Supersymmetric GUTs

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Outline

Evolution of SUSY GUTs

- 4 dimensional SUSY GUTs
- 5 dimensional orbifold GUTs
- String orbifolds ⟷ orbifold GUTs
- Conclusions

Outline

- 4 dimensional SUSY GUTs
- 5 dimensional orbifold GUTs
- String orbifolds ⟺ orbifold GUTs
- Conclusions

The Goal of Beyond SM physics
Reduce # fundamental parameters

18 parameters in Standard Model

- 13 charged fermion masses & mixing
- \+ 3 gauge couplings
- \+ 2 W & Higgs mass

27 including neutrino masses & mixing

28 including QCD θ parameter

29 including G_N

Supersymmetric Grand Unified Theories

- $M_Z \ll M_G$ "natural"
- Explains charge quantization & Families
- Unification of gauge couplings
- Yukawa coupling unification
- + family symmetry ➡ fermion mass hierarchy
- Neutrino masses via See-Saw
- LSP - dark matter candidate
- Baryogenesis via leptogenesis
- SUSY desert ➡ LHC probes physics at M_{Pl}
- SUSY GUTs "natural extension of SM"

$$q = \begin{pmatrix} u \\ d \end{pmatrix}, u^c, d^c, l = \begin{pmatrix} \nu \\ e \end{pmatrix}, e^c, \nu^c$$

$$\mathrm{T}_A\, q = \frac{1}{2}\lambda_A\, q, \quad \mathrm{T}_A\, u^c = -\frac{1}{2}\lambda_A^T\, u^c, \quad \mathrm{T}_A\, d^c = -\frac{1}{2}\lambda_A^T\, d^c$$

$$\mathrm{T}_A\, l = \mathrm{T}_A\, e^c = \mathrm{T}_A\, \nu^c = 0$$

$$[\mathrm{T}_A, \mathrm{T}_B] = i\, f_{ABC}\, \mathrm{T}_C \qquad \mathrm{SU(3)}$$

$$T_a\, q = \frac{1}{2}\tau_a\, q, \quad T_a\, l = \frac{1}{2}\tau_a\, l$$

$$T_a\, u^c = T_a\, d^c = T_a\, e^c = T_a\, \nu^c = 0$$

$$[T_a,\ T_b] = i\ \varepsilon_{abc}\ T_c \qquad \mathrm{SU(2)}$$

$$Y\ q = \frac{1}{3}q, \ \ Y\ u^c = -\frac{4}{3}u^c, \ \ Y\, d^c = \frac{2}{3}d^c$$

$$Y\ l = -l, \ \ Y\ e^c = +2e^c, \ \ Y\ \nu^c = 0$$

$$\mathrm{U(1)}_Y$$

$$Q_{EM} = T_3 + \frac{Y}{2}$$

All interactions of fermions & Higgs with gauge bosons fixed by their charges

$$D_\mu = (\partial_\mu + ig_s \mathrm{T}_A G_{\mu A} + igT_a W_{\mu a} + ig' \frac{Y}{2} B_\mu)$$

$$L_{gauge-fermion} = [l^* \, i\overline{\sigma}_\mu D^\mu \, l + \cdots] + [-\frac{1}{2} Tr(G_{\mu\nu} G^{\mu\nu}) + \cdots]$$

The standard Dirac form of the SM can be obtained by defining 4 component Dirac field

$$\Psi_e = \begin{pmatrix} e \\ i\sigma_2 \, (e^c)^* \end{pmatrix}$$

$$H_u = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}, \qquad H_d = \begin{pmatrix} \bar{h}^0 \\ \bar{h}^- \end{pmatrix}$$

$$\mathrm{T}_A\, H_u = \mathrm{T}_A\, H_d = 0$$

$$T_a\, H_u = \frac{1}{2}\tau_a\, H_u,\ \ T_a\, H_d = \frac{1}{2}\tau_a\, H_d$$

$$Y\ H_u = +\ H_u,\ \ Y\ H_d = -\ H_d$$

$$L_{gauge-Higgs} = |\,D_\mu H_u\,|^2 + |\,D_\mu H_d\,|^2 + V_{Higgs}$$

$$-L_{Yukawa} = \lambda_e^{ij}\ l_i\ e_j^c\ H_d + \lambda_d^{ij}\ q_i\ d_j^c\ H_d + \lambda_u^{ij}\ q_i\ u_j^c\ H_u + \lambda_\nu^{ij}\ l_i\ \nu_j^c\ H_u - \frac{1}{2} M_{ij}\ \nu_i^c\ \nu_j^c$$

The MSSM is obtained by defining chiral superfields

$$Q=\begin{pmatrix}U\\D\end{pmatrix},\ U^c,\ D^c,\ L=\begin{pmatrix}V\\E\end{pmatrix},\ E^c,\ V^c$$

$$E(y,\theta)=\tilde{e}(y)+\sqrt{2}(\theta\, e(y))+(\theta\,\theta)F_e(y)$$

$$y^\mu=x^\mu-i\theta\,\sigma^\mu\overline{\theta},\qquad (\theta\, e(y))\equiv\theta^\alpha\ e(y)_\alpha$$

$$L_{gauge-matter}=\int d^4\theta\,[L_i^\dagger\ exp(-2\,[g_sV_g+gV_W+g'\frac{Y}{2}V_B])\,L_i+\cdots]$$

$$+\int d^2\theta\,[\frac{1}{8g_s^2}Tr(W_g{}^\alpha\,W_{g\alpha})+\cdots]+h.c.$$

$$L_{Yukawa}=\int d^2\theta\ [\lambda_e^{ij}\ L_i\ E_j^c\ H_d+\lambda_d^{ij}\ Q_i\ D_j^c\ H_d$$

$$+\lambda_u^{ij}\ Q_i\ U_j^c\ H_u+\lambda_\nu^{ij}\ L_i\ V_j^c\ H_u$$

$$-\frac{1}{2}M_{ij}\,V_i^c\,V_j^c-\mu\,H_u\ H_d\,]+h.c.$$

Grand Unification & Charge Quantization

$$q = \begin{pmatrix} u & u & u \\ d & d & d \end{pmatrix} \qquad u^c = \begin{pmatrix} u^c & u^c & u^c \end{pmatrix} \qquad d^c = \begin{pmatrix} d^c & d^c & d^c \end{pmatrix}$$

$$l = \begin{pmatrix} \nu \\ e \end{pmatrix} \qquad e^c$$

$$Q = \begin{pmatrix} u & u & u & \nu \\ d & d & d & e \end{pmatrix} \qquad Q^c = \begin{pmatrix} u^c & u^c & u^c & \nu^c \\ d^c & d^c & d^c & e^c \end{pmatrix}$$

$$Q_{EM} = \tfrac{1}{2}\left(B - L\right) + T_L^3 + T_R^3$$

Pati-Salam - lepton # = 4th color

Quarks & leptons fit into two irreducible reps.

$$Q = (q\ l), \quad Q^c = (q^c\ l^c) \qquad \mathrm{SU}(4)_C \otimes \mathrm{SU}(2)_L \otimes SU(2)_R$$

$$q^c = \begin{pmatrix} u^c \\ d^c \end{pmatrix}, \quad l^c = \begin{pmatrix} \nu^c \\ e^c \end{pmatrix} \qquad (4,2,1) \oplus (\bar{4},1,\bar{2})$$

The two Higgs doublets

$$H = (H_d\ H_u) \qquad (1,2,\bar{2})$$

$$\lambda\ \ Q^c\, H\, Q \quad \Rightarrow \quad \lambda_t = \lambda_b = \lambda_\tau = \lambda_{\nu_\tau} \equiv \lambda$$

Yukawa coupling unification

Grand Unification – SO(10) GUT

State	Y $= \frac{2}{3}\Sigma(C) - \Sigma(W)$	Color C spins	Weak W spins
$\bar{\nu}$	0	− − −	− −
ē	2	− − −	+ +
u_r		+ − −	− +
d_r		+ − −	+ −
u_b	$\frac{1}{3}$	− + −	− +
d_b		− + −	+ −
u_y		− − +	− +
d_y		− − +	+ −
$\bar{u}_r$		− + +	− −
$\bar{u}_b$	$-\frac{4}{3}$	+ − +	− −
$\bar{u}_y$		+ + −	− −
$\bar{d}_r$		− + +	+ +
$\bar{d}_b$	$\frac{2}{3}$	+ − +	+ +
$\bar{d}_y$		+ + −	+ +
ν	−1	+ + +	− +
e		+ + +	+ −

Georgi
Fritzsch & Minkowski

spinor repsn. of SO(10)

tensor product of 5 spin ½ w/ even no. + signs

Aside: SO(10) group theory

The defining representation

$10_i, \; i = 1, \cdots, 10$

$SO(10) = \left\{ O: \text{ real } 10 \times 10 \text{ matrices; } O^T O = 1; \det O = 1 \right\}$

$10'_i \; = \; O_{ij} 10_j$

Infinitesimal transf.-

$O = 1 + i\tilde{\omega} \quad \text{with} \quad \tilde{\omega}^T = -\tilde{\omega}$

$\tilde{\omega}_{ij} \equiv \omega_{ab} \Sigma_{ij}^{ab} \quad \text{with} \quad \Sigma_{ij}^{ab} = i(\delta_i^a \delta_j^b - \delta_j^a \delta_i^b) \;$ generators

$\omega_{ab} = -\omega_{ba} \quad 45$ parameters

Lie algebra

$$[\Sigma^{ab}, \Sigma^{cd}]_{ik} \equiv \Sigma^{ab}_{ij}\Sigma^{cd}_{jk} - \Sigma^{cd}_{ij}\Sigma^{ab}_{jk} = [\Sigma^{ad}_{ik}\delta_{bc} - \Sigma^{ac}_{ik}\delta_{bd} + \Sigma^{bc}_{ik}\delta_{ad} - \Sigma^{bd}_{ik}\delta_{ac}]$$

$$45'_{ij} = O_{ik}O_{jl}45_{kl} \quad \text{or} \quad 45'_{ij} = (O45O^T)_{ij}$$

Define spinor representation of SO(10)

$$\Gamma_i^\dagger = \Gamma_i,\ \{\Gamma_i, \Gamma_j\} = 2\delta_{ij} \qquad 2^5 \times 2^5 \text{ matrices}$$

$$\Gamma_{11} \equiv \prod_{i=1}^{10}\Gamma_i,\ \ \{\Gamma_{11}, \Gamma_i\} = 0\ \ \forall i,\ \ \Gamma_{11}^2 = 1 \quad \Rightarrow \text{ eig. } \pm 1$$

$$\Sigma_{ij} = \frac{i}{4}[\Gamma_i, \Gamma_j] \qquad [\Gamma_{11}, \Sigma_{ij}] = 0$$

$$16 \oplus \overline{16}$$

Define $A_\alpha = \frac{\Gamma_{2\alpha-1} + i\Gamma_{2\alpha}}{2}$, $A_\alpha^\dagger = \frac{\Gamma_{2\alpha-1} - i\Gamma_{2\alpha}}{2}$ $,\alpha = 1,\cdots,5$

$$\{A_\alpha, A_\beta\} = 0,\ \{A_\alpha, A_\beta^\dagger\} = \delta_{\alpha\beta}$$

$$\mathscr{L}_{SO(10)} = \{Q_A,\ \Delta_{\alpha\beta},\ \Delta_{\alpha\beta}^\dagger,\ X\}$$

$Q_A = A_\alpha^\dagger \frac{\lambda_{\alpha\beta}^a}{2} A_\beta,\ A = 1,\cdots,24 \quad [Q_A, Q_B] = i\, f_{ABC}\, Q_C$

where $\lambda_{\alpha\beta}^A$ 5×5 traceless, hermitian matrices of SU(5)

$$\Delta_{\alpha\beta} = A_\alpha A_\beta = -\Delta_{\beta\alpha},\ \Delta_{\alpha\beta}^\dagger = A_\alpha^\dagger A_\beta^\dagger = -\Delta_{\beta\alpha}^\dagger$$

$$X = -2\sum_{\alpha=1}^{5}(A_\alpha^\dagger A_\alpha - \frac{1}{2})$$

$$\text{Define} \quad |0\rangle \equiv |\mathbf{1}\rangle \quad \ni \quad A_\alpha |0\rangle \equiv 0$$

$$\Rightarrow \qquad Q_A |0\rangle \equiv 0$$

$$\Delta^\dagger_{\alpha\beta} |0\rangle = |\mathbf{10}\rangle_{\alpha\beta}\,, \quad \epsilon^{\alpha\beta\gamma\delta\lambda} \Delta^\dagger_{\alpha\beta} \Delta^\dagger_{\gamma\delta} |0\rangle = |\overline{\mathbf{5}}\rangle^\lambda$$

$$SO(10) \to SU(5) \otimes U(1)_X$$

$$\to \ \mathrm{SO}(6) \otimes \mathrm{SO}(4) \approx \mathrm{SU}(4)_C \otimes SU(2)_L \otimes SU(2)_R$$

$$\to \ \mathrm{SU}(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$$

$$\to SU(5)' \otimes U(1)' \quad \text{flipped SU(5)}$$

Grand Unification – SO(10) GUT

Multiplet	State	$Y = \frac{2}{3}\Sigma(C) - \Sigma(W)$	Color C spins	Weak W spins
1	$\bar{\nu}$	0	$- - -$	$- -$
10	$\bar{e}$	2	$- - -$	$+ +$
	u_r	$\frac{1}{3}$	$+ - -$	$- +$
	d_r		$+ - -$	$+ -$
	u_b		$- + -$	$- +$
	d_b		$- + -$	$+ -$
	u_y		$- - +$	$- +$
	d_y		$- - +$	$+ -$
	$\bar{u}_r$	$-\frac{4}{3}$	$- + +$	$- -$
	$\bar{u}_b$		$+ - +$	$- -$
	$\bar{u}_y$		$+ + -$	$- -$
$\bar{5}$	$\bar{d}_r$	$\frac{2}{3}$	$- + +$	$+ +$
	$\bar{d}_b$		$+ - +$	$+ +$
	$\bar{d}_y$		$+ + -$	$+ +$
	ν	-1	$+ + +$	$- +$
	e		$+ + +$	$+ -$

Georgi
Fritzsch & Minkowski

spinor repsn. of SO(10)

tensor product of 5 spin ½ w/ even no. + signs

Georgi & Glashow

SU(5)

SU(5) group theory

$$SU(5) = \{U \mid U = 5 \times 5 \text{ complex matrix; } U^{\dagger}U = 1;\ detU = 1\}$$

$$5'^{\alpha} = U^{\alpha}{}_{\beta}\ 5^{\beta}$$

$$U = \exp(iT_A\ \omega_A) \quad \text{where}$$

$$Tr(T_A) = 0,\ T_A^{\dagger} = T_A,\ A = 1, \ldots, 24$$

$$[T_A, T_B] = i\ f_{ABC}\ T_C$$

f_{ABC} structure constants of SU(5)

Choose normalization: $Tr(\mathrm{T}_A^f\ \mathrm{T}_B^f) = \frac{1}{2}\delta_{AB}$

$$SU(3): \qquad \mathrm{T}_A = \begin{pmatrix} \frac{1}{2}\lambda_A & 0 \\ 0 & 0 \end{pmatrix}, \ A = 1, \ldots, 8$$

$$SU(2): \qquad T_A = \begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{2}\tau_{(A-20)} \end{pmatrix}, \ A = 21, 22, 23$$

$$U(1)_Y: \quad T_{24} = \sqrt{\frac{3}{5}} \begin{pmatrix} -1/3 & 0 & 0 & & \\ 0 & -1/3 & 0 & & 0 \\ 0 & 0 & -1/3 & & \\ & 0 & & 1/2 & 0 \\ & & & 0 & 1/2 \end{pmatrix} \equiv \sqrt{\frac{3}{5}}\,\frac{Y}{2}$$

$$T_A, \ A = 9, \ldots, 20: \quad \frac{1}{2}\begin{pmatrix} & & & 1 & 0 \\ & 0 & & 0 & 0 \\ & & & 0 & 0 \\ 1 & 0 & 0 & & \\ 0 & 0 & 0 & & 0 \end{pmatrix}, \quad \frac{1}{2}\begin{pmatrix} & & & -i & 0 \\ & 0 & & 0 & 0 \\ & & & 0 & 0 \\ i & 0 & 0 & & \\ 0 & 0 & 0 & & 0 \end{pmatrix}$$

$$5^{\alpha} = \begin{pmatrix} d^a \\ l^i \end{pmatrix}, a = 1, 2, 3;\ i = 4, 5$$

$$d^a = (3, 1, \text{-}2/3)\ ,\ \ l^i = (1, 2, +1)$$

$$\bar{5}_{\alpha} = \begin{pmatrix} d_a^c \\ l_i^c \end{pmatrix}, a = 1, 2, 3;\ i = 4, 5$$

$$d^c = (\bar{3}, 1, 2/3),\ \ l^c = (1, \bar{2}, -1)$$

$$10^{\alpha\beta} = -10^{\beta\alpha} \propto 5_1^{\alpha}\ 5_2^{\beta} - 5_2^{\alpha} 5_1^{\beta}$$

$$10^{ab} \equiv \epsilon^{abc} (u^c)_c = (\bar{3}, 1, -4/3)$$

$$10^{ai} \equiv q^{ai} = (3, 2, 1/3) \qquad\qquad 10^{ij} \equiv \epsilon^{ij} e^c = (1, 1, +2)$$

