSY are much weaker, but “most” of parameter space in the MSSM yields a Standard-Model-like lightest Higgs boson.

Impact of the LEP2 bound on m_{h^0}

Recall that in the decoupling limit:

$$m_{h^0}^2 = m_Z^2 \cos^2(2\beta) + \frac{3}{4\pi^2} y_t^2 m_t^2 \left[\ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \right) + C_{\tilde{t}\text{-mixing}} \right] + \ldots$$

For $\cos^2(2\beta) \approx 1$ and $C_{\tilde{t}\text{-mixing}} \approx 0$, we therefore need, roughly:

$$\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \gtrsim 600\ \mathrm{GeV}.$$

This suggests a pessimistic attitude toward discovering squarks at the Tevatron.

However, there are many ways out:

- Top-squark mixing gives positive contributions to m_{h^0}, up to $C_{\tilde{t}\text{-mixing}} = 3$.
- Enlarging the Higgs sector, for example by adding a singlet Higgs supermultiplet, also gives positive contributions to m_{h^0}.
- Enlarging the squark sector, with new Yukawa couplings, can raise m_{h^0}.
- The other squarks might be much lighter than the top squarks.
- If h^0 has non-standard couplings, maybe it is lighter but hid from LEP2.

Tevatron Signals for SUSY in $p\overline{p}$ collisions at $\sqrt{s} = 1.96$ TeV

Trilepton + $\not\!\!E_T$ Signal at the Tevatron

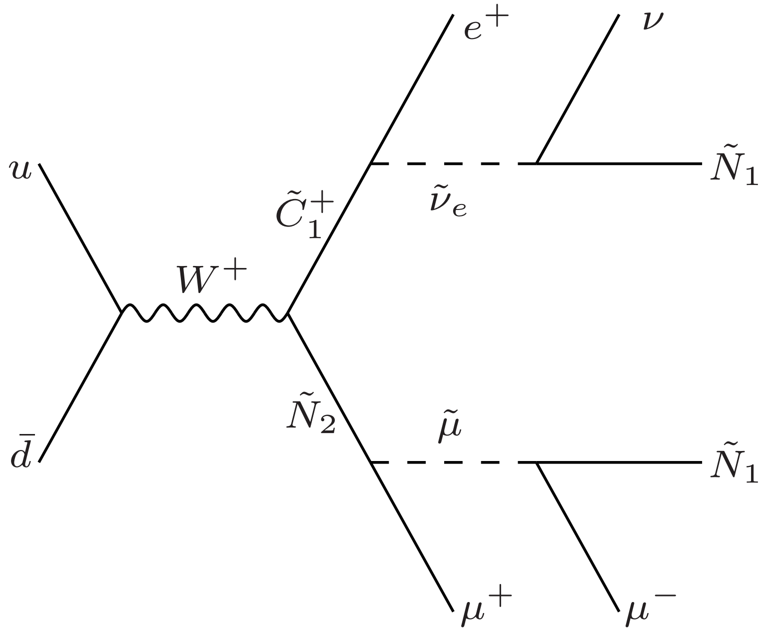
This signal arises if one can produce a pair of wino-like sparticles

$$p\bar{p} \to \tilde{C}_1^{\pm}\tilde{N}_2,$$

which then each decay leptonically with a significant branching fraction,

$$\tilde{N}_2 \to \ell^+\ell^-\tilde{N}_1, \qquad\qquad \tilde{C}_1^{\pm} \to \ell^{\pm}\nu\tilde{N}_1$$

