Superconformal symmetry and inflation in the NMSSM

Einhorn and Jones, 0912.2718

Ferrara, Kallosh, A.L., Marrani and Van Proeyen, 1004.0712

H.M. Lee, 1005.2735

S. Ferrara, R. Kallosh, A. L., A. Marrani, and A. Van Proeyen, 1008.2942

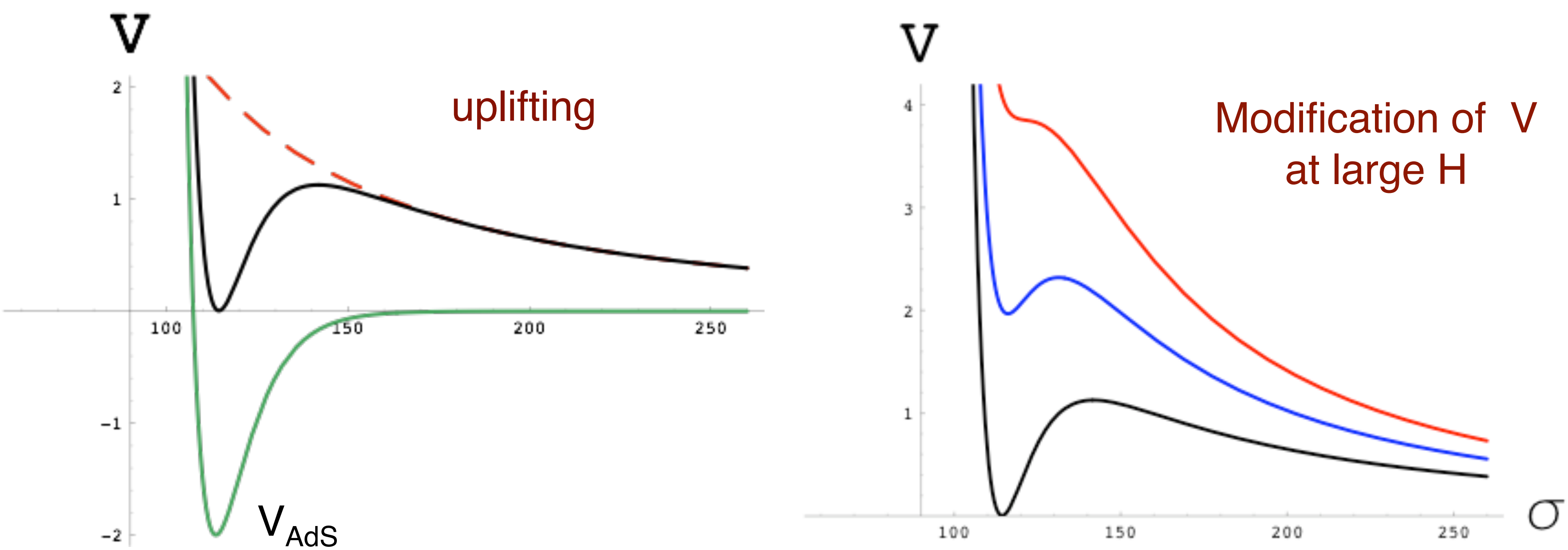
R. Kallosh, A. L., 1008.3375

Before we discuss supergravity,
a short note on string cosmology:

String Cosmology and the Gravitino Mass

Kallosh, A.L. 2004

The height of the KKLT barrier is smaller than $|V_{AdS}| = m^2_{3/2}$. The inflationary potential V_{infl} cannot be much higher than the height of the barrier. Inflationary Hubble constant is given by $H^2 = V_{infl}/3 < m^2_{3/2}$.



Constraint on the Hubble constant in this class of models:

$$H < m_{3/2}$$

Can we avoid these conclusions?

Chaotic inflation is string theory (Silverstein and Westphal, 2008) requires $m_{3/2} > H \sim 10^{13}$ GeV.

In more complicated theories one can have $H \gg m_{3/2}$. But it requires fine-tuning (Kallosh, A.L. 2004, Badziak, Olechowski, 2007)

In models with large volume of compactification (Quevedo et al) the situation is even more dangerous: $H < m_{3/2}^{3/2} < 1\ KeV$

It is possible to solve this problem, but it is rather nontrivial, and, once again, it requires fine tuning.