To summarize -

$$\bar{5}_\alpha = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ -e \\ \nu \end{pmatrix}, \; 10^{\alpha\beta} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & u^1 & d^1 \\ -u_3^c & 0 & u_1^c & u^2 & d^2 \\ u_2^c & -u_1^c & 0 & u^3 & d^3 \\ -u^1 & -u^2 & -u^3 & 0 & e^c \\ -d^1 & -d^2 & -d^3 & -e^c & 0 \end{pmatrix}$$

Gauge coupling unification

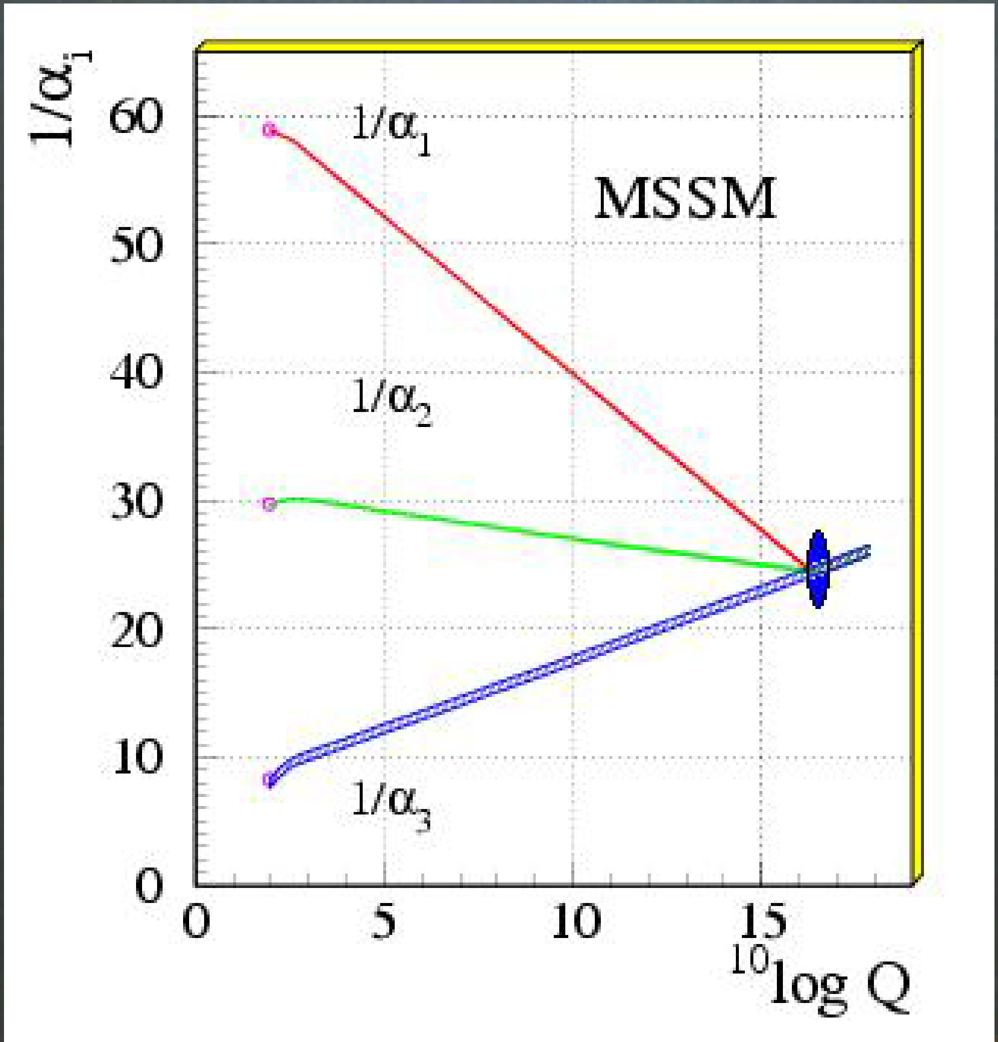
Boundary conditions at M_{GUT}

$$g_G T_A G_\mu^A \supset g_s \mathrm{T}_A G_\mu^A + g T_a W_\mu^a + g' \frac{Y}{2} B_\mu$$

$$g_s = g_3 \,,\; g = g_2 \,,\; g' = \sqrt{3/5}\, g_1$$

$$\ni\; g_3 = g_2 = g_1 = g_G \,, \qquad \sin^2\theta_W = \frac{g'^2}{g^2 + g'^2} = \frac{3}{8}$$

Gauge coupling unification ★



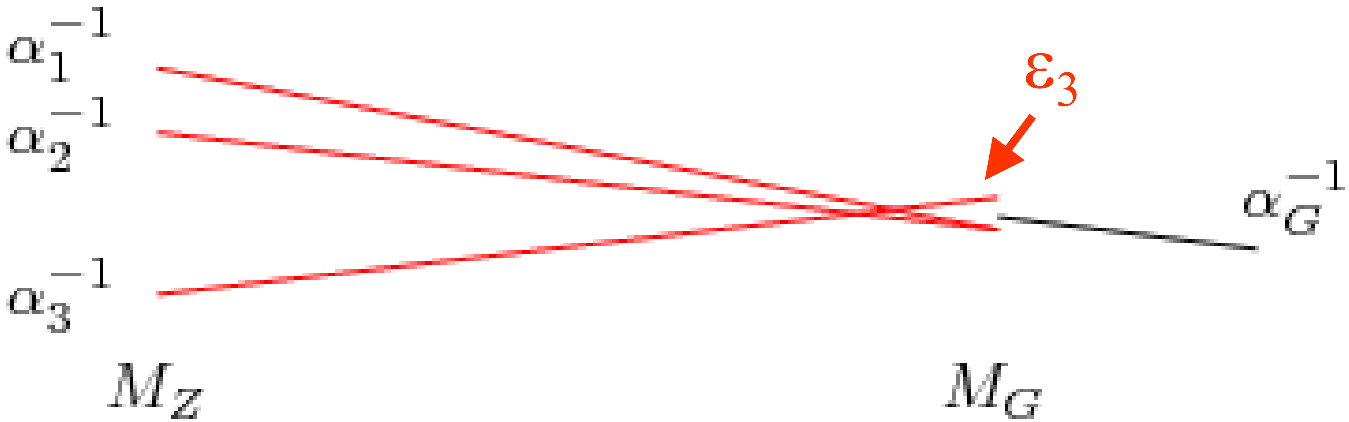
Dimopoulos, Raby
& Wilczek
PRD24, 1681 (1981)

LEP data !!

Amaldi, de Boer
& Furstenau,
PLB260, 447 (1991)

— Dimopoulos, S.R. & Wilczek; Dimopoulos & Georgi

* Only evidence for SUSY



- Significant GUT threshold corrections from Higgs and GUT breaking sectors

Def: $M_G \iff \alpha_1(M_G) = \alpha_2(M_G) \equiv \tilde{\alpha}_G$

Using two loop RGE from M_Z to M_G, find

$$M_G \approx 3 \times 10^{16} \text{ GeV}$$
$$\alpha_G^{-1} \approx 24$$

Good fit requires:

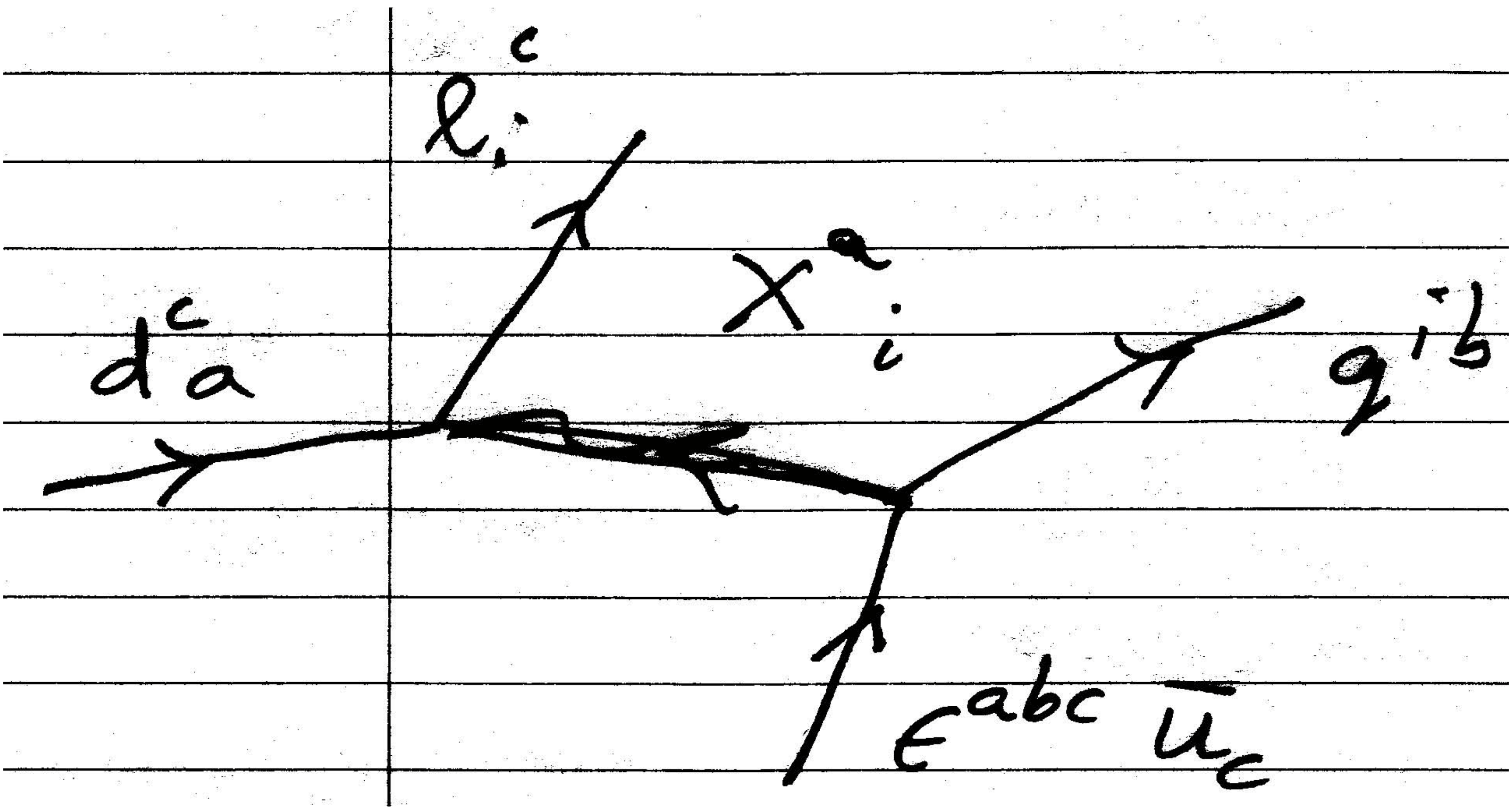
$$\epsilon_3 \equiv \frac{(\alpha_3(M_G) - \tilde{\alpha}_G)}{\tilde{\alpha}_G} \sim \text{-3\% to -4\%}$$

This assumes degenerate squarks and sleptons and degenerate gauginos at GUT scale.

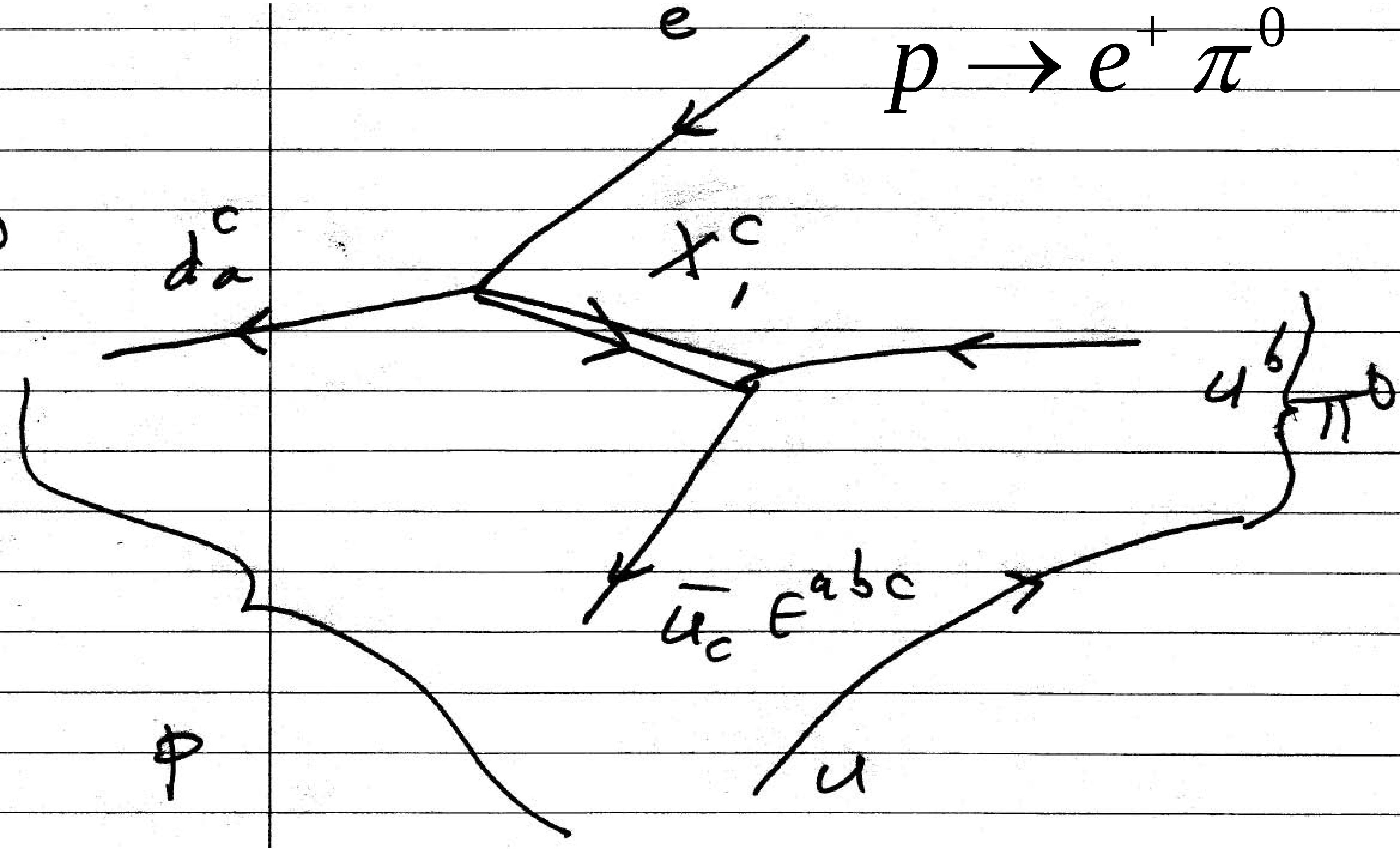
If, for example, $M_3 << M_2 \simeq M_1$ then can have $\varepsilon_3 \geq 0$.