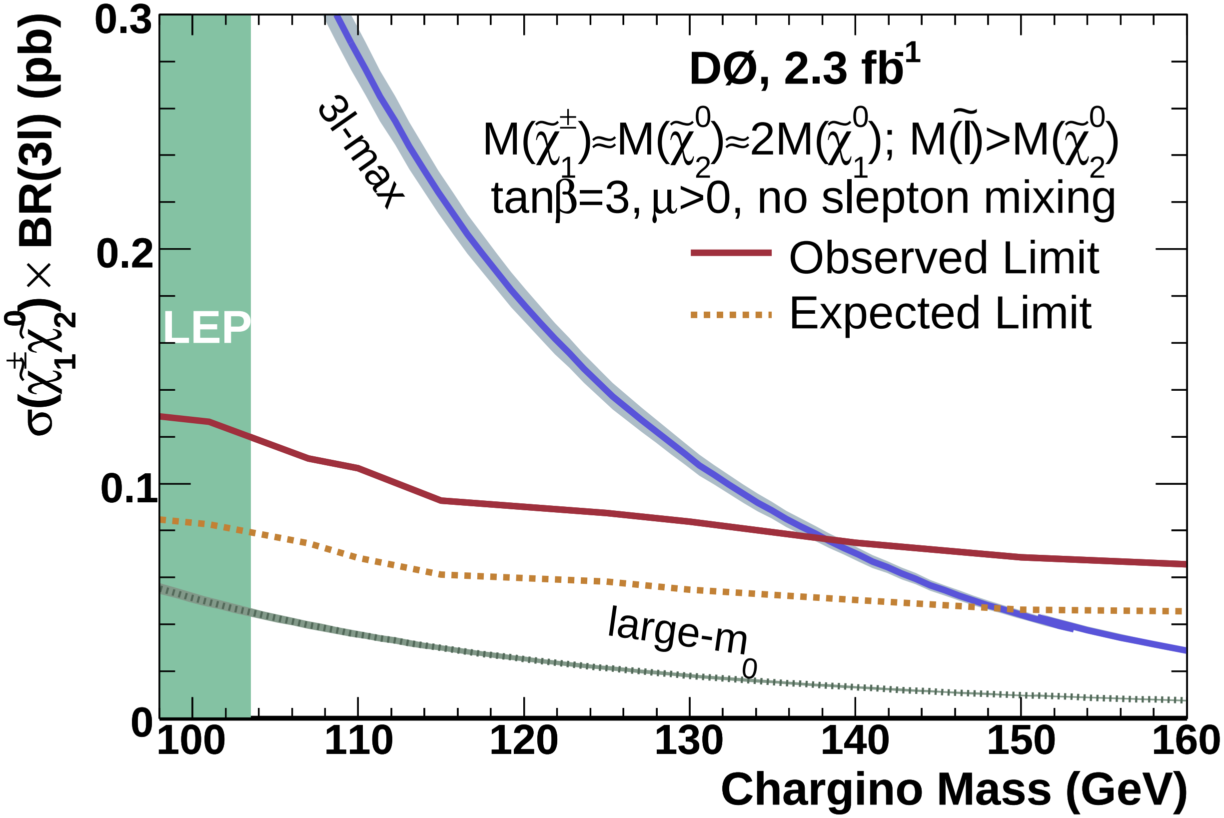
With no hard jets in the event, and three identified leptons, the Standard Model backgrounds are small. Here is a typical Feynman diagram for the whole event:



$$p\bar{p} \to \ell^+\ell^-\ell'^{\pm} + \not\!\!E_T$$

Decays of $\tilde{C}_1^{\pm}$ and $\tilde{N}_2$ through virtual squarks and/or virtual h^0 kill the signal. Decays through Z, W hurt the signal. Decays through sleptons, as shown, help the signal.

Recent Tevatron trilepton search results (from DØ 0901.0646; CDF similar)



This is for an mSUGRA-like model scenario. The greatest sensitivity comes when $m_{\tilde{\ell}}$ is slightly larger than $m_{\tilde{N}_2}$, so that lepton yields are maximized ("3ℓ-max").

A stronger limit follows if squarks are taken heavier than mSUGRA predicts.

A weaker limit follows for larger $\tan\beta$; leptons tend to be τ's.

Multi-Jets + $\not\!\!E_T$ at Tevatron, LHC

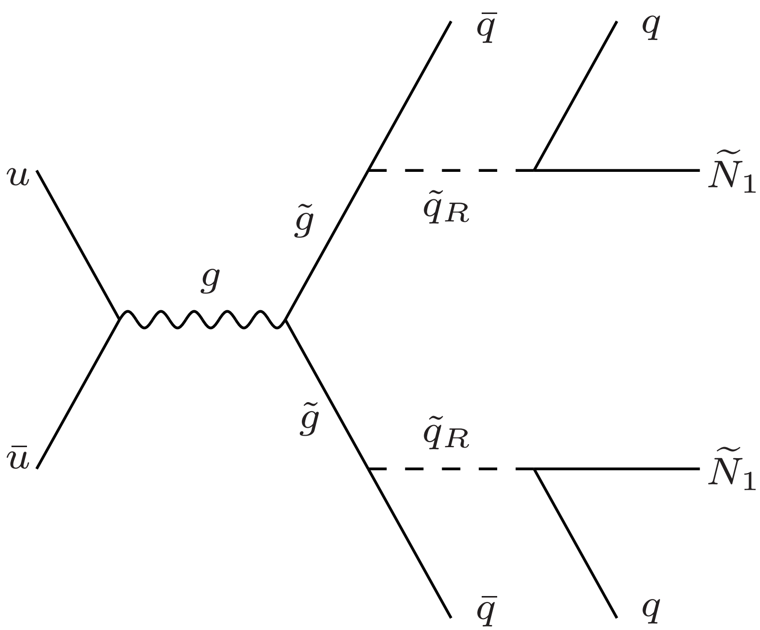
Another strategy: look for events with gluino-gluino, gluino-squark, and squark-squark pair production:

$$p\overline{p} \longrightarrow \tilde{g}\tilde{g},\ \tilde{g}\tilde{q},\ \tilde{q}\tilde{q}$$

followed by decays **without** leptons:

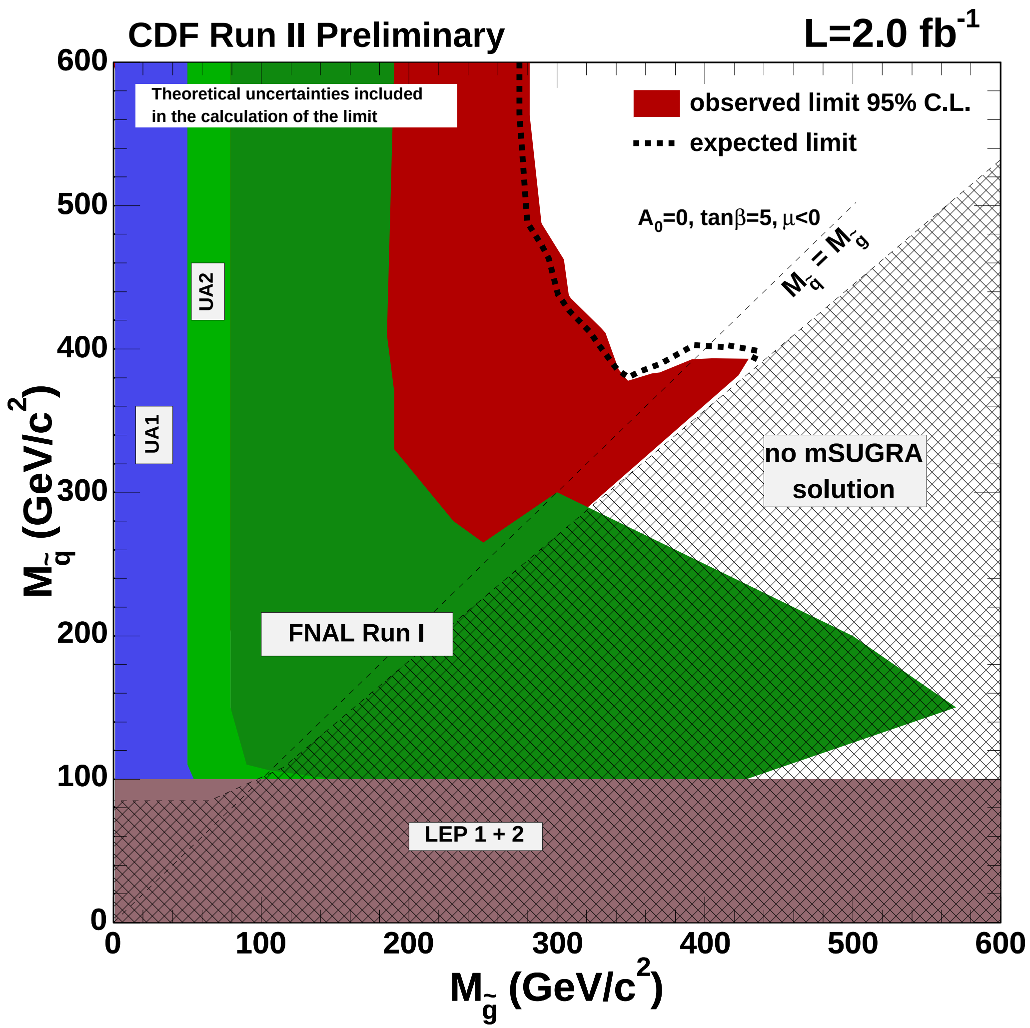
$$\tilde{g} \longrightarrow q\bar{q}\widetilde{N}_1, \qquad\qquad \tilde{q} \longrightarrow q\widetilde{N}_1$$

A typical Feynman diagram for the whole event:



By **vetoing** on isolated, energetic leptons, the Standard Model backgrounds with $\not\!\!E_T$ from $W \longrightarrow \ell\nu$ are reduced.