Conlon, Kallosh, A.L., Quevedo, 2008

2010 update:

One can make this problem less dangerous, but nevertheless the problem with light gravitino remains unsolved.

He, Kachru, Westphal, 2010

Tensor Modes and GRAVITINO

$$r \sim 10^8 H^2$$

$$H \leq M_{3/2}$$

$$r \leq 10^8 \ M_{3/2}^2$$

Kallosh, A.L. 2007

$$r \sim 10^{-2} \longrightarrow M_{3/2} \sim 10^{13} GeV$$

superheavy gravitino

$$M_{3/2} \sim 1 TeV \longrightarrow r \sim 10^{-24}$$

unobservable

If we find tensor modes AND light superpartners of normal particles at LHC, it might have as important consequences for string theory as the discovery of the cosmological constant

Superconformal symmetry and inflation in the NMSSM

- Superconformal symmetry and SUGRA
- NMSSM
- Inflation in NMSSM and beyond

A toy model: conformal symmetry

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}\partial_\mu\phi\partial_\nu\phi\, g^{\mu\nu} + \frac{\phi^2}{12}R(g) - \frac{1}{2}\partial_\mu h\partial_\nu h\, g^{\mu\nu} - \frac{h^2}{12}R(g) - \frac{\lambda}{4}h^4\right]$$

invariant under local conformal transformations

$$g'_{\mu\nu} = e^{-2\sigma(x)}g_{\mu\nu}\,, \qquad \phi' = e^{\sigma(x)}\phi\,, \qquad h' = e^{\sigma(x)}h$$

gauge fixing (fixing the conformal compensator ϕ):

$$\phi(x) = \sqrt{6}\,M_p$$

$$\mathcal{L}_{\mathrm{E}} + \mathcal{L}_{\mathrm{conf}} = \sqrt{-g}\,\frac{M_p^2}{2}R(g) - \sqrt{-g}\left[\frac{1}{2}\partial_\mu h\partial_\nu h\, g^{\mu\nu} + \frac{h^2}{12}R(g) + \frac{\lambda}{4}h^4\right]$$

Einstein gravity

matter part remains conformal invariant

One can always go to the Einstein frame, where $h^2 R$ is absent, but this will hide conformal invariance of the matter action.

We could add the terms $h\phi^3$, $h^2\phi^2$, $h^3\phi$, or ϕ^4 to the original theory without breaking its conformal invariance. After the gauge fixing, these terms produce enormously large terms $hM_p{}^3$, $h^2M_p{}^2$, h^3M_p, $M_p{}^4$. These terms strongly break conformal invariance and introduce Planckian parameters.

If we do not want to break conformal symmetry too strongly, we should require that matter is decoupled from the conformal compensator, or at least that its action does not contain terms proportional to ϕ. Symmetry breaking by terms inversely proportional to ϕ will be suppressed by the smallness of the gravitational coupling.

Superconformal approach to supergravity

To study matter interactions in SUGRA one may start with the superconformal theory coupled to supergravity, then fix the conformal compensator.

However, in the standard textbook approach, the resulting theory was formulated in the Einstein frame, and the original superconformal symmetry was well hidden.

It was difficult to formulate supergravity in Jordan frame and use advantages of superconformal invariance.

Textbook version:
Einstein frame

Kaku, Townsend and van Nieuwenhuizen 1977
Cremmer, Julia, Scherk, Ferrara, Girardello and van Nieuwenhuizen 1979
Barbieri, Ferrara, Nanopoulos and Stelle 1982
Cremmer, Ferrara, Girardello and Van Proeyen 1983
Girardi, Grimm, Muller and Wess 1984
Weinberg, QFT Volume III, 2000

Einstein frame, by gauge fixing of the conformal compensator

Kallosh, Kofman, A.L. and Van Proeyen 2000

$$-\frac{1}{6}\mathcal{N}(X,\bar{X})R \quad \Longrightarrow \quad \frac{1}{2}M_p^2 R$$