See Raby, Ratz, Schmidt-Hoberg arXiv:0911.4249 (hep-ph)

Proton decay – dim 6 operators



$$p \to e^{+} \pi^{0}$$



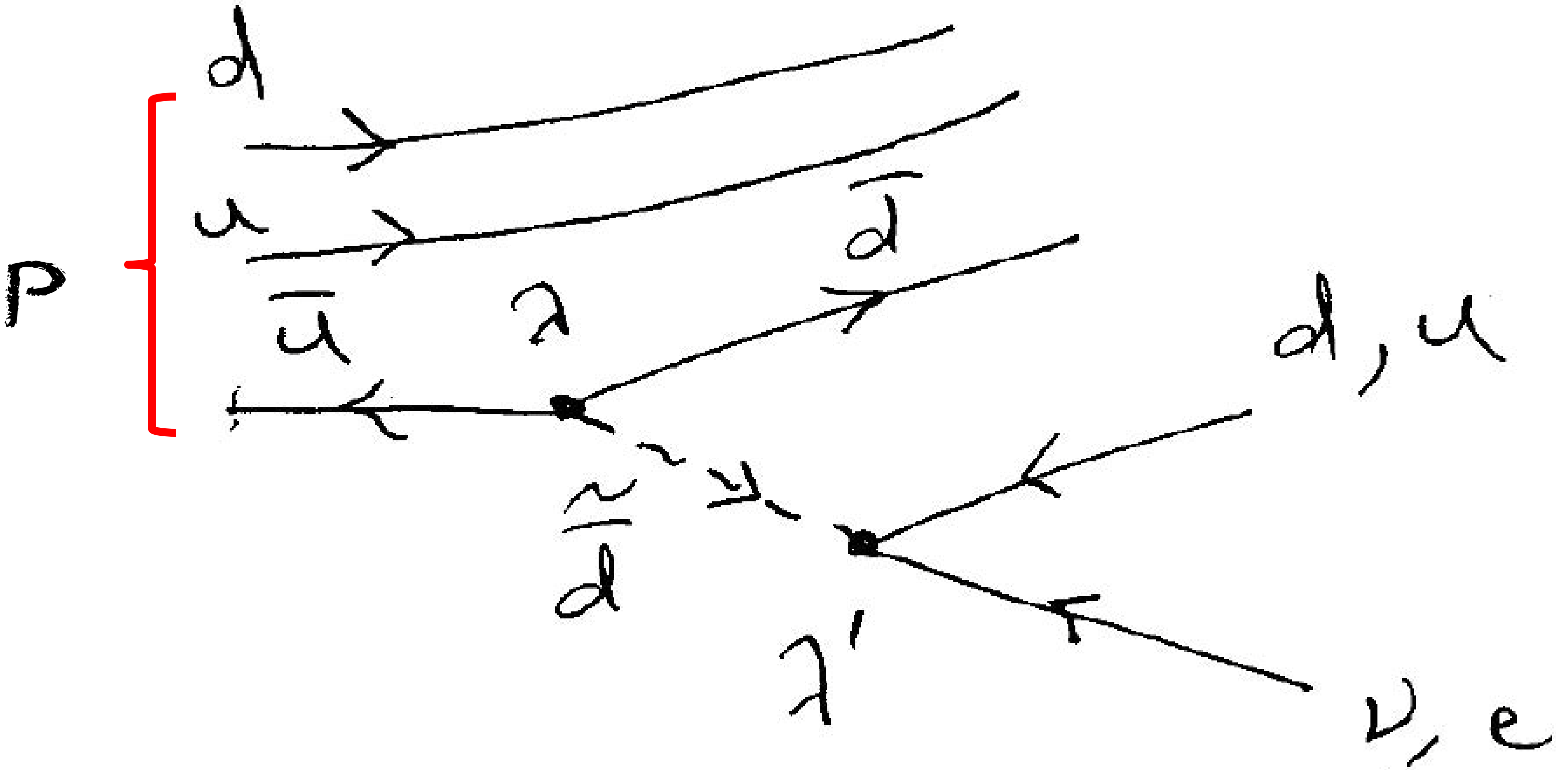
$$\tau_p \big/ Br(p \to e^+\pi^0) \geq 1\times 10^{34}\, yrs \qquad Super-Kamiokande$$

$$\sim \quad \frac{M_G^4}{g_G^4\, m_p^5}$$

$$\approx \quad 10^{36\pm 2} \;\; yrs \qquad Theory$$

Proton decay – dim 4 operators

$$10\,\bar{5}\,\bar{5} \supset \lambda\; U^c D^c D^c + \lambda'\; \mathrm{Q\,L}\, D^c \;+ \lambda''\, E^c\, \mathrm{L\,L}$$



$$p \to \pi^+\;\pi^0\;\bar{\nu}\;\text{ or }\;(\pi^0\;\pi^0\,,\pi^+\;\pi^-)\,e^+$$

$$\Gamma_p \approx \frac{(\lambda\ \lambda')^2\, m_p^5}{\tilde{m}^4}$$

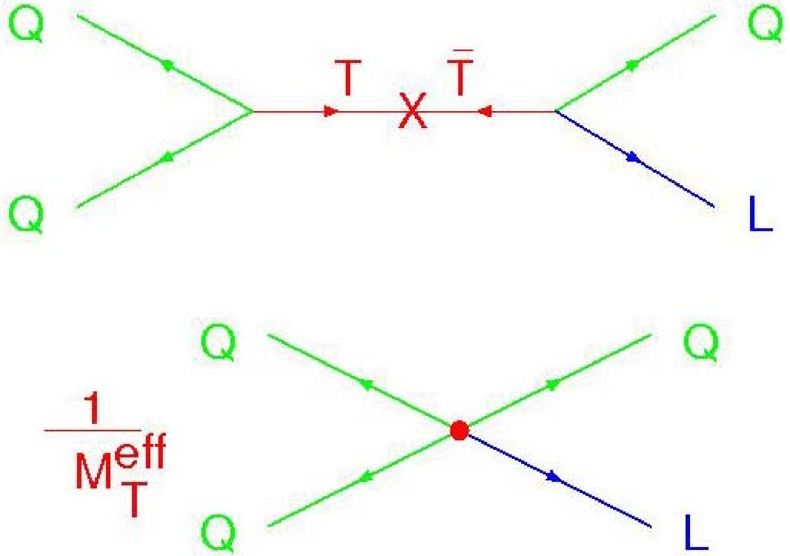
$$\frac{\lambda\lambda'}{\tilde{m}^2} < \frac{g_G^2}{M_G^2} \leq 10^{-32}\, GeV^{-2} \quad \Rightarrow \quad \lambda\lambda' < 10^{-26} \left(\frac{\tilde{m}}{1TeV} \right)^2$$

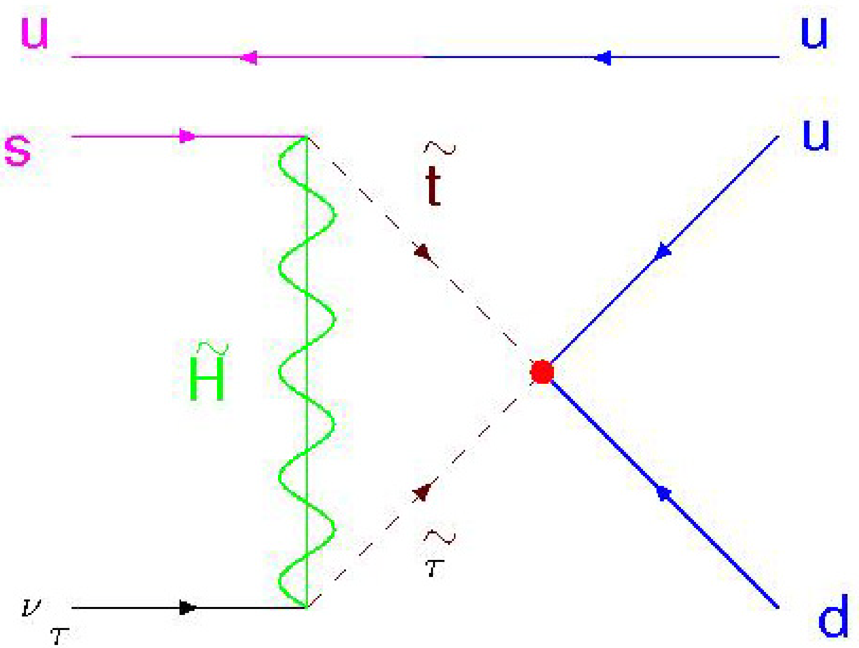
$\mathbb{Z}_2$ matter parity excludes dim 4 operators

F $\rightarrow$ -F, H $\rightarrow$ H

Proton decay - dim 5 operators

$$10\,10\,10\,\bar{5} \supset \mathrm{Q\,Q\,Q\,L} + U^c U^c D^c E^c$$





$$(LF) \propto \frac{\lambda_t \; \lambda_\tau}{16\pi^2} \frac{\sqrt{\mu^2 + M_{1/2}^2}}{m_{16}^2}$$

$$A(p \to K^+ \overline{\nu}) \propto \frac{c\;c}{M_T^{eff}} \, (\mathrm{LF})$$

c ~ Yukawa couplings

minimize LF

maximize M_T^{eff}

$$M_T = \begin{pmatrix} 0 & M_G \\ M_G & X \end{pmatrix} \;\Rightarrow\; \frac{1}{M_T^{eff}} \equiv \frac{X}{M_G^2}, \quad M_D = \begin{pmatrix} 0 & 0 \\ 0 & X \end{pmatrix}$$

Thus for $X << M_G$ obtain $M_T^{eff} >> M_G$

$$Eg. \quad W \supset 10\; 45\; 10' + X\,(10')^2$$

$$\langle 45 \rangle \sim M_G\,(B-L)$$

$$\epsilon_3 = \epsilon_3^{\text{Higgs}} + \epsilon_3^{\text{GUT breaking}} + \cdots$$

$$\epsilon_3^{\text{Higgs}} = \frac{3\alpha_G}{5\pi} \ln\left(\frac{M_T^{eff}}{M_G}\right)$$

Model	Minimal SU_5	SU_5 "Natural" D/T [17]	Minimal SO_{10} [18]
$\epsilon_3^{\text{GUTbreaking}}$	0	-7.7%	-10%
$\epsilon_3^{\text{Higgs}}$	-4%	$+3.7\%$	$+6\%$
M_T^{eff} [GeV]	2×10^{14}	3×10^{18}	6×10^{19}

Minimal SU_5: Proton decays toooo fast !!

$$\tau_{(p\to K^+\bar{\nu})} > 2.3\times 10^{33}\ yrs\ \ (92ktyr)\ \ \text{at}\ \ 90\%CL$$

$$\sim (1/3 - 3)\times 10^{34}\ \ yrs \qquad Theory$$

Quark & Lepton masses and mixing

Hierarchical $\lambda\, q_3\, \overline{u_3}\, H_u \qquad \lambda \sim \mathrm{O}(1)$

$$q_i\, \overline{u_j}\, H_u \left(\frac{\mathrm{S}}{\mathrm{M_P}}\right)^{n(i,j)}$$

Froggatt-Nielsen

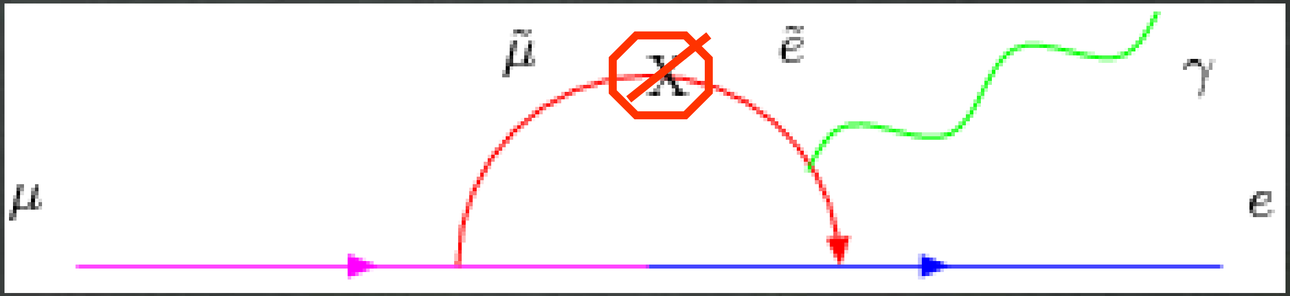
Flavor symmetry breaking

$U(1)$ or non-Abelian SU(2), SU(3)

$\mathrm{S}_3 \approx D_3,\ D_4,\ A_4,\ \Delta(27),\ \Delta(54)$

Fermion mass hierarchy ⟹ FCNC

Non-abelian family symmetry



→ Suppresses flavor violation

Fermion Masses in SO(10)

λ 16 10 16 top, bottom, tau, ν_τ

Yukawa unification

"Minimal" SO(10) Model

1) $$W \supset 16\,10\,16 + 16\,\overline{126}\,16 + 16\,120\,16 + 210^3 + 210^2 + 210\,126\,\overline{126} + \ldots$$

Aulakh, Babu, Bajc, Chen, Mahanthappa, Mohapatra, Senjanovic, ...

Renormalizable ! - large rep's & few predictions

"Minimal" SO(10) Model II

$$2) \quad W \supset 16\,10\,16 + 16\,10\,\frac{45}{M}\,16 + \cdots$$

Albright, Anderson, Babu, Barr, Barbieri, Berezhiani, Blazek, Carena, Dermisek, Dimopoulos, Hall, Pati, Raby, Romanino, Rossi, Starkman, Wagner, Wilczek, Wiesenfeldt, Willenbrock

Effective higher dimension operators,

Small rep's + Many predictions !!

Possible UV completion to strings !!

Yukawa Unification & Soft SUSY breaking

Blazek, Dermisek & Raby PRL 88, 111804 (2002)

PRD 65, 115004 (2002)

Baer & Ferrandis, PRL 87, 211803 (2001)

Auto, Baer, Balazs, Belyaev, Ferrandis & Tata

JHEP 0306:023 (2003)

Tobe & Wells **NPB 663, 123 (2003)**

Dermisek, Raby, Roszkowski & Ruiz de Austri

JHEP 0304:037 (2003)

JHEP 0509:029 (2005)

Baer, Kraml, Sekmen & Summy

JHEP 0803:056 (2008)

JHEP 0810:079 (2008)

$$\lambda \ \ 16_3 \ 10 \ 16_3$$

$$\lambda_t = \lambda_b = \lambda_\tau = \lambda_{\nu_\tau} \equiv \lambda$$

Note, CANNOT predict top mass due to large SUSY threshold corrections to bottom and tau mass

So instead use Yukawa unification to predict soft SUSY breaking masses !!

$MSO_{10}SM^{TM}$

$$\lambda \;\; 16_3 \; 10 \; 16_3$$

$$\lambda_t = \lambda_b = \lambda_\tau = \lambda_{\nu_\tau} \equiv \lambda$$

Fit t,b,tau requires

$$A_0 \approx -2 m_{16} \quad m_{10} \approx \sqrt{2} m_{16}$$

$$m_{16} > \textit{few} \;\text{TeV} \qquad \mu, M_{1/2} << \; m_{16}$$

$$\tan\beta \approx 50$$

Radiative EWSB requires

$$\Delta m_H^2 \equiv \frac{\left(m_{H_d}^2 - m_{H_u}^2\right)}{2m_{10}^2} \approx 13\%$$

Roughly ½ comes from RG running from

$M_G \to m_{\nu_\tau}$

Blazek, Dermisek & Raby "Just so"

Varying $A_0,\ m_{10},\ \Delta m_H^2$

$\Rightarrow\ m_A$ arbitrary

Note, this is difficult in bottom – up approach !!