Tevatron jets$+\not\!\!E_T$ search (from CDF Note 9229, 0811.2512; DØ similar)



Again, this is for an mSUGRA-like model scenario.

Like-Charge Dileptons + $\not{E}_T$ at Tevatron (may also be strong at LHC!)

Exploit the fact mentioned earlier that gluinos cascade decay into leptons of either charge with equal probability:

$$p\overline{p} \longrightarrow \tilde{g}\tilde{g} \longrightarrow \text{(jets)} + \ell^{\pm}\ell^{\pm} + \not{E}_T.$$

Multi-b-jets + $\not{E}_T$ at Tevatron

Produce gluinos that decay into bottom quark and bottom squark:

$$p\overline{p} \longrightarrow \tilde{g}\tilde{g} \longrightarrow (b\tilde{b}_1)(\overline{b}\tilde{b}_1^*) \longrightarrow (b\overline{b}\tilde{N}_1)(b\overline{b}\tilde{N}_1) \longrightarrow (b\overline{b})(b\overline{b}) + \not{E}_T.$$

Light Top Squarks at Tevatron

$$\begin{aligned} p\overline{p} \longrightarrow \tilde{t}_1\tilde{t}_1^* \quad &\longrightarrow \quad (c\tilde{N}_1)(\overline{c}\tilde{N}_1) \longrightarrow jj + \not{E}_T, \qquad \text{or} \\ &\longrightarrow \quad (b\tilde{C}_1^+)(\overline{b}\tilde{C}_1^-) \longrightarrow b\overline{b}\ell^+\ell'^- + \not{E}_T \end{aligned}$$

These and other searches are ongoing. . .

LHC Signals for SUSY in pp collisions at $\sqrt{s} = 7$ TeV

The LHC is a gluon-gluon and gluon-quark collider, to first approximation.

The dominant production cross-sections are:

$$pp \longrightarrow \tilde{g}\tilde{g},\ \tilde{g}\tilde{q},\ \tilde{q}\tilde{q}$$

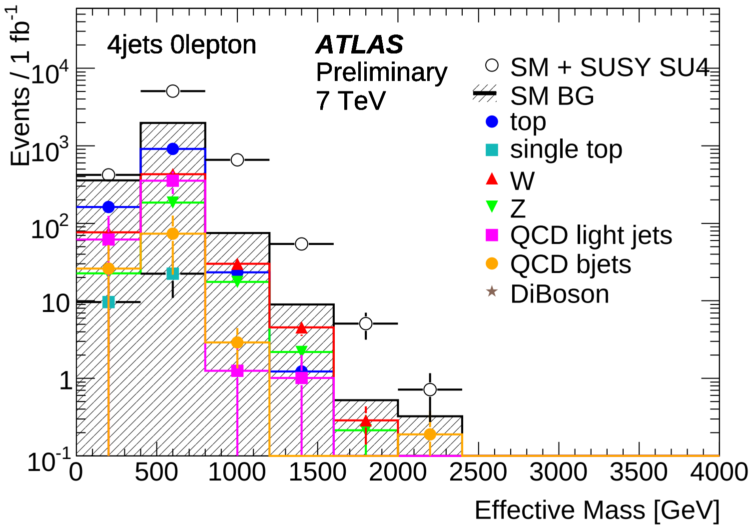
leading to high-p_T jets, possibly leptons, and large $E_T^{\rm miss}$.