New version:
Jordan frame

Ferrara, Kallosh, A.L., Marrani and Van Proeyen, 1004.0712

$$-\frac{1}{6}\mathcal{N}(X,\bar{X})\,R \quad \Longrightarrow \quad -\frac{1}{6}\Phi(z,\bar{z})\,R$$

Jordan frame
CSS

Ferrara, Kallosh, A.L., Marrani and Van Proeyen, 1008.2942

CSS (canonical superconformal supergravity) is a special class of SUGRA models with canonical kinetic terms and a potential as in a global SUSY

Canonical Superconformal Supergravity

Top – down approach: We found a class of supergravity models in the Jordan frame, in which, under certain conditions, superconformal invariance of the matter part of the action remains unbroken. Such theories are very simple: kinetic terms are canonical, and scalar potential is the same as in global SUSY: $V = |\partial W|^2$

No such terms as e^K, $-3|W|^2$, $K^{i\bar{k}}$, $K_{,i}W$, **they all cancel!**

We call it **Canonical Superconformal Supergravity (CSS)**.

The main condition: matter fields are decoupled from conformal compensator.

Bottom - up approach: One can embed SUSY to SUGRA in many different ways. For theories with scale-invariant superpotentials and canonical kinetic terms, such as the scale-invariant NMSSM, there is a special choice of the Kahler potential which allows to embed SUSY to the CSS. This embedding is trivial: One simply adds to supergravity the action of the global SUSY conformally coupled to gravity. The scalar potential and kinetic terms remain the same as in the global SUSY.

CSS embedding of scale-free NMSSM

$$W_{\rm Higgs} = -\lambda S H_u \cdot H_d + \frac{\rho}{3} S^3$$

$$\hat{\mathcal{L}}_{\rm total} = \hat{\mathcal{L}}_{\rm supergravity} + \hat{\mathcal{L}}_{\rm sc}^{m}$$

$$\hat{\mathcal{L}}^{\rm m}_{\rm sc} = \sqrt{-g_J}\left[-\frac{R}{6}\left(S\bar{S} + H_u H_u^\dagger + H_d H_d^\dagger\right) - D_\mu H^u\, D^\mu H_u^\dagger - D_\mu H^d\, D^\mu H_d^\dagger - D_\mu S\, D^\mu S^\dagger - \hat{V}_J\right]$$

$$\hat{V}_J = \left|\frac{\partial W}{\partial S}\right|^2 + \left|\frac{\partial W}{\partial H_u}\right|^2 + \left|\frac{\partial W}{\partial H_d}\right|^2 + \frac{g'^2}{8}(|H_u^0|^2 - |H_d^0|^2)^2 + \frac{g^2}{8}(H_u^\dagger \vec{\tau} H_u + H_d^\dagger \vec{\tau} H_d)^2$$

Mass parameters are forbidden by superconformal invariance, which results in the scale-free structure of the superpotential in the NMSSM.

No Higgs mass, no μ-term, no tadpoles, no nonrenormalizable terms.

One must break superconformal invariance and introduce masses, but one can do it softly, due to gravitational interactions, hidden sector, and anomalies.

To break the superconformal invariance, we modify the frame function

$$\Phi = -3 + |S|^2 + |H_u|^2 + |H_d|^2 + \frac{3}{2}\chi(H_u \cdot H_d + \text{h.c.})$$

This is equivalent to the introduction of nonminimal coupling to gravity

$$-\frac{1}{4}\chi(H_u \cdot H_d + \text{h.c.})\, R$$

It can be also described by the Kahler potential

$$\mathcal{K}_\chi(z,\bar{z}) = -3\log\left[1 - \frac{1}{3}\left(S\bar{S} + H_u H_u^\dagger + H_d H_d^\dagger\right) - \tfrac{3}{2}\chi(H_u \cdot H_d + \text{h.c.})\right]$$

The last term induces the effective μ-term by the Giudice-Masiero mechanism, due to the interaction with a hidden sector:

$$W_{\text{eff}} = -\lambda\, S\, H_u \cdot H_d + \frac{\rho}{3} S^3 + \frac{3}{2}\,\chi\, m_{3/2} H_u \cdot H_d$$

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$$\mu_{\text{eff}} = \frac{3}{2}\chi m_{3/2} - \lambda \langle S \rangle$$

$\mathbb{Z}_3$ -noninvariant contribution to the scalar potential:

$$\Delta V = \frac{3}{2} B_\mu \, \chi m_{3/2} \, (H_u \cdot H_d + h.c.)$$

Our estimates suggest that this contribution breaks $\mathbb{Z}_3$ symmetry strongly enough to solve the domain wall problem in the NMSSM.