$MSO_{10}SM$

- Gauge coupling unification
- Yukawa unification
- Inverted scalar mass hierarchy

Bagger, Feng, Polonsky & Zhang
PLB473, 264 (2000)

- Suppresses flavor & CP violation
- Nucleon decay

$MSO_{10}SM$

1. Extend to 3 family model in 4D

   (Dermisek & Raby PLB 622, 327 (2005))

2. Extend to orbifold GUT in 5D

   (HD Kim, Schradin & Raby,

   JHEP 0301, 056 (2003) & JHEP 0505, 036 (2005))

3. Extend to heterotic string in 10D

   compactified on $Z_3 x Z_2$ orbifold

   (Kobayashi, Raby & Zhang,

   PLB 593, 262 (2004) & NPB 704, 3 (2005))

χ^2 analysis – 3rd family

Dermisek, Raby, Roszkowski and Ruiz de Austri

JHEP 09 (2005) 029

11 Parameters :

$M_G \;\; \alpha_G \;\; \varepsilon_3 \;\; \lambda$

$\mu \;\; M_{1/2} \;\; A_0 \;\; \tan\beta \;\; m_{16} \;\; m_{10} \;\; \Delta m_H$

9 Observables (included in χ^2) :

$\alpha_{EM} \;\; G_\mu \;\; \alpha_s \;\; M_Z \;\; M_W \;\; \rho_{NEW}$

$M_t \;\; m_b(m_b) \;\; M_\tau$

Results

1. Dark Matter & WMAP
2. $B_s \to \mu^+ \mu^-$
3. Light Higgs mass – m_h
4. Upper bound on m_A

   ⇒ Lower bound on $BR(B_s \to \mu^+ \mu^-)$

Dark Matter & WMAP

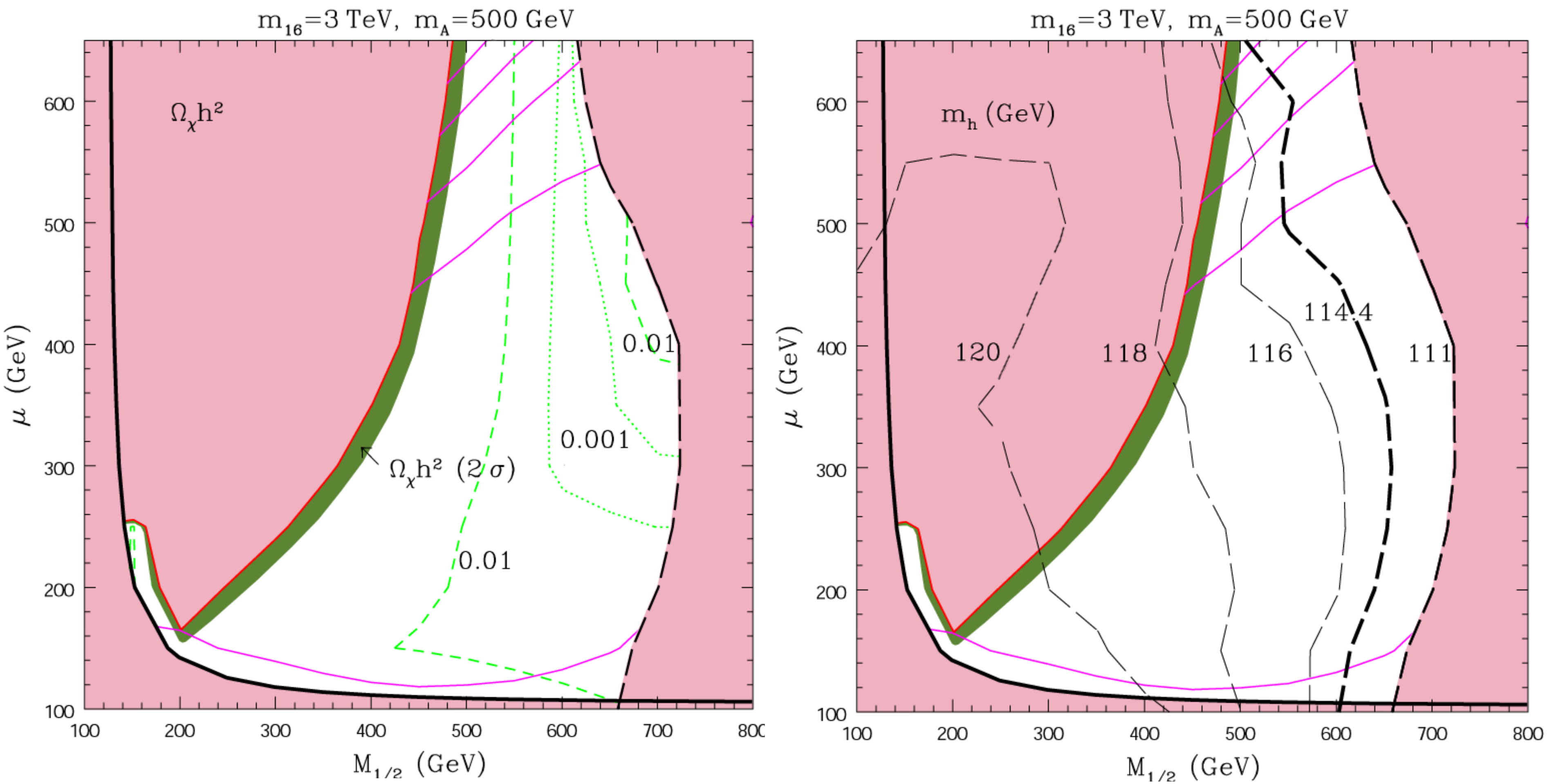
Neutralino LSP

$$\chi_1 \chi_1 \rightarrow A \rightarrow f \bar{f}$$

$$A b \bar{b} \propto \tan\beta$$

A : broad resonance, m_A arbitrary

Results - m_{16} & m_A fixed



Green band consistent with WMAP

Light Higgs mass contours

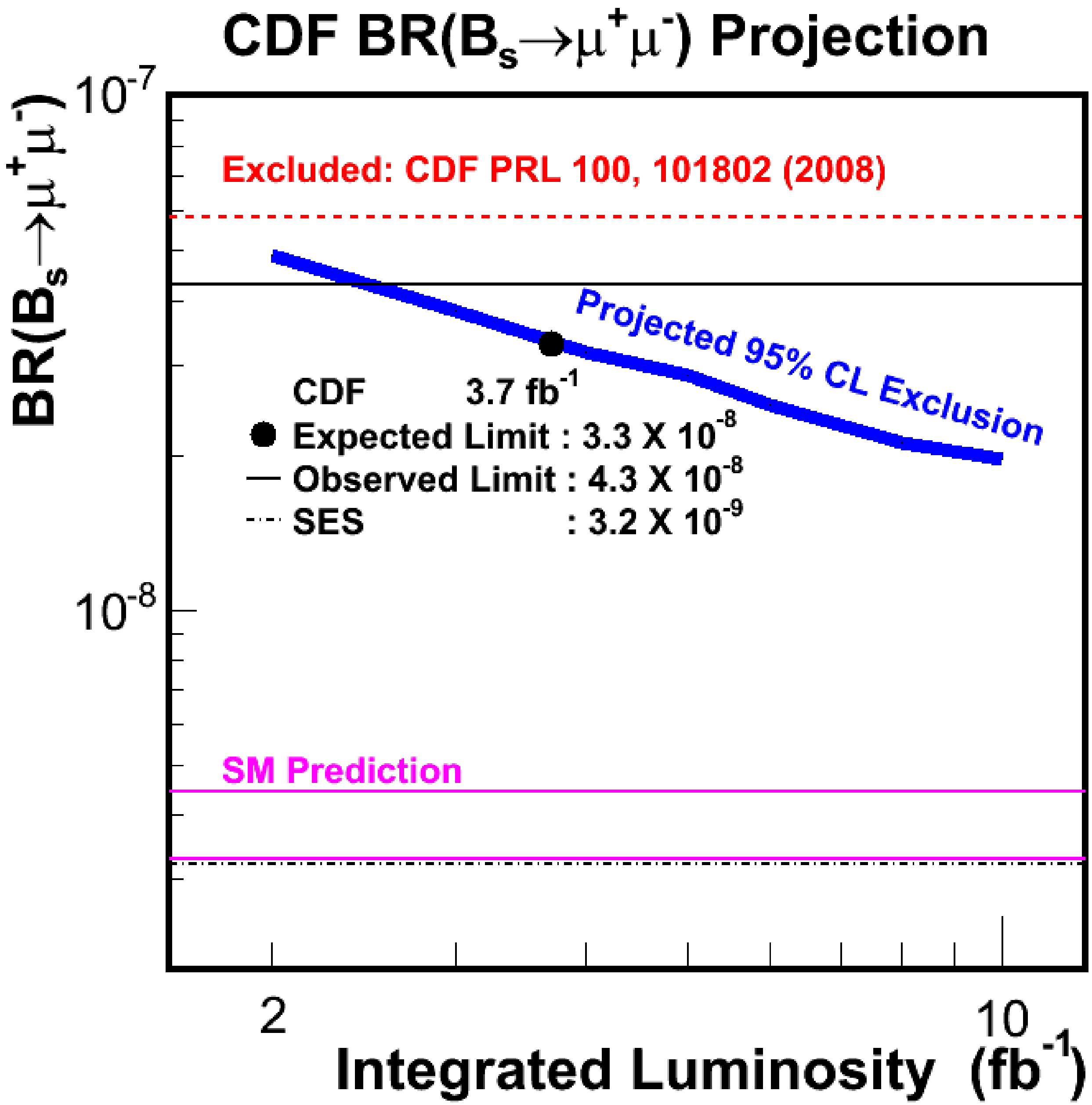
Br(B_s -> $\mu^+ \mu^-$)

SM : 3×10^{-9}

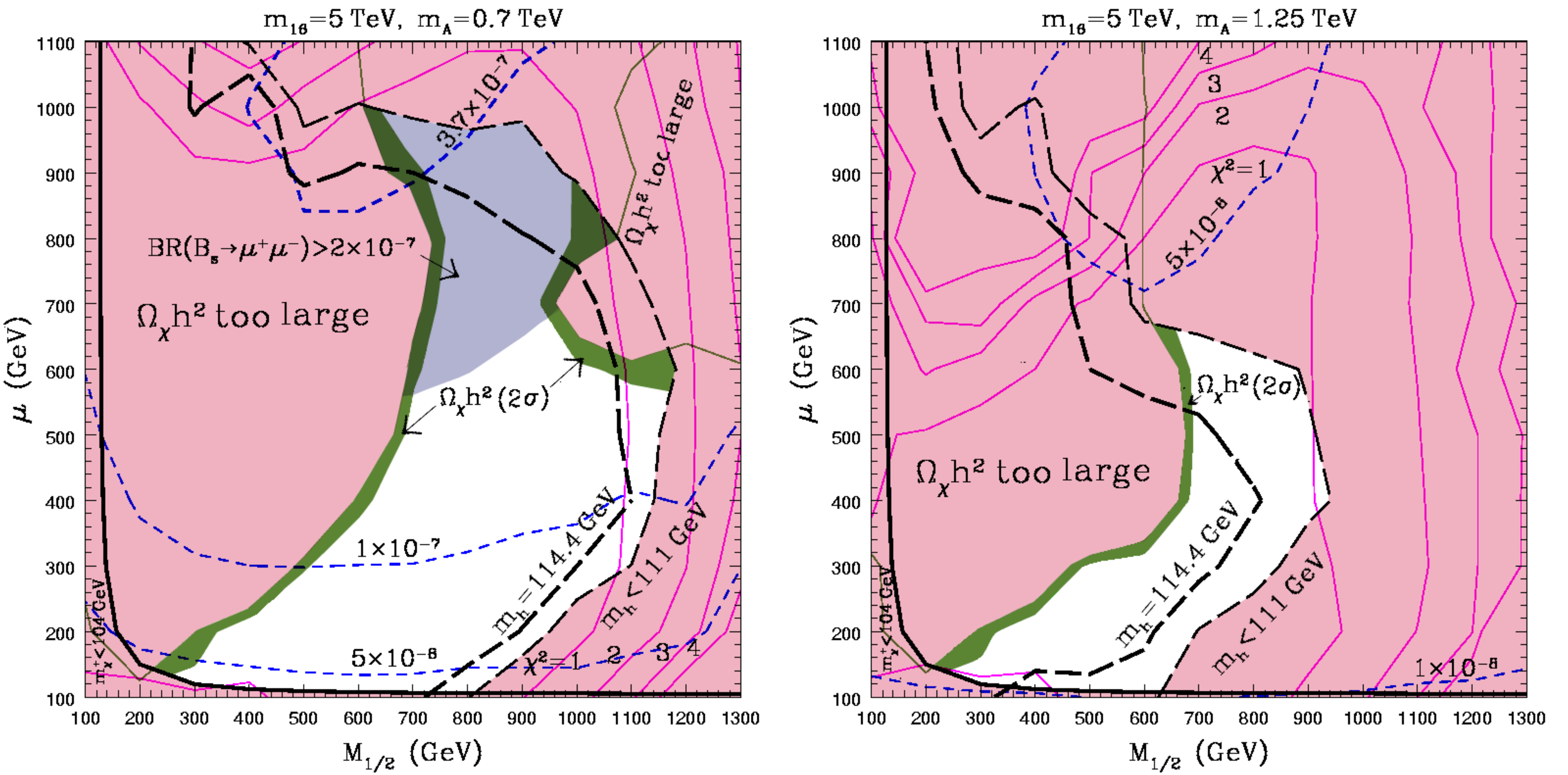
MSSM : ~ $(\tan \beta)^6 / m_A{}^4$

DZero < 4.7×10^{-8} (95% CL) w/ 5.0 fb^{-1}
"expected sensitivity"

CDF bound < 4.3×10^{-8} (95% CL) w/ 3.7 fb^{-1}


CDF BR($B_s \to \mu^+\mu^-$) Projection
Excluded: CDF PRL 100, 101802 (2008)
Projected 95% CL Exclusion
CDF 3.7 fb^{-1}
Expected Limit : 3.3 X 10^{-8}
Observed Limit : 4.3 X 10^{-8}
SES : 3.2 X 10^{-9}
SM Prediction
10^{-7}
10^{-8}
BR($B_s \to \mu^+\mu^-$)
2
10
Integrated Luminosity (fb^{-1})

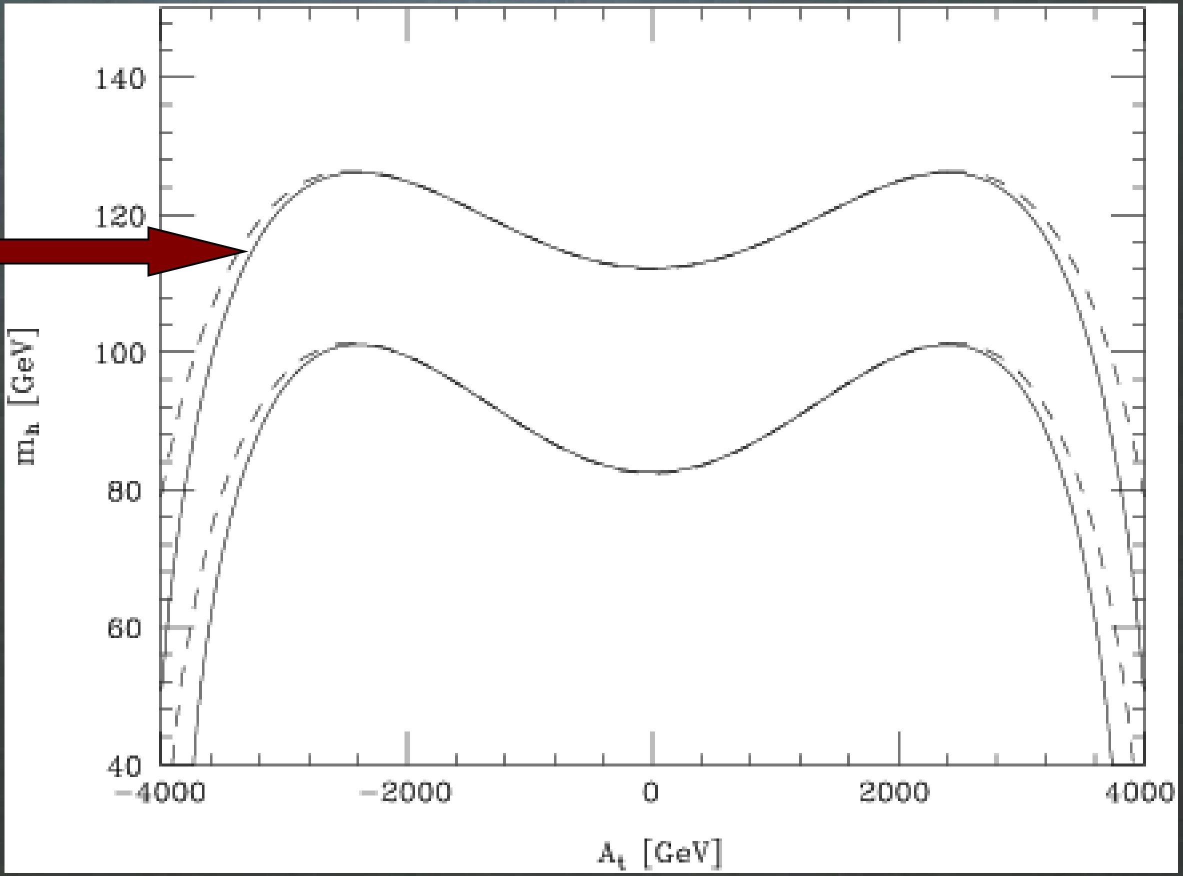
Results – m_{16} & m_A fixed



Constant χ^2 contours

Constant $B_s \to \mu^+ \mu^-$ contours

Light Higgs mass



Carena, Quiros & Wagner '95

$|A_t|$ increases → m_h decreases

Light Higgs mass

$$\delta m_b / m_b \propto \frac{\alpha_3 \mu M_{\tilde{g}} \tan\beta}{m_{\tilde{b}}^2} + \frac{\lambda_t^2 \mu A_t \tan\beta}{m_{\tilde{t}}^2} + \log corr.$$