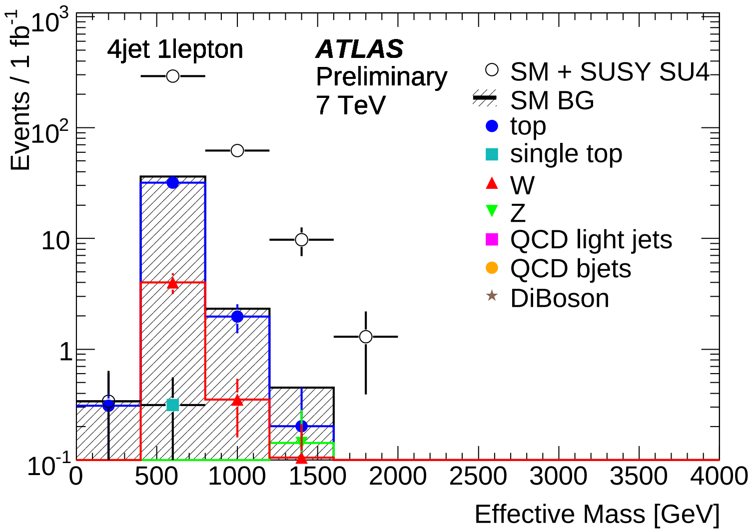
Best early searches may come from channels with exactly 0 or exactly 1 isolated lepton.

The **effective mass** is a useful discovery discriminant:

$$M_{\rm eff} = \sum_{i\leq 4} p_T^{{\rm jet},i} + \sum_i p_T^{{\rm lepton},i} + \not{E}_T .$$

From ATLAS-PHYS-PUB-2010-010:





$$M_{\text{eff}} = \sum_{i \le 4} p_T^{\text{jet},i} + \sum_i p_T^{\text{lepton},i} + \not{E}_T.$$

is also a rough superpartner mass estimate.

Cuts include:

$p_T^{\text{jets}} > 100, 40, 40, 40$ GeV

$\not{E}_T > \max(0.2 M_{\text{eff}},\ 80\ \text{GeV})$

$\Delta\phi(\not{E}_T, \text{jet}) > 0.2$

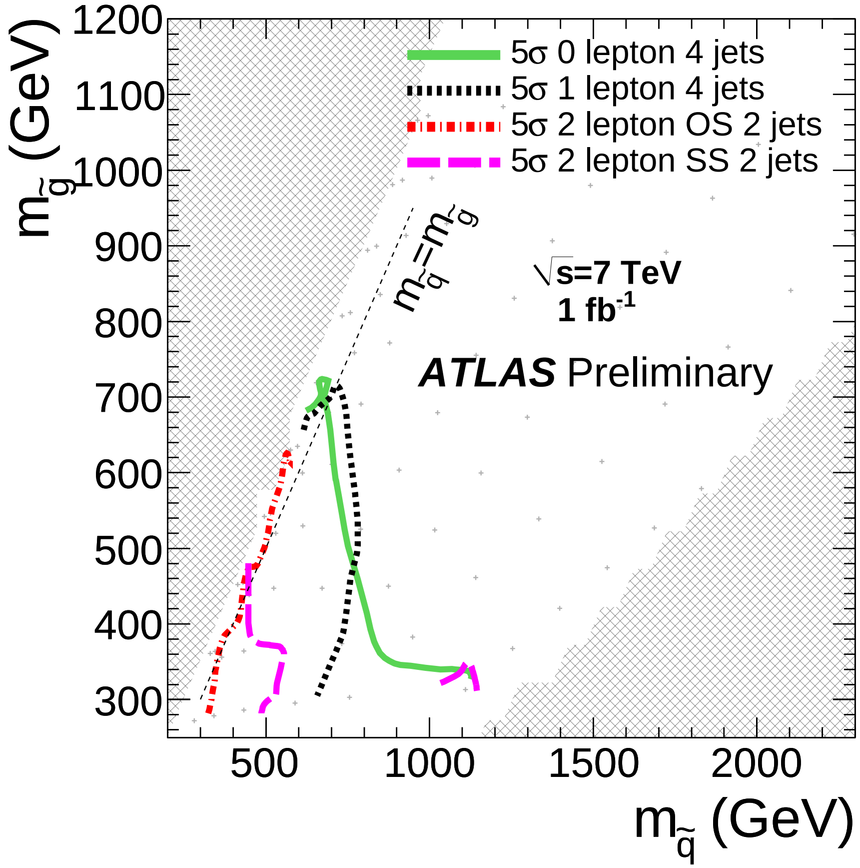
"SUSY SU4" is a benchmark model with $m_{\tilde{g}} \approx m_{\tilde{q}} \approx 410$ GeV.

Requiring one lepton with $p_T > 20$ GeV reduces background more than signal, especially pure-QCD part.

Yesterday's discoveries (top, W, Z) are today's background.