Inflation in NMSSM

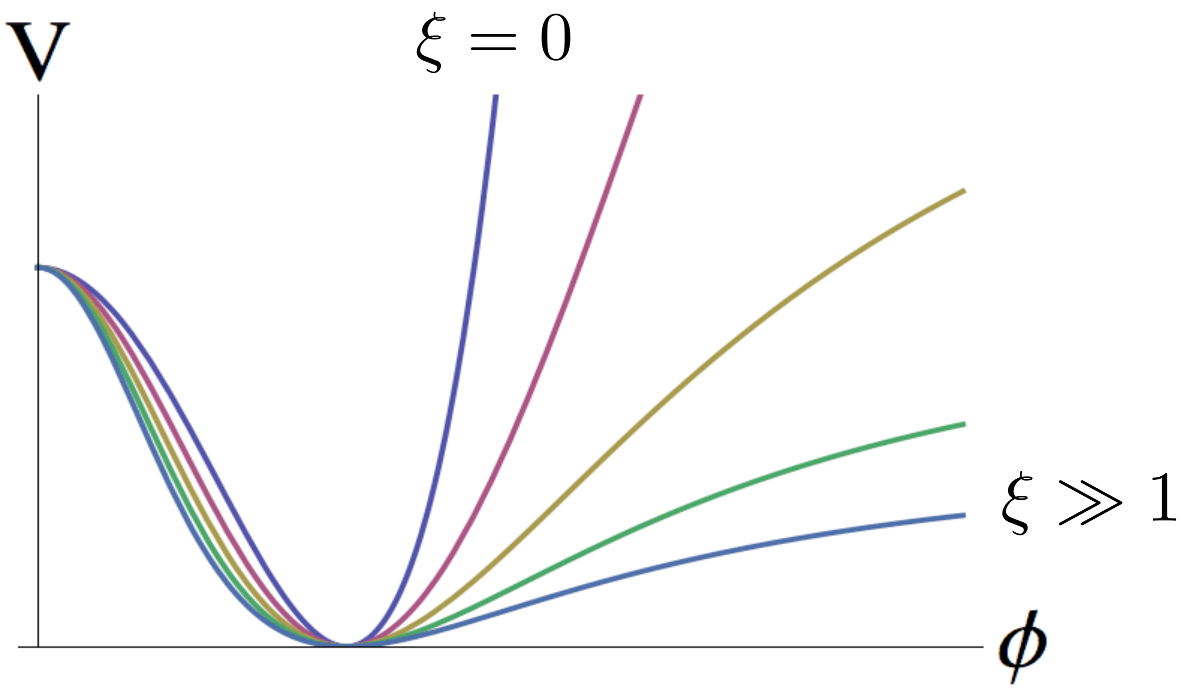
Basic idea (Bezrukov, Shaposhnikov 2007, non-SUSY) : Consider the standard model with

$$V(\phi) = \frac{\lambda^2}{4}(\phi^2 - v^2)^2$$

and add nonminimal coupling of the Higgs field to gravity $\frac{\xi}{2}\phi^2 R$

The potential in the Einstein frame becomes flat at large values of the field:

$$V(\phi) = \frac{\lambda^2}{4}\frac{(\phi^2 - v^2)^2}{(1+\xi\phi^2)^2}$$



One can achieve it in NMSSM

Einhorn and Jones, 0912.2718

$$W_{\text{Higgs}} = -\lambda S H_u \cdot H_d + \frac{\rho}{3} S^3$$

$$\mathcal{K}_\chi(z,\bar{z}) = -3\log\left[1 - \frac{1}{3}\left(S\bar{S} + H_u H_u^\dagger + H_d H_d^\dagger\right) - \tfrac{3}{2}\chi(H_u \cdot H_d + \text{h.c.})\right]$$