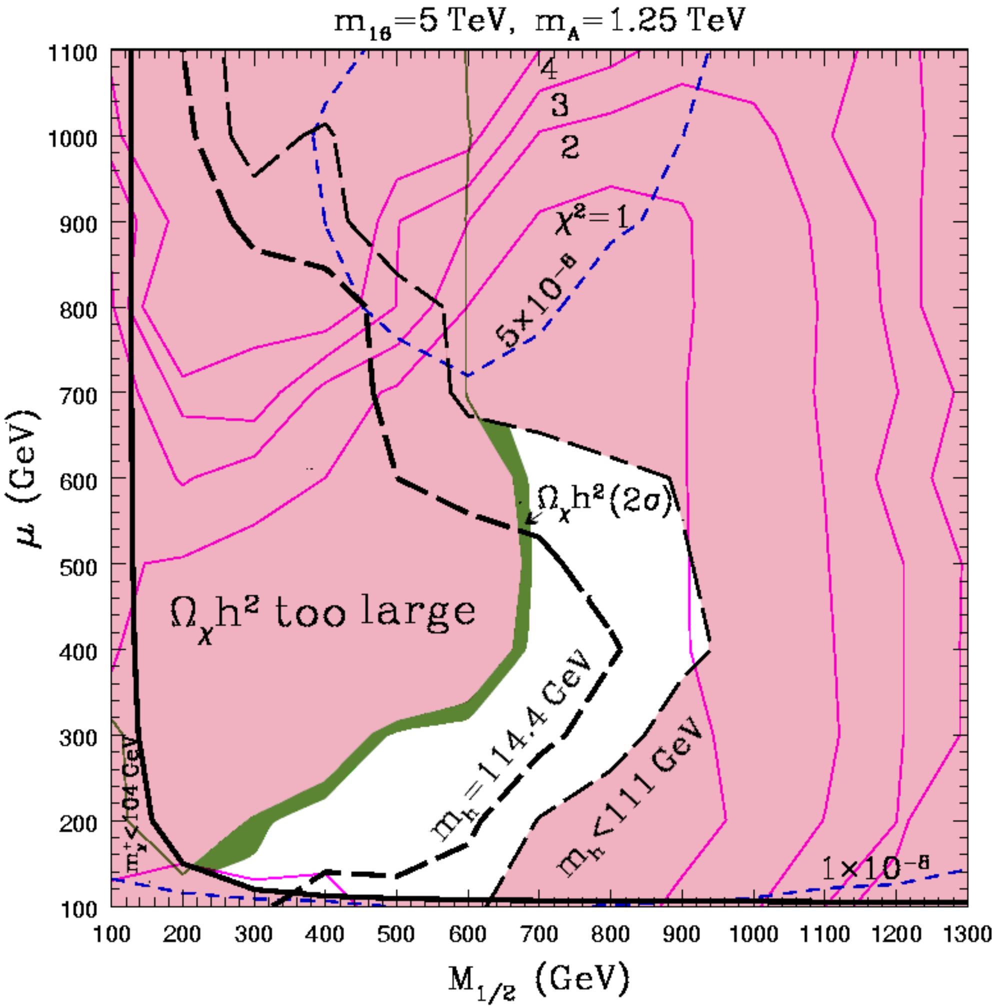
$$\delta m_b / m_b \le -2\%$$

Needed to fit data

$M_{1/2}$ increases → $-A_t$ increases

→ m_h decreases

$m_A(max) \longrightarrow Br(B_s \to \mu^+ \mu^-)(min)$



$m_A \le 1.3$ TeV

$Br(B_s \to \mu^+ \mu^-)$

$\ge 10^{-8}$

Fermilab ??

Summary - $MSO_{10}SM$

1. Gauge & Yukawa unification
2. Suppresses flavor & CP viol. & N decay
3. Dark Matter consistent w/WMAP
4. $114 < m_h < 121$ GeV
5. $m_A < 1.3$ TeV

   ⇒ $BR(B_s \to \mu^+ \mu^-) > 10^{-8}$

3 Family SO(10) + family symmetry

Dermisek & Raby PLB 622:327 (2005)

Dermisek, Harada & Raby PRD74, 035011 (2006)

Albrecht, Altmannshofer, Buras, Guadagnoli & Straub JHEP 0710:055 (2007)

3 family SO_{10} SUSY Model

- $D_3 \times U(1)$ Family Symmetry
- Superpotential
- Yukawa couplings
- χ^2 analysis
- Charged fermion masses & mixing
- Neutrino masses & mixing

Superpotential for charged fermion Yukawa couplings

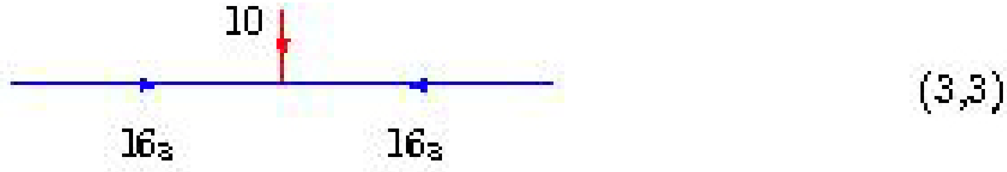
$$W_{ch.\,fermions} = 16_3 10 16_3 + 16_a 10 \chi_a$$

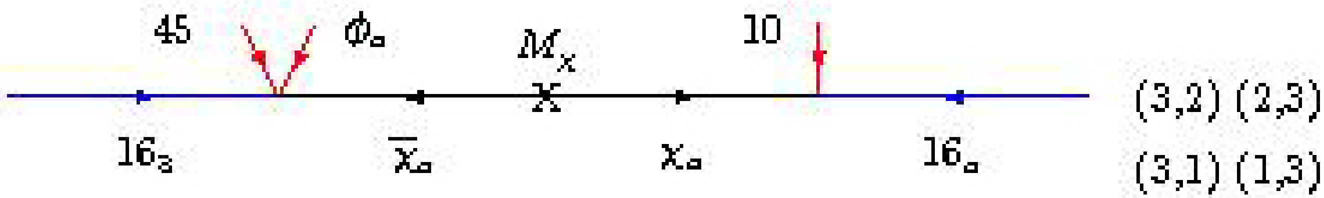
$$+ \overline{\chi_a} \left(M_\chi \chi_a + 45 \frac{\phi_a}{M} 16_3 + 45 \frac{\tilde{\phi}_a}{M} 16_a + A 16_a \right)$$

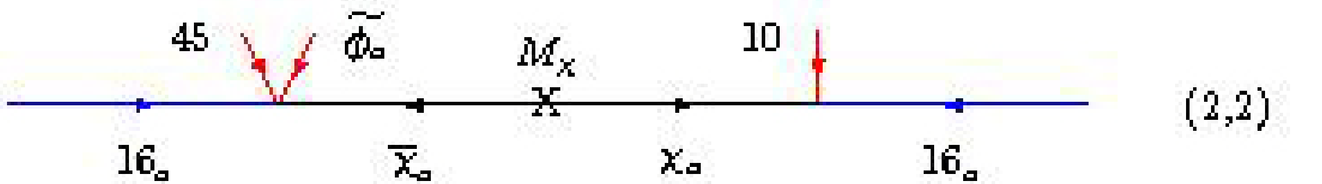
$$\langle \phi \rangle = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \qquad \langle \tilde{\phi} \rangle = \begin{pmatrix} 0 \\ \tilde{\phi}_2 \end{pmatrix} \qquad \langle 45 \rangle = (B - L) M_G$$

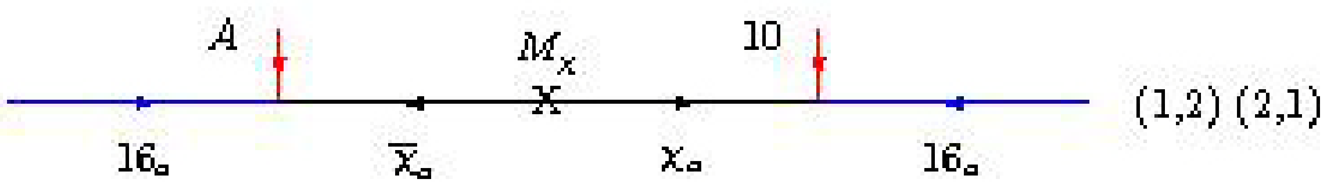
Familon VEVs assumed

Effective higher dimension operators









SO(10) x (D_3 x U(1) family sym.)
Yukawa Unification for 3rd Family

7 real para's
\+ 4 phases

\+ 3 real Majorana Neutrino masses

Dermisek & Raby
PLB 622:327 (2005).

$$Y_u = \begin{pmatrix} 0 & \epsilon' \rho & -\epsilon \xi \\ -\epsilon' \rho & \tilde{\epsilon} \rho & -\epsilon \\ \epsilon \xi & \epsilon & 1 \end{pmatrix} \lambda$$

$$Y_d = \begin{pmatrix} 0 & \epsilon' & -\epsilon \xi \sigma \\ -\epsilon' & \tilde{\epsilon} & -\epsilon \sigma \\ \epsilon \xi & \epsilon & 1 \end{pmatrix} \lambda$$

$$Y_e = \begin{pmatrix} 0 & -\epsilon' & 3 \epsilon \xi \\ \epsilon' & 3 \tilde{\epsilon} & 3 \epsilon \\ -3 \epsilon \xi \sigma & -3 \epsilon \sigma & 1 \end{pmatrix} \lambda$$

$$Y_\nu = \begin{pmatrix} 0 & -\epsilon' \omega & \frac{3}{2} \epsilon \xi \omega \\ \epsilon' \omega & 3 \tilde{\epsilon} \omega & \frac{3}{2} \epsilon \omega \\ -3 \epsilon \xi \sigma & -3 \epsilon \sigma & 1 \end{pmatrix} \lambda$$

Extend to neutrino sector

$$W_{neutrino} = \overline{16}\left(\lambda_2 N_a 16_a + \lambda_3 N_3 16_3\right) + \tfrac{1}{2}\left(S_a N_a N_a + S_3 N_3 N_3\right)$$

$$\langle S_a \rangle = M_a \qquad \langle S_3 \rangle = M_3 \qquad \langle \overline{16} \rangle = v_{16}$$

Assume 3 new real para's

$$W_{neutrino} = \nu m_\nu \bar{\nu} + \bar{\nu} V N + \tfrac{1}{2} N M_N N$$

$$Y_\upsilon = \begin{pmatrix} 0 & -\varepsilon'\omega & \tfrac{3}{2}\varepsilon\xi\omega \\ \varepsilon'\omega & 3\tilde{\varepsilon}\omega & \tfrac{3}{2}\varepsilon\omega \\ -3\varepsilon\xi\sigma & -3\varepsilon\sigma & 1 \end{pmatrix} \lambda$$

$$\omega = 2\sigma / (2\sigma - 1)$$

$$m_\nu = Y_\nu \frac{v}{\sqrt{2}} \sin\beta$$

$$V = v_{16} \begin{pmatrix} 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

$$M_N = diag\begin{pmatrix} M_1 & M_2 & M_3 \end{pmatrix}$$

$$M_\nu = U_e^T \left(m_\nu \left(V^T\right)^{-1} M_N V^{-1} m_\nu^T \right) U_e$$

Using χ^2 analysis, fit

15 charged fermion & 4 neutrino low energy observables with

11 arbitrary Yukawa & 3 Majorana mass parameters

⇒ 4 & 1 d. o. f. or 5 predictions

Global χ^2 analysis

Sector	#	Parameters
gauge	3	α_G, M_G, ϵ_3,
SUSY (GUT scale)	5	m_{16}, $M_{1/2}$, A_0, m_{H_u}, m_{H_d},
textures	11	ϵ, ϵ', λ, ρ, σ, $\tilde{\epsilon}$, ξ,
neutrino	3	M_{R_1}, M_{R_2}, M_{R_3},
SUSY (EW scale)	2	$\tan\beta$, μ

24 parameters at GUT scale

compared to SM – 27 parameters

CMSSM – 32 parameters

Global χ^2 analysis - good fits to

- charged fermion masses & mixing angles
- neutrino masses & mixing angles
- naturally satisfies Lepton Flavor Violation and electron electric dipole moment bounds

Dermisek, Harada & Raby

PRD74, 035011 (2006)

$m_{16} = 4$ TeV

$\mu = 300$ GeV

$M_{1/2} = 200$ GeV

Observable (masses in GeV)	Data (σ)	Theory	Pull
$G_\mu \times 10^5$	1.16637 (0.1%)	1.16638	< 0.01
α_{EM}^{-1}	137.036 (0.1%)	137.035	< 0.01
$\alpha_s(M_Z)$	0.1187 (0.002)	0.1174	0.37
M_t	172.7 (2.9)	173.11	0.02
$m_b(M_b)$	4.25 (0.25)	4.49	0.94
$M_b - M_c$	3.4 (0.2)	3.61	**1.16**
$m_c(m_c)$	1.2 (0.2)	1.16	0.03
m_s	0.105 (0.025)	0.107	0.01
m_d/m_s	0.0521 (0.0067)	0.0638	**3.09**
$Q^{-2} \times 10^3$	1.934 (0.334)	1.815	0.12
M_τ	1.777 (0.1%)	1.777	< 0.01
M_μ	0.10566 (0.1%)	0.10566	< 0.01
$M_e \times 10^3$	0.511 (0.1%)	0.511	< 0.01
V_{us}	0.22 (0.0026)	0.2193	0.06
V_{cb}	0.0413 (0.0015)	0.0410	0.03
V_{ub}	0.00367 (0.00047)	0.00316	**1.15**
V_{td}	0.0082 (0.00082)	0.00824	< 0.01
ϵ_K	0.00228 (0.000228)	0.00234	0.08
$\sin(2\beta)$	0.687 (0.064)	0.6435	0.46
$\Delta m_{31}^2 \times 10^3$	2.3 (0.6)	2.382	0.01
$\Delta m_{21}^2 \times 10^5$	7.9 (0.6)	7.880	< 0.01
$\sin^2\theta_{12}$	0.295 (0.045)	0.289	0.01
$\sin^2\theta_{23}$	0.51 (0.13)	0.532	0.03
TOTAL χ^2			**7.65**

Results for $7 < \chi^2 < 8$

$$m_{16} = 4 \text{ TeV}$$

$$114 < m_h < 129 \text{ GeV}$$

$$0.010 < \sin^2 2\vartheta_{13} < 0.015$$

$$1 < \mathrm{BR}(\mu \to e\gamma) \times 10^{14} < 7$$

$$2 < d_e(e\text{ cm}) \times 10^{29} < 5$$

χ^2 analysis including B physics

Albrecht, Altmannshofer, Buras, Guadagnoli, & Straub
JHEP 0710:055 (2007)

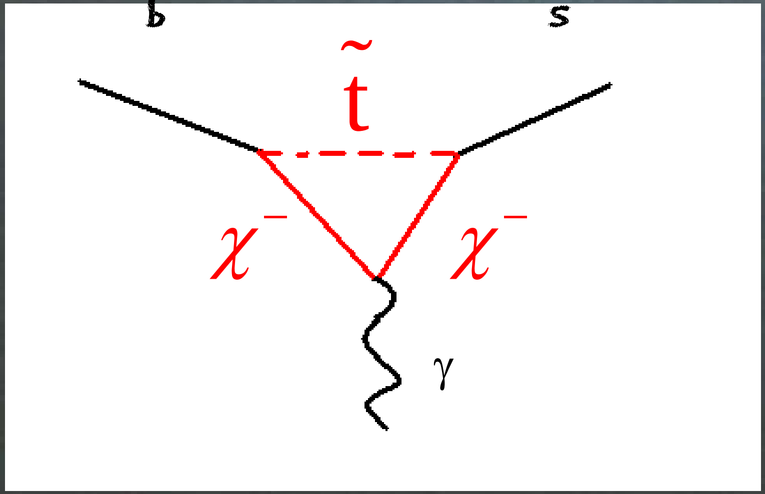
Find good fits to quark, charged lepton
& neutrino masses and mixing angles
& test flavor violation in b physics

some tension between $b \rightarrow s\,\gamma$ & $b \rightarrow s\, l^+ l^-$

⇒ $m_{16} \sim 10$ TeV

What is the tension ?