5σ discovery reach estimate, 1 fb^{-1} at $\sqrt{s} = 7$ TeV:

ATLAS-PHYS-PUB-2010-010

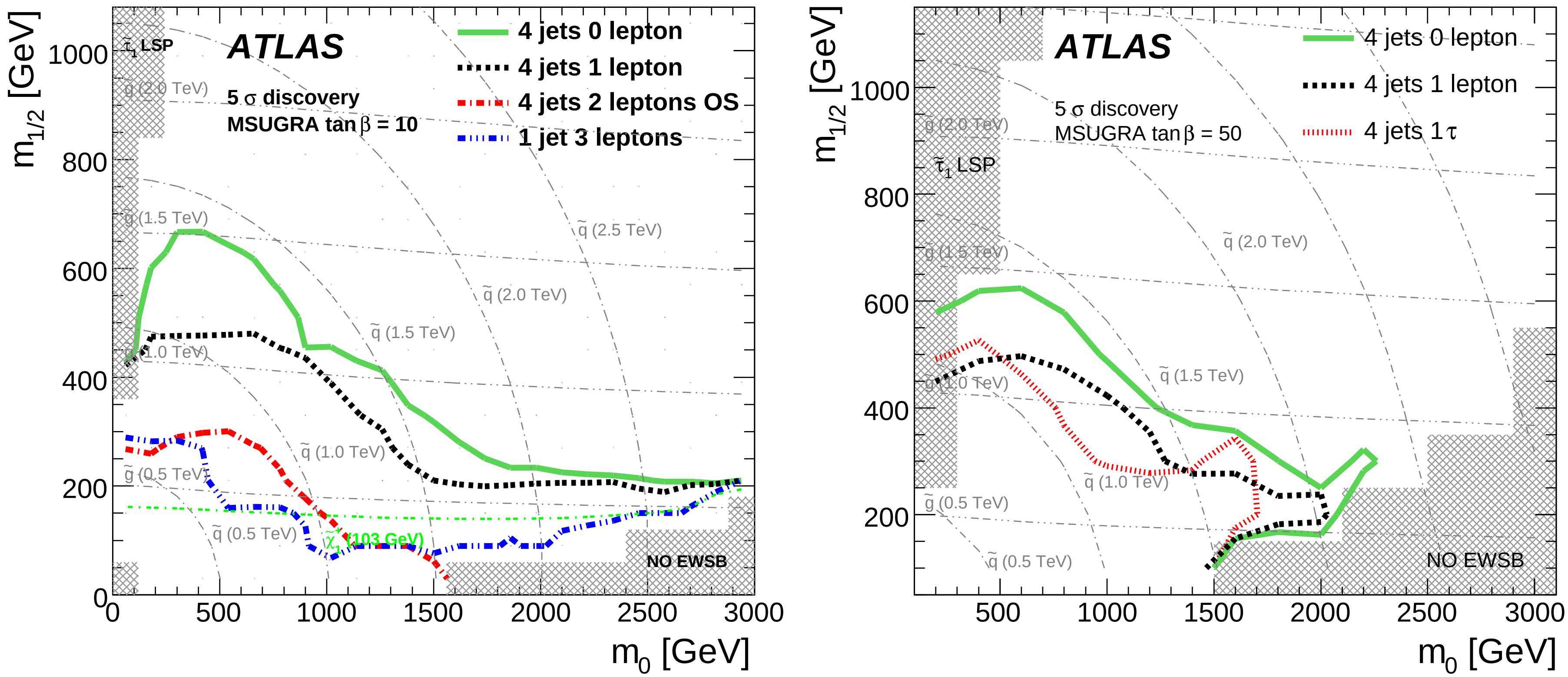


Unfortunately, we should NOT be too surprised if SUSY is not found with 1 fb^{-1} at $\sqrt{s} = 7$ TeV.

BUT

Higher luminosity runs at higher energy should decisively settle the question of whether SUSY is the solution to the hierarchy problem.

ATLAS 5σ discovery reach estimate for mSUGRA, for only 1 fb^{-1} at $\sqrt{s} = 14$ TeV



From ATLAS “CSC book” 0901.0512.

Discovery reach extends well beyond 1 TeV for both gluino and squarks.

Can LHC discover the lightest Higgs boson h^0 in SUSY events?

When kinematically allowed, the wino-like neutralino decays to the LSP and the lightest Higgs boson:

$$\tilde{N}_2 \to \tilde{N}_1 h^0 \qquad (\mathrm{BR} \approx 90\%)$$

This is very common; essentially generic! In the context of trilepton searches at the Tevatron, this kills the signal, and has been called the “spoiler mode”.