This theory has the same potential in the Higgs direction as the Bezrukov-Shaposhnikov model, but it is unstable in the S – direction

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Fortunately, it can be stabilized by adding a term $-S^4$ to the Kahler potential

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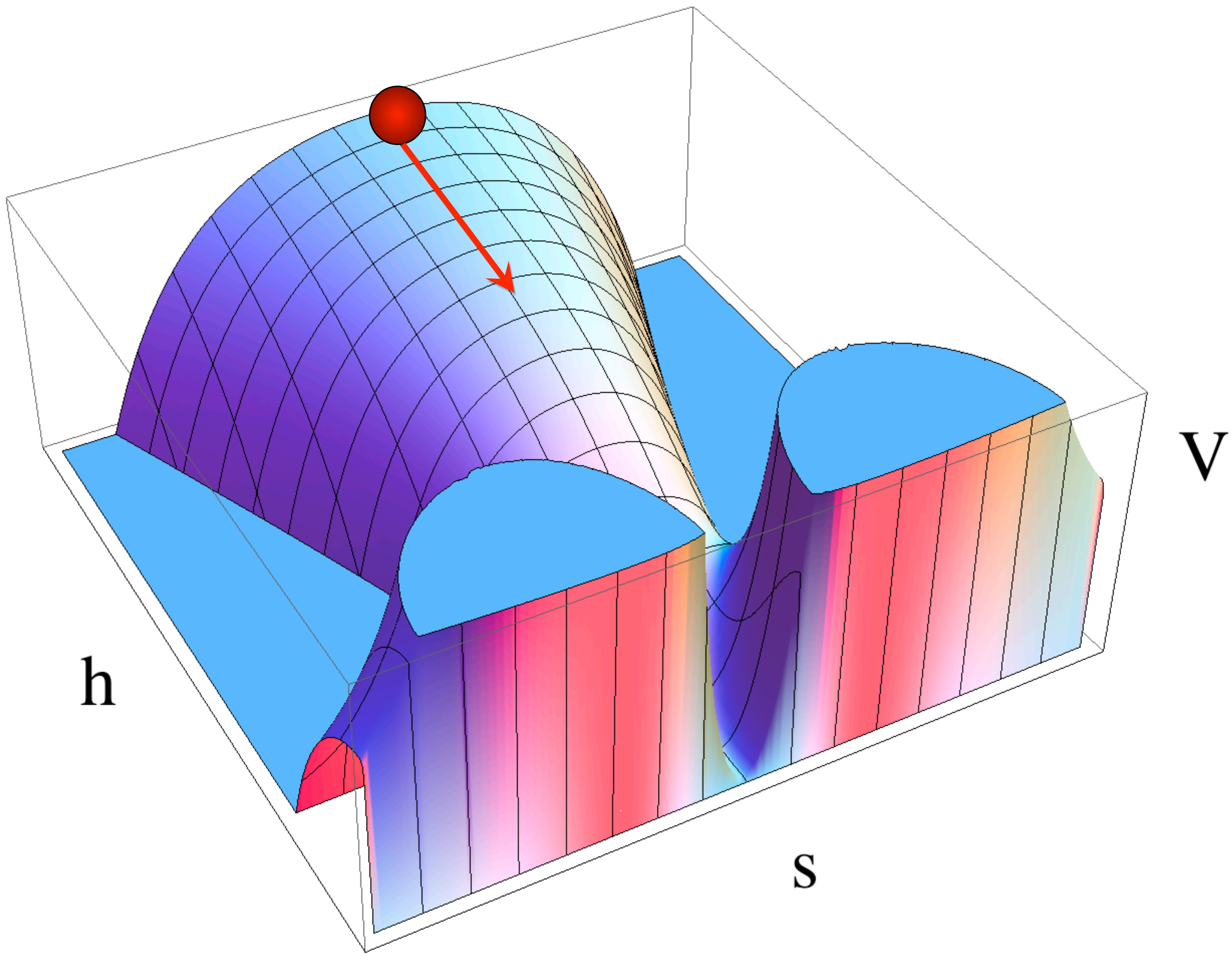