$$C_7^{\chi^+} \propto \mu A_t \tan\beta \times \mathrm{sign}\left(C_7^{SM}\right) \approx -2C_7^{SM}$$



$$A_t < 0$$

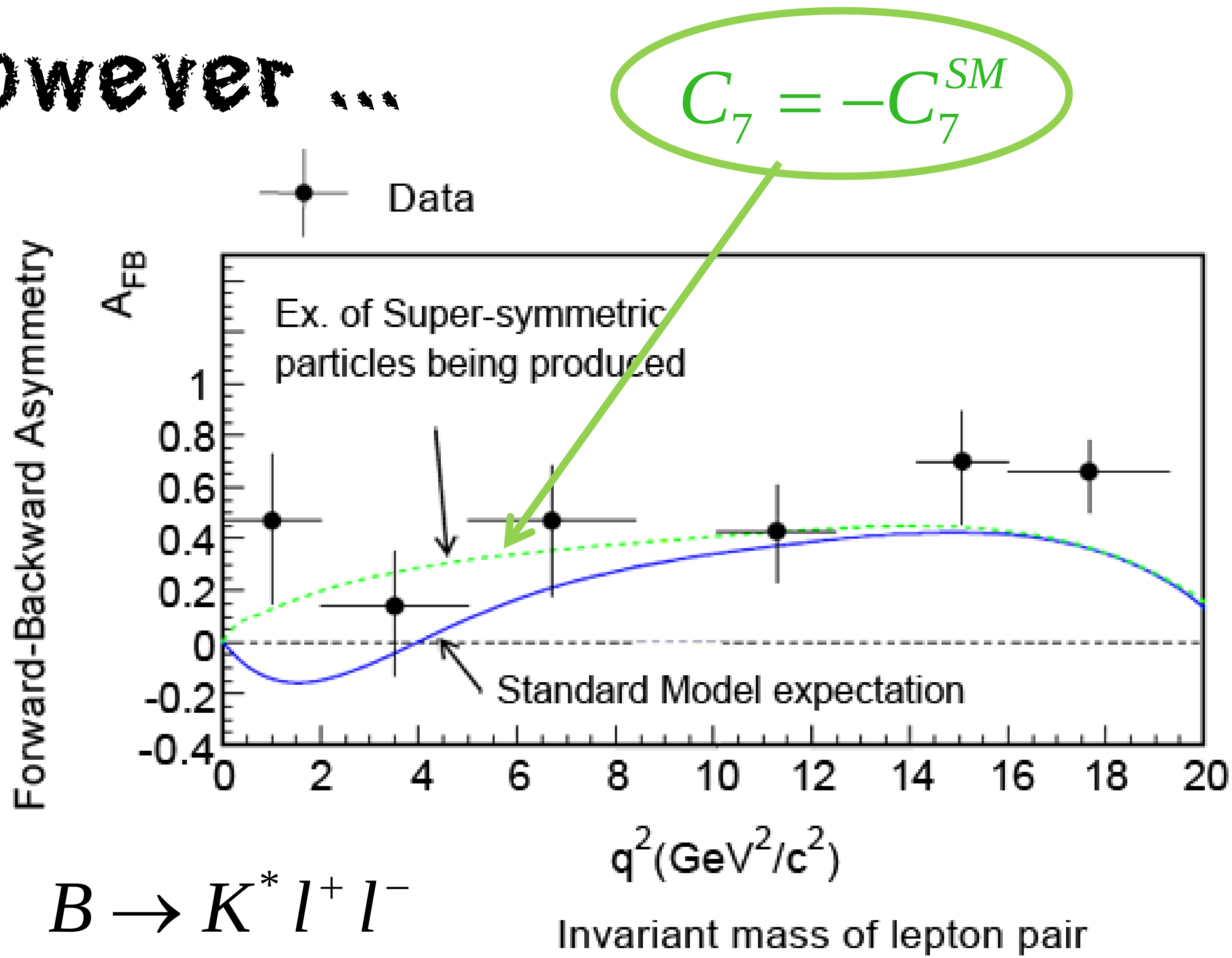
$$\Downarrow$$

$$C_7 = C_7^{SM} + C_7^{\chi^+} \approx -C_7^{SM}$$

$BR(B \to X_s\, l^+\, l^-)$ favors $C_7 \approx +C_7^{SM}$

at $\sim 2\sigma$

However ...



$$B \to K^* l^+ l^-$$

Belle arXiv:0904.0770

Observable	Exp. value	Fit value	Pull (σ)
M_W	80.403	80.6	0.5
M_Z	91.1876	90.7	1.1
$G_F \times 10^5$	1.16637	1.16	0.3
$1/\alpha_{em}$	137.036	136.8	0.4
$\alpha_s(M_Z)$	0.1176	0.117	0.2
M_t	170.9	170.6	0.2
$m_b(m_b)$	4.2	4.22	0.3
$m_c(m_b)$	1.25	1.14	1.2
$m_s(2\,\mathrm{GeV})$	0.095	0.107	0.5
$m_d(2\,\mathrm{GeV})$	0.005	0.00741	1.2
$m_u(2\,\mathrm{GeV})$	0.00225	0.00461	**3.1**
M_τ	1.777	1.78	0.1
M_μ	0.10566	0.106	0.1
M_e	0.000511	0.000511	0.0
$\|V_{us}\|$	0.2258	0.225	0.6
$\|V_{ub}\| \times 10^3$	4.1	3.26	**2.1**
$\|V_{cb}\|$	0.0416	0.0416	0.1
$\sin 2\beta$	0.675	0.639	1.4
$\Delta m^2_{31} \times 10^{21}$	2.6	2.6	0.0
$\Delta m^2_{21} \times 10^{23}$	7.9	7.9	0.0
$\sin^2 2\theta_{12}$	0.852	0.852	0.0
$\sin^2 2\theta_{23}$	0.996	1.0	0.2
$\epsilon_K \times 10^3$	2.229	2.33	0.4
$\mathrm{BR}(B \to X_s\gamma) \times 10^4$	3.55	2.86	1.3
$\mathrm{BR}(B \to X_s\ell^+\ell^-) \times 10^6$	1.6	1.62	0.0
$\Delta M_s/\Delta M_d$	35.05	31.1	1.1
$\mathrm{BR}(B^+ \to \tau^+\nu) \times 10^4$	1.31	0.517	1.7
		total χ^2:	**27.4**

Albrecht et al.

JHEP 0710:055 (2007)

m_{16}	4000	6000	10000
μ	378	953	1200
$\mathrm{BR}(B_s \to \mu^+\mu^-) \times 10^8$	8.6	7.7	2.1
$\hat{s}_0$	0.022	0.13	0.14
$\mathrm{BR}(\mu \to e\gamma) \times 10^{13}$	0.36	0.021	0.0026
$\delta a_\mu^{\mathrm{SUSY}} \times 10^{10}$	+5.8	+1.6	+0.52
M_{h_0}	126	129	129
M_A	507	559	842
$m_{\tilde{t}_1}$	640	1172	1903
$m_{\tilde{b}_1}$	895	1475	2366
$m_{\tilde{\tau}_1}$	1510	2419	3933
$m_{\tilde{\chi}^0_1}$	60	60	60
$m_{\tilde{\chi}^+_1}$	115	119	120
$m_{\tilde{g}}$	462	478	506

Required $C_7 = +C_7^{SM}$

$MSO_{10}SM$ & Large $\tan(\beta)$

- Fits WMAP
- Predicts light Higgs with mass of order 120 – 130 GeV
- Predicts lighter 3rd and heavy 1st & 2nd gen. squarks and sleptons (inverted scalar mass hier.)
- LFV bounds satisfied
- Enhances $Br(B_s \longrightarrow \mu^+ \mu^-)$
- Suppresses $Br(B \longrightarrow \tau \nu)$ & ΔM_{B_s}
- $B \longrightarrow X_s \gamma$, $X_s l^+ l^-$ tension ??

LHCb

$MSO_{10}SM$ & Large tan(β)

- predicts light gluinos ~ 300 - 500 GeV
- predicts light charginos and neutralinos

LHC

$MSO_{10}SM$: Beautiful symmetry

Many experimental tests !!

Problems of SUSY GUTs

- GUT symmetry breaking
- Higgs doublet-triplet splitting

Missing partner mechanism SU(5)

$$W \supset 75^3 + M\ 75^2 + \mathrm{H}_u\, 75\, H_{50} + \overline{H}_d\, 75\, H_{\overline{50}} + X\, H_{\overline{50}}\, H_{50}$$

Missing VEV mechanism SO(10)

$$W \supset 45^4 + M\ 45^2 + \mathrm{X}\left(\overline{16}\ 16\right)^2 + F\left(X\right)$$

$$+\overline{16}'\left(S\,45 + S_1\right)\ 16 + \overline{16}'\left(S\,45 + S_2\right)\ 16' + 10\,45\,10' + X'\left(10'\right)^2$$

Outline

- 4 dimensional SUSY GUTs
- **5 dimensional orbifold GUTs**
- String orbifolds ⟺ orbifold GUTs
- Conclusions

Orbifold GUTs in 5 or 6 dimensions

GUT symmetry breaking

Doublet-Triplet splitting

Kawamura; Hall & Nomura; Contino, Pilo, Rattazzi & Trincherini; Altarelli, Feruglio & Masina; Dermisek & Mafi; H.D. Kim & Raby; Asaka, Buchmuller & Covi; Lee; Hebecker & March-Russell

H.D.Kim, Raby & Schradin JHEP 0505:036 (2005)

Hall & Nomura SU(5) GUT on M x $S^1/(Z_2 \times Z'_2)$

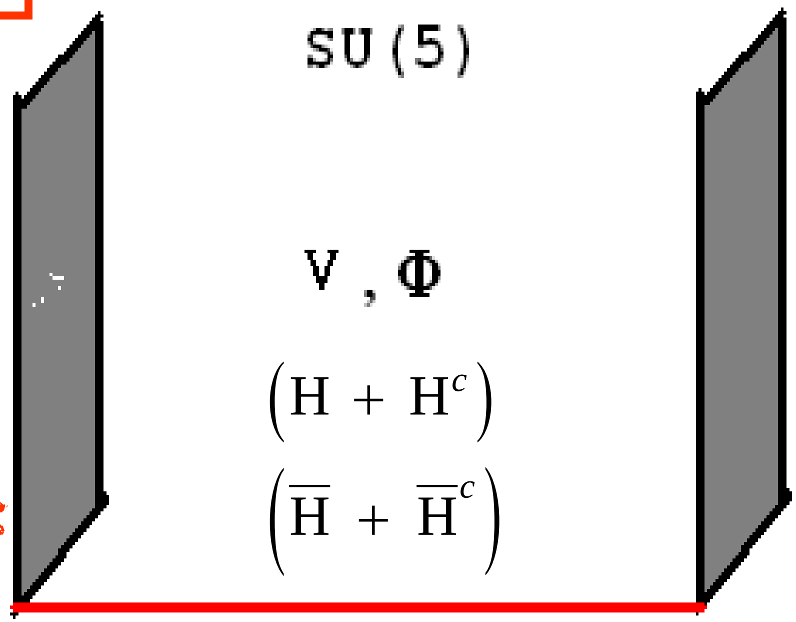
GUT symmetry breaking
Higgs D-T splitting
via orbifold BCs

b-τ Yukawa unification

$10+\overline{5}$

3rd family

Brane states



0 π R

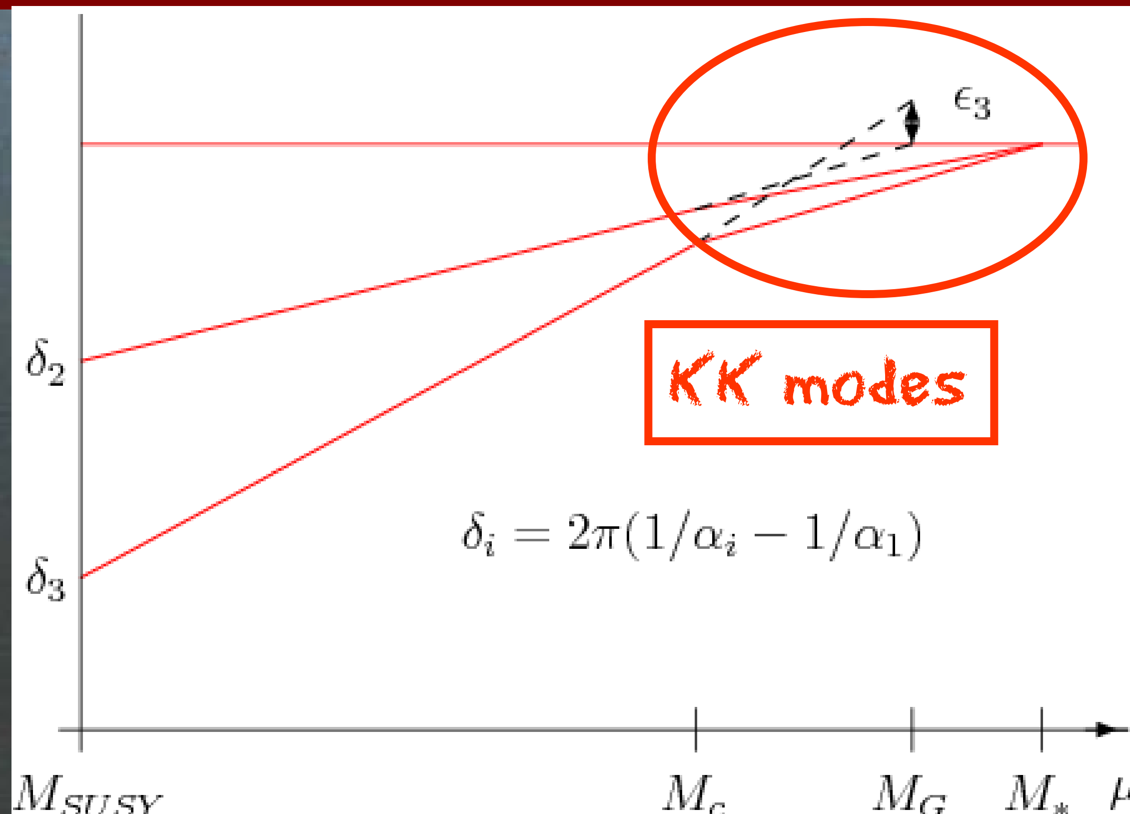
SU(5) Bulk states SM

1st & 2nd families in bulk

Suppress proton decay

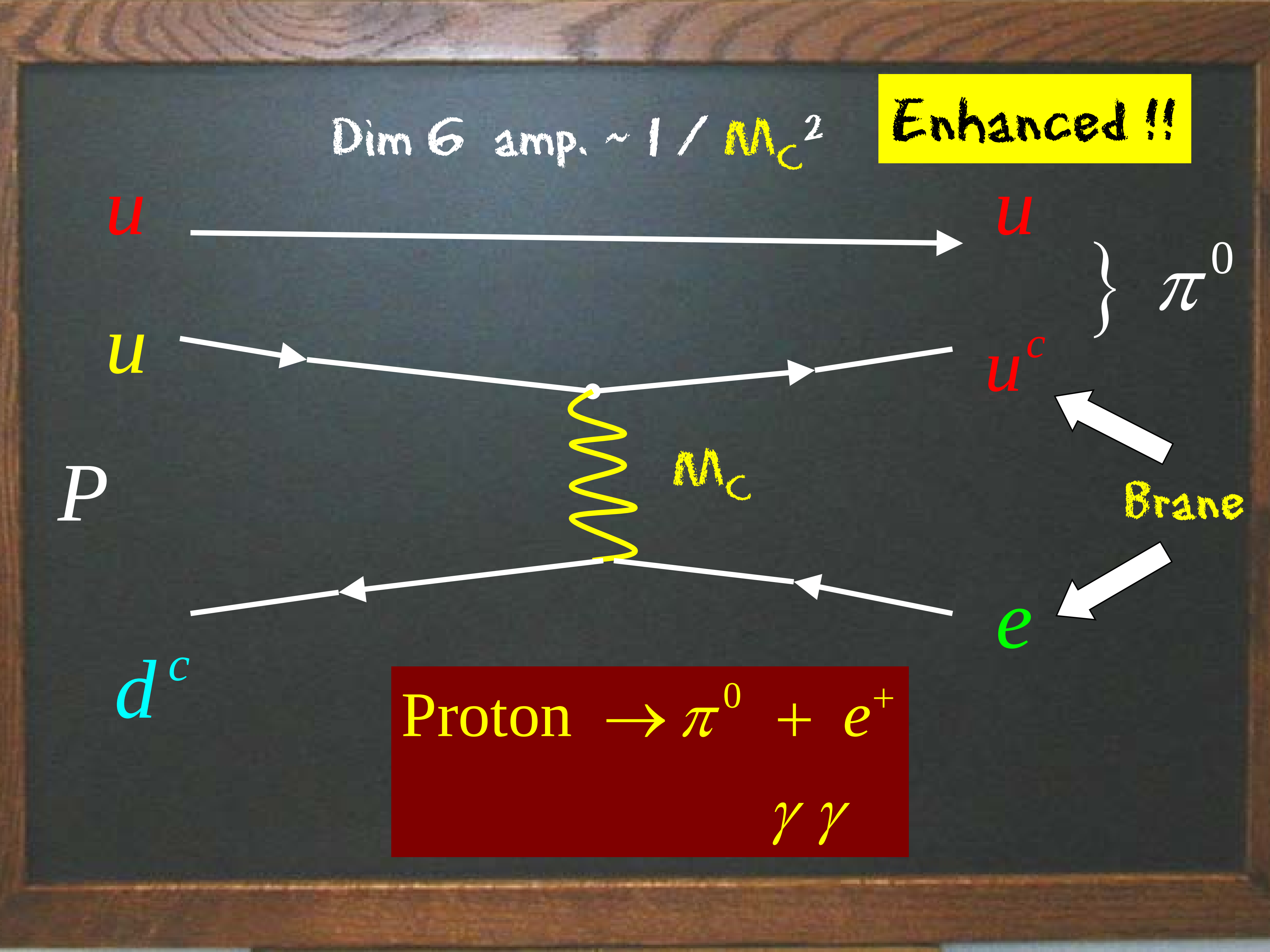
G.C. Unif. & Proton decay > 4D

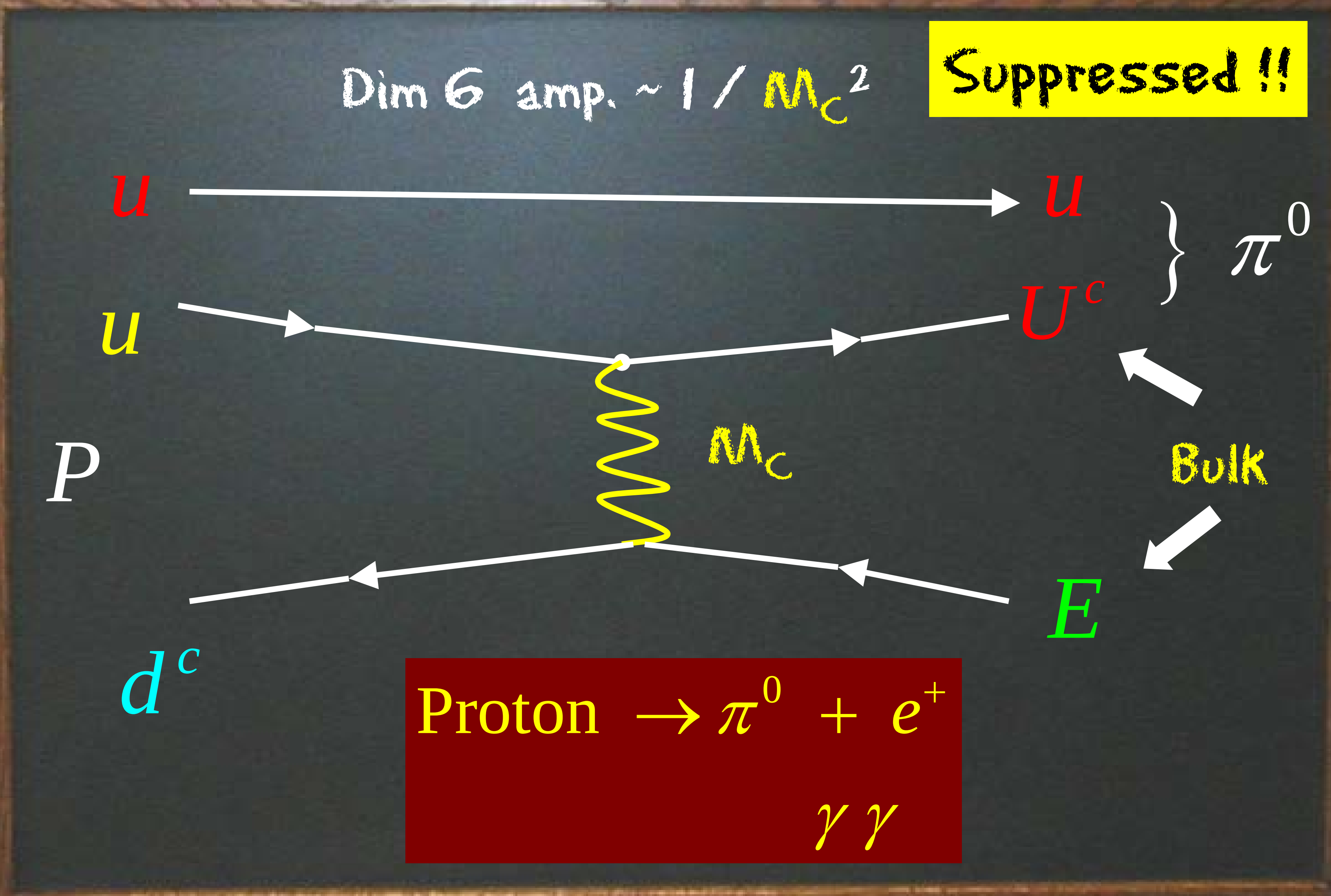
5D Orbifold GUT



$M_c \sim 1/\pi R < M_G < M_*$

Dienes et al., Hall & Nomura, Kim & S.R., Feruglio et al.


Dim 6 amp. ~ 1 / M_c^2
Enhanced !!
u
u
π^0
u^c
P
M_c
Brane
e
d^c
Proton $\rightarrow \pi^0 + e^+$
$\gamma\gamma$


Dim 6 amp. ~ 1 / M_c^2
Suppressed !!
u
u
π^0
u
U^c
M_c
P
Bulk
E
d^c
Proton $\rightarrow \pi^0 + e^+$
$\gamma\,\gamma$

Orbifold GUTs in 5 or 6 dimensions :

- GUT breaking via Orbifold "Parity"
- Higgs doublet - triplet splitting via Orbifold "P"
- NO proton decay via Dim 5 operators due to R symmetry
- Proton decay via Dim 6 operators can be suppressed, BUT typically enhanced (model dependent)
- Matter localization determines Yukawas

Outline

- 4 dimensional SUSY GUTs
- 5 dimensional orbifold GUTs
- String orbifolds ⟺ orbifold GUTs
- Conclusions

UV completion

Orbifold GUTs in 5 or 6 dimensions
Derived from heterotic string in 10 D

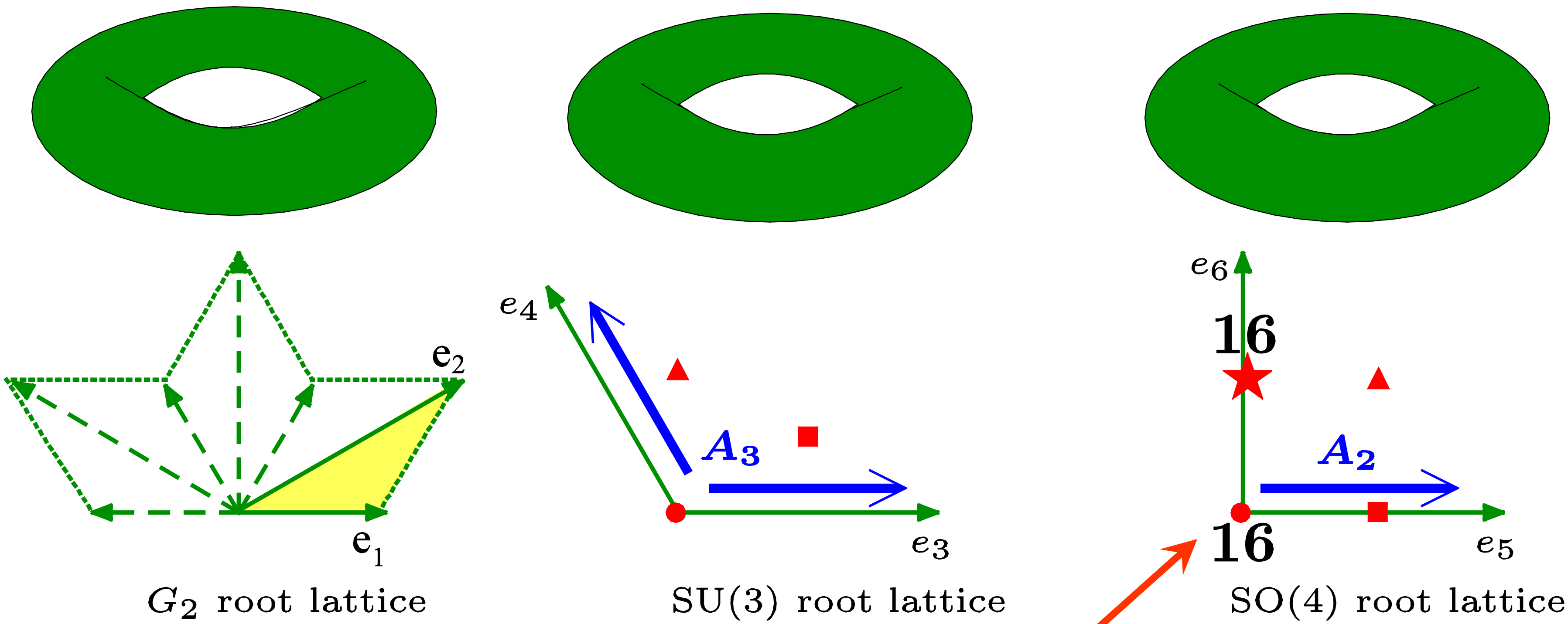
Kobayashi, Raby & Zhang; Forste, Nilles, Vaudrevange & Wingerter; Buchmuller, Hamaguchi, Lebedev & Ratz; JE Kim, JH Kim & Kyae; Buchmuller, Ludeling & Schmidt

Road to the MSSM with R-parity
Lebedev et al. - 0708.2691(hep-th)

Find 15 models from orbifold compactification of E(8)xE(8) heterotic string

- ✓ MSSM spectrum at low energy
- ✓ Exact R parity
- ✓ Light Higgs
- ✓ Non-trivial charged fermion masses
- ✓ Neutrino masses via See-Saw

Compactify E(8)xE(8) heterotic string on $(T^2)^3/(Z_3 \times Z_2)$ + Wilson lines (A_2, A_3)



Local SO(10) GUT

D_4 family symmetry

$$D_4 = \{\pm 1, \pm\sigma_1, \pm\sigma_3, \mp i\sigma_2\}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} : f_1 \leftrightarrow f_2 \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} : f_2 \leftrightarrow - f_2$$

geometry

string selection rule

$$\begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$$ doublet

$f_3,\ \mathrm{H}_u,\ \mathrm{H}_d$ singlets

For detailed list of all discrete non-abelian symmetries from orbifold compactification of the heterotic string, see

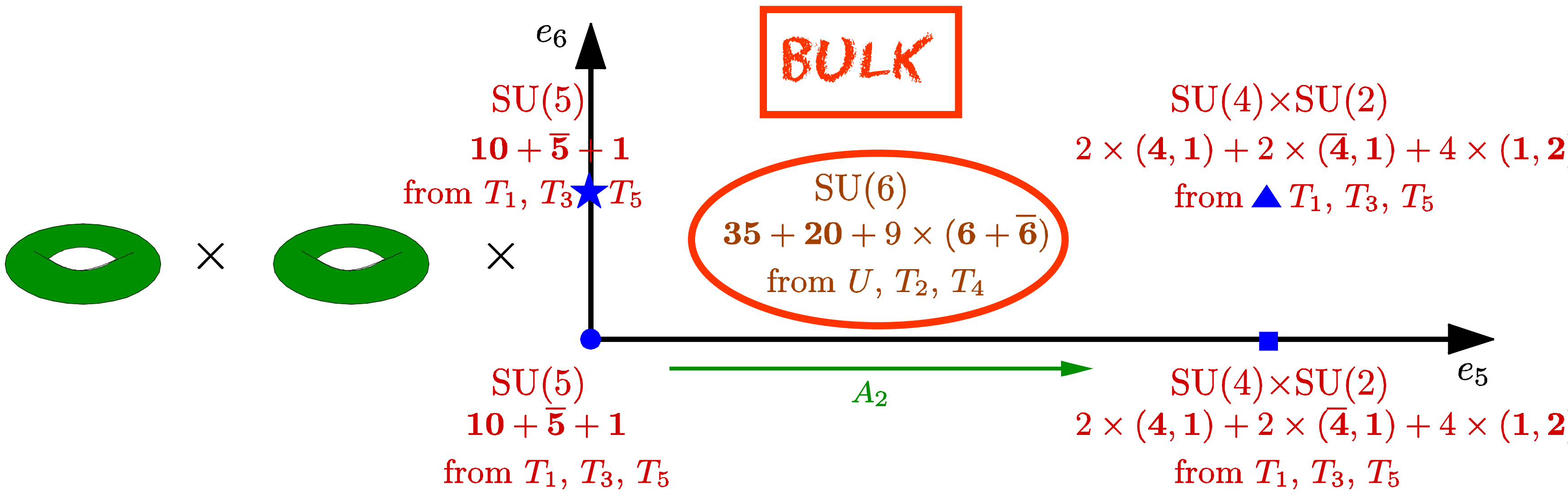
Kobayashi, Nilles, Ploeger, SR & Ratz
hep-ph/0611020

For D_4 phenomenology, see

Ko, Kobayashi, Park & SR
arXiv:0704.2807

Compactify E(8)xE(8) heterotic string on $(T^2)^3/(Z_3)$ + A_3 only ($R_5 \gg l_s$)

⇒ SU(6) orbifold GUT



After Z_2

SU(5) brane

$Z'_2 = Z_2 + A_2$

SU(4)xSU(2) brane

Bulk states

$SU(6) \to SU(5)$

$$35 \to 24 + 5 + \overline{5} + 1 \supset H_u + H_d$$

Gauge - Higgs unification !

$$20 + 20^c \to \left(10 + \overline{10}\right) + \ldots \supset Q = \begin{pmatrix} t \\ b \end{pmatrix} + t^c + \tau^c$$

$$2\left(6 + 6^c\right) \to L = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} + b^c$$

3rd family

Tree level coupling

$$20\ 35\ 20^c \supset Q\ H_u\ t^c$$

"Benchmark" model I - Spectrum

#	irrep	label	#	irrep	label
3	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1})_{(1/3,1/3)}$	q_i	3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(-4/3,-1/3)}$	$\bar{u}_i$
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(2,1)}$	$\bar{e}_i$	8	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(0,*)}$	m_i
4	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(2/3,-1/3)}$	$\bar{d}_i$	1	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{(-2/3,1/3)}$	d_i
4	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(-1,-1)}$	ℓ_i	1	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(1,1)}$	$\bar{\ell}_i$
1	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(-1,0)}$	ϕ_i	1	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(1,0)}$	$\bar{\phi}_i$
6	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(2/3,2/3)}$	$\bar{\delta}_i$	6	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{(-2/3,-2/3)}$	δ_i
14	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(1,*)}$	s_i^+	14	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1,*)}$	s_i^-
16	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,1)}$	$\bar{n}_i$	13	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,-1)}$	n_i
5	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{(0,1)}$	$\bar{\eta}_i$	5	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{(0,-1)}$	η_i
10	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{(0,0)}$	h_i	2	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{2})_{(0,0)}$	y_i
6	$(\mathbf{1},\mathbf{1};\mathbf{4},\mathbf{1})_{(0,*)}$	f_i	6	$(\mathbf{1},\mathbf{1};\overline{\mathbf{4}},\mathbf{1})_{(0,*)}$	$\bar{f}_i$
2	$(\mathbf{1},\mathbf{1};\mathbf{4},\mathbf{1})_{(-1,-1)}$	f_i^-	2	$(\mathbf{1},\mathbf{1};\overline{\mathbf{4}},\mathbf{1})_{(1,1)}$	$\bar{f}_i^+$
4	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,\pm 2)}$	χ_i	32	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,0)}$	s_i^0
2	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1/3,2/3)}$	$\bar{v}_i$	2	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/3,-2/3)}$	v_i

Table 1: Spectrum. The quantum numbers under $SU(3) \times SU(2) \times [SU(4) \times SU(2)']$ are shown, hypercharge and B-L charge appear as subscript.

$\tilde{S}$

Yukawa couplings

$$Y_u = \begin{pmatrix} \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 \\ \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 \\ \tilde{s}^6 & \tilde{s}^6 & 1 \end{pmatrix}, \qquad Y_d = \begin{pmatrix} \tilde{s}^5 & \tilde{s}^5 & 0 \\ \tilde{s}^5 & \tilde{s}^5 & 0 \\ \tilde{s}^6 & \tilde{s}^6 & 0 \end{pmatrix},$$

$$Y_e = \begin{pmatrix} \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^6 \\ \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^6 \\ \tilde{s}^6 & \tilde{s}^6 & 0 \end{pmatrix}$$

Effective higher dimension operators !!

Neutrino masses via See-Saw !!

Conclusions

Evolution of SUSY GUT Model Building

Bottom up

- 4D SUSY GUT + family symmetry

  test predictions via global analysis
- Lift to 5D (or 6D) orbifold GUT

Top down

- Derive from heterotic string,

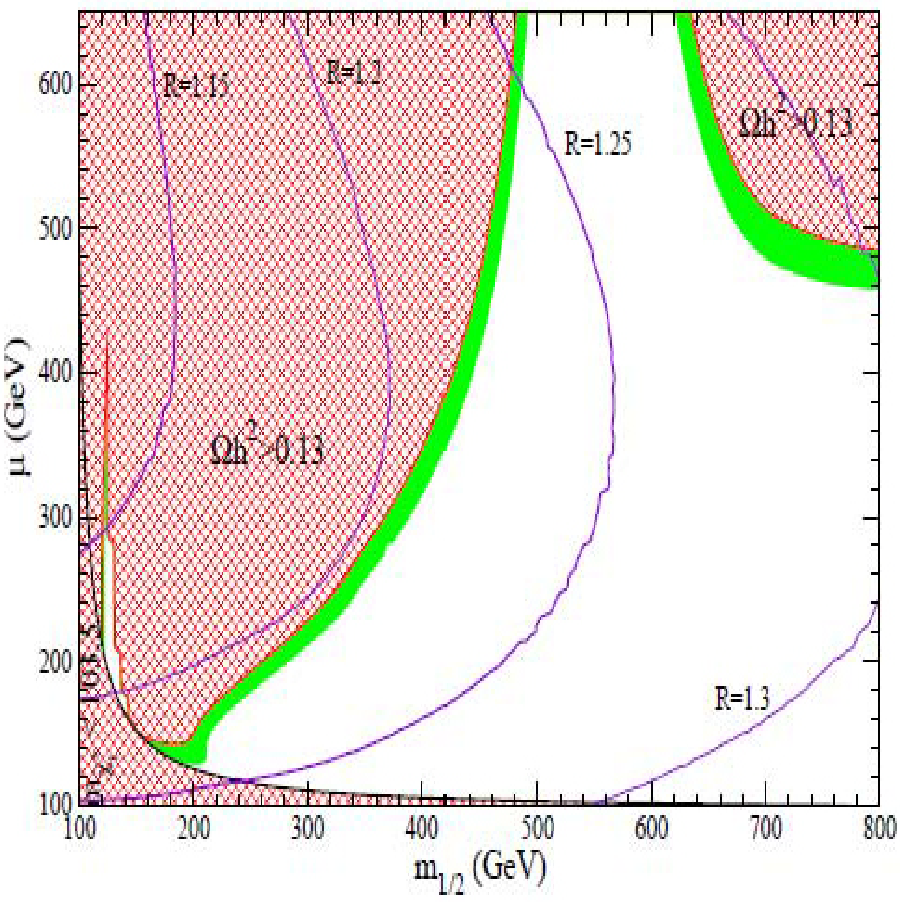
  M or F theory **includes Gravity !!**

Backup Slides

Baer, Kraml, Sekmen and Summy

JHEP 03 (2008) 056

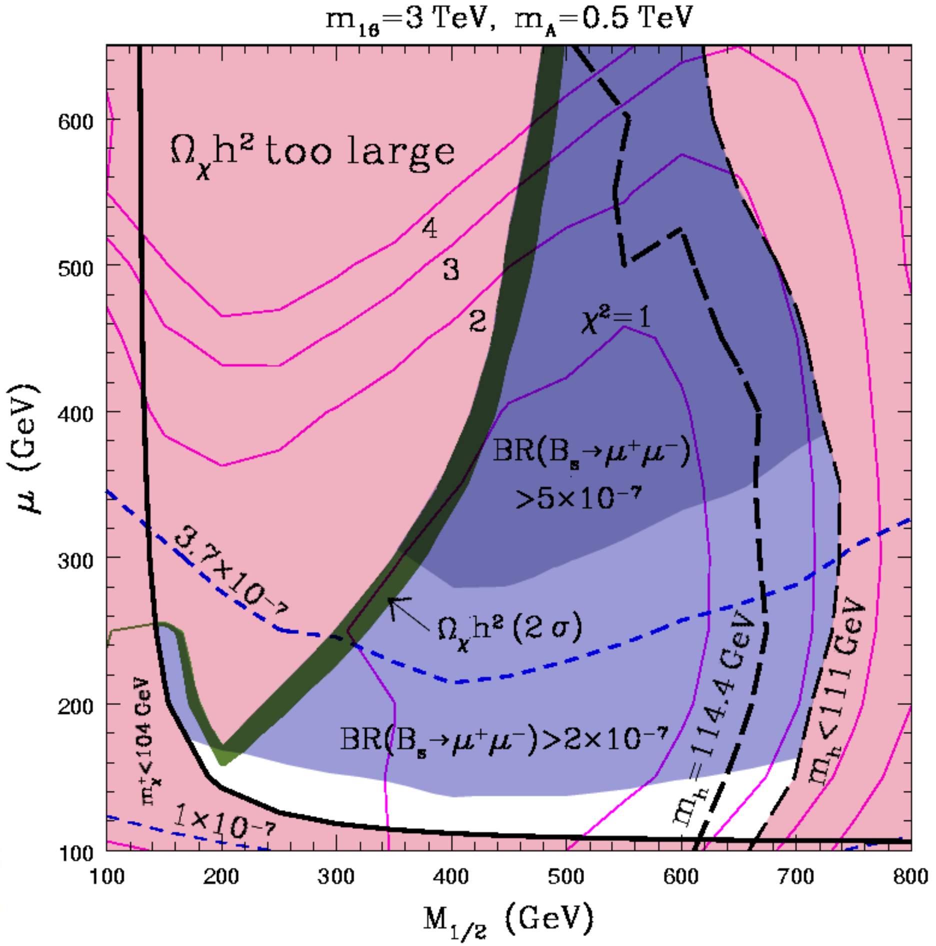
Bottom - Up and Down and Up ...



$$R = \frac{\max(f_t, f_b, f_\tau)}{\min(f_t, f_b, f_\tau)}$$

Dermisek, Raby, Roszkowski and Ruiz de Austri JHEP 09 (2005) 029

Top - Down



$$\chi^2$$

$$\Delta m_H^2 \equiv \frac{\left(m_{H_d}^2 - m_{H_u}^2\right)}{2m_{10}^2} \approx 13\%$$

BDR "Just so"

$$m_i^2 = Q_X^i D_X + \left(m_i^0\right)^2$$

$$m_{16}^{(3)} \leq m_{16}^{(1,2)}$$

Baer, Kraml & Sekmen

arXiv:0908.0134

"DR3"

	Q	U^c	D^c	L	E^c	V^c	|	H_U	H_D
Q_X^i	1	1	-3	-3	1	5	|	-2	2
m_i^0		m_{16}					|	m_{10}	

Varying BCs at M_{GUT}
3rd Family only

Balazs & Dermisek JHEP 0306:024 (2003)

non universal gaugino masses

Altmannshofer, Guadagnoli, Raby & Straub

$$\lambda_t \neq \lambda_b = \lambda_\tau$$

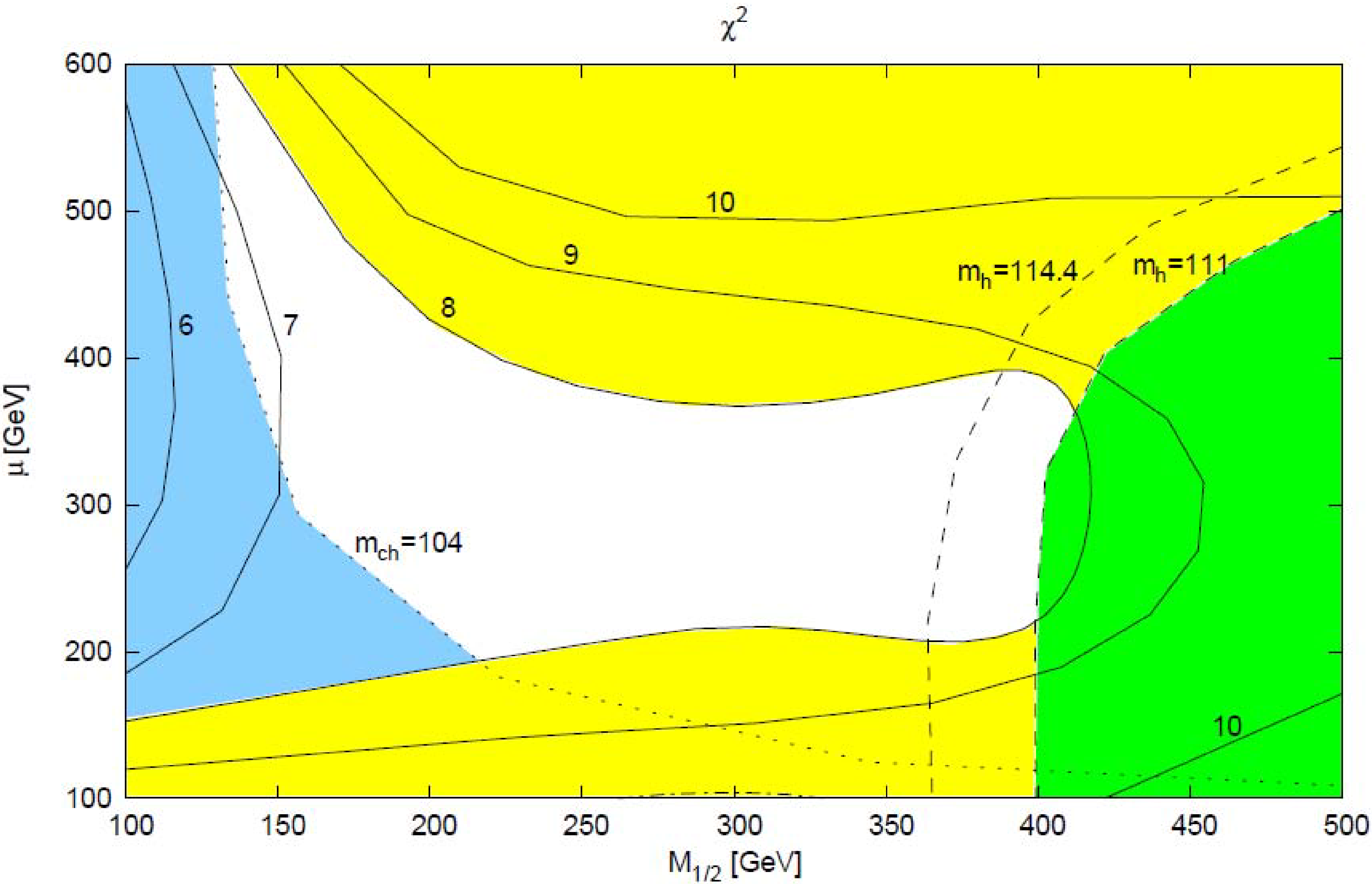
PLB 668, 385 (2008)

Baer, Kraml & Sekmen arXiv:0908.0134 (hep-ph)

D term splitting & $m_{16}^{(3)} \neq m_{16}^{(1,2)}$

Guadagnoli, Raby & Straub arXiv:0907.4709 (hep-ph)

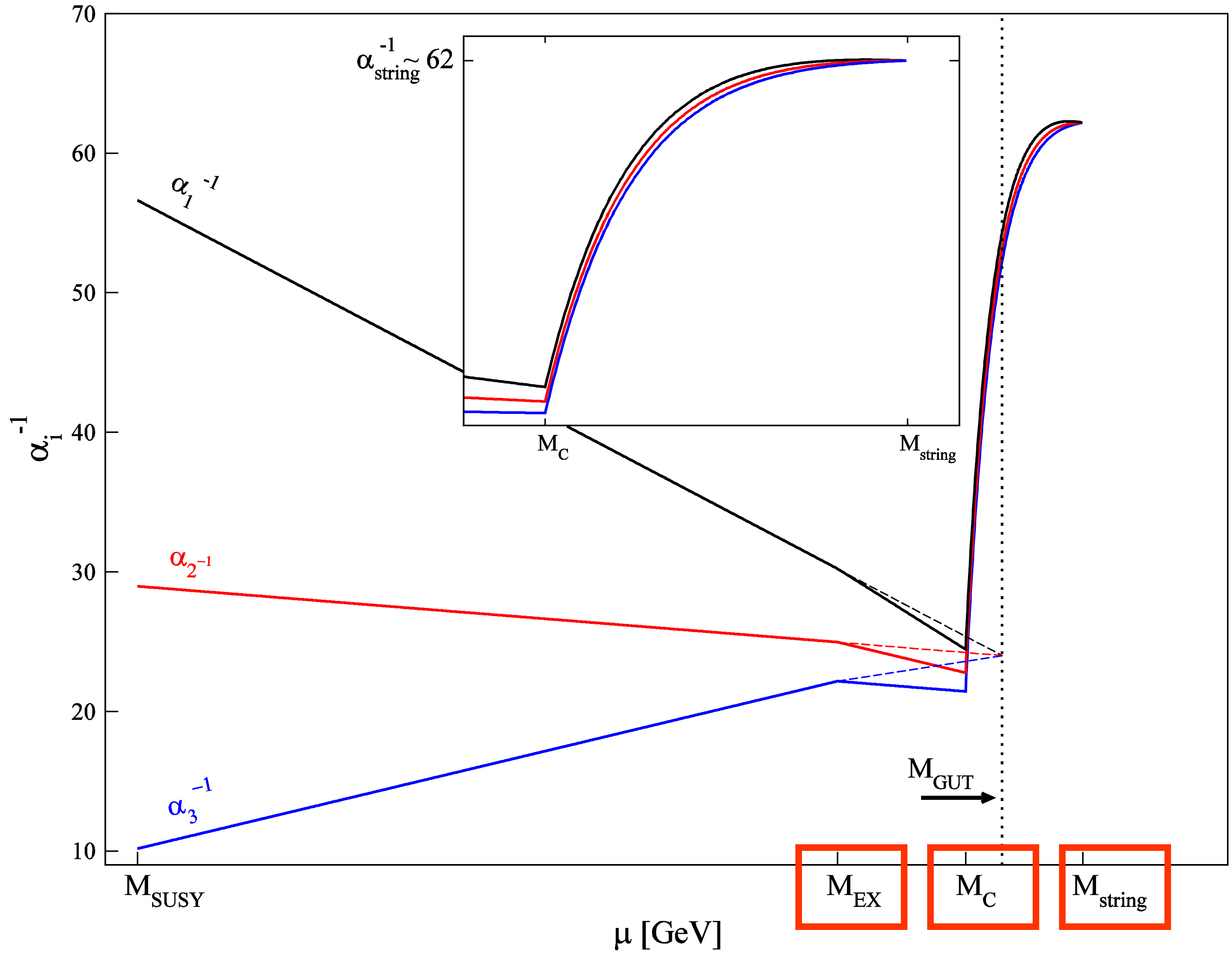
$$A_t \neq A_b \quad \& \quad m_{H_U}^2 \neq m_{H_D}^2$$



χ^2
600
500
400
300
200
100
100
150
200
250
300
350
400
450
500
$M_{1/2}$ [GeV]
μ [GeV]
6
7
8
9
10
10
m_h=114.4
m_h=111
m_{ch}=104

Gauge coupling unification

Dundee, SR & Wingerter arXiv:0805.4186



Proton decay - Dim. 6 operators