However, at LHC, one may be able to exploit this decay to find the Higgs boson, for example from squark/gluino production followed by:

$$\tilde{q}_L \to q\tilde{N}_2 \to qh^0\tilde{N}_1$$

From ATLAS “CSC book” 0901.0512:

Look for $h^0 \to b\bar{b}$ from

the decay $\tilde{N}_2 \to h^0 \tilde{N}_1$

in a sample of SUSY events.

mSUGRA model (SU9) with

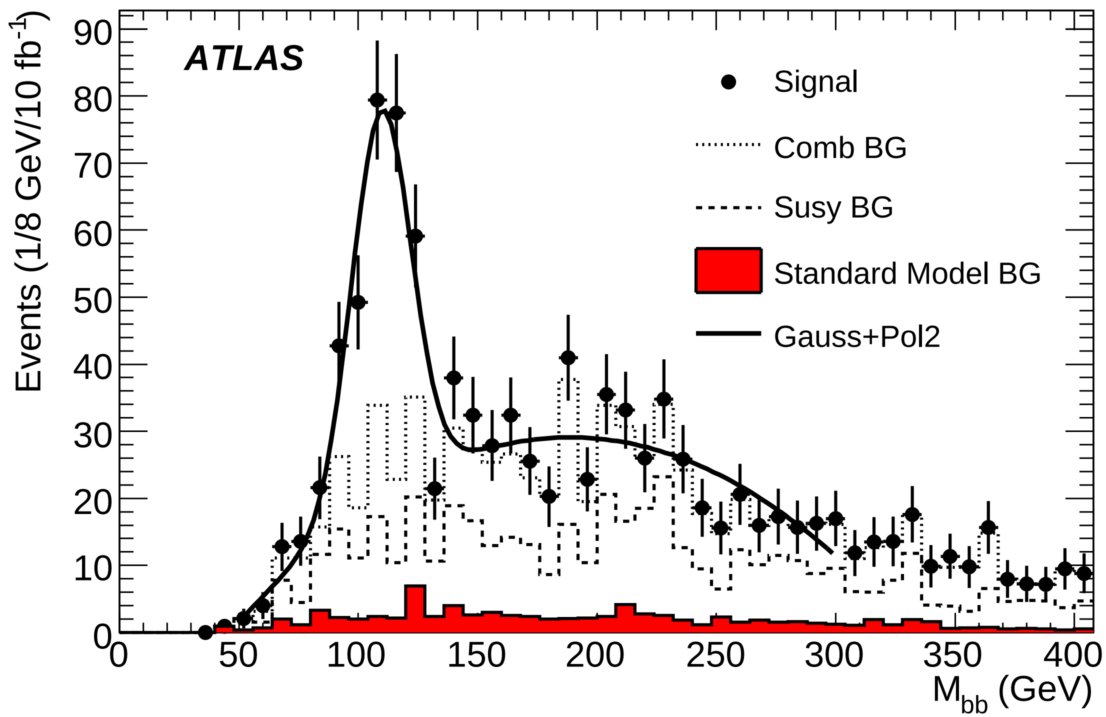
$m_{\tilde{g}} = 1000$ GeV,

$m_{\tilde{q}_L} = 950$ GeV,

$m_{\tilde{N}_2} = 325$ GeV,

$m_{\tilde{N}_1} = 173$ GeV,

$m_{h^0} = 114.5$ GeV



Integrated luminosity = 10 fb^{-1}

Cuts: $\not{E}_T > 300$ GeV,

$p_T^{b_1}, p_T^{b_2} > 50$ GeV,

$p_T^{j_1}, p_T^{j_2} > 100$ GeV,

veto on isolated leptons.

I have not covered many important topics, including:

- Superfields and superspace methods
- Next-to-Minimal Supersymmetric Standard Model
- Gauge-Mediated SUSY Breaking (GMSB)
- Little hierarchy problem
- R-parity violation and alternatives to R-parity
- Dirac gauginos
- Anomaly-Mediated SUSY Breaking (AMSB)
- Hidden Valley and Quirky Models
- Models with extra vectorlike supermultiplets
- Supergravity
- . . .

A bold prediction in two parts:

- Supersymmetry will be discovered at the LHC.
- Some feature of it will be unexpected by (nearly) everyone.

Expect the unexpected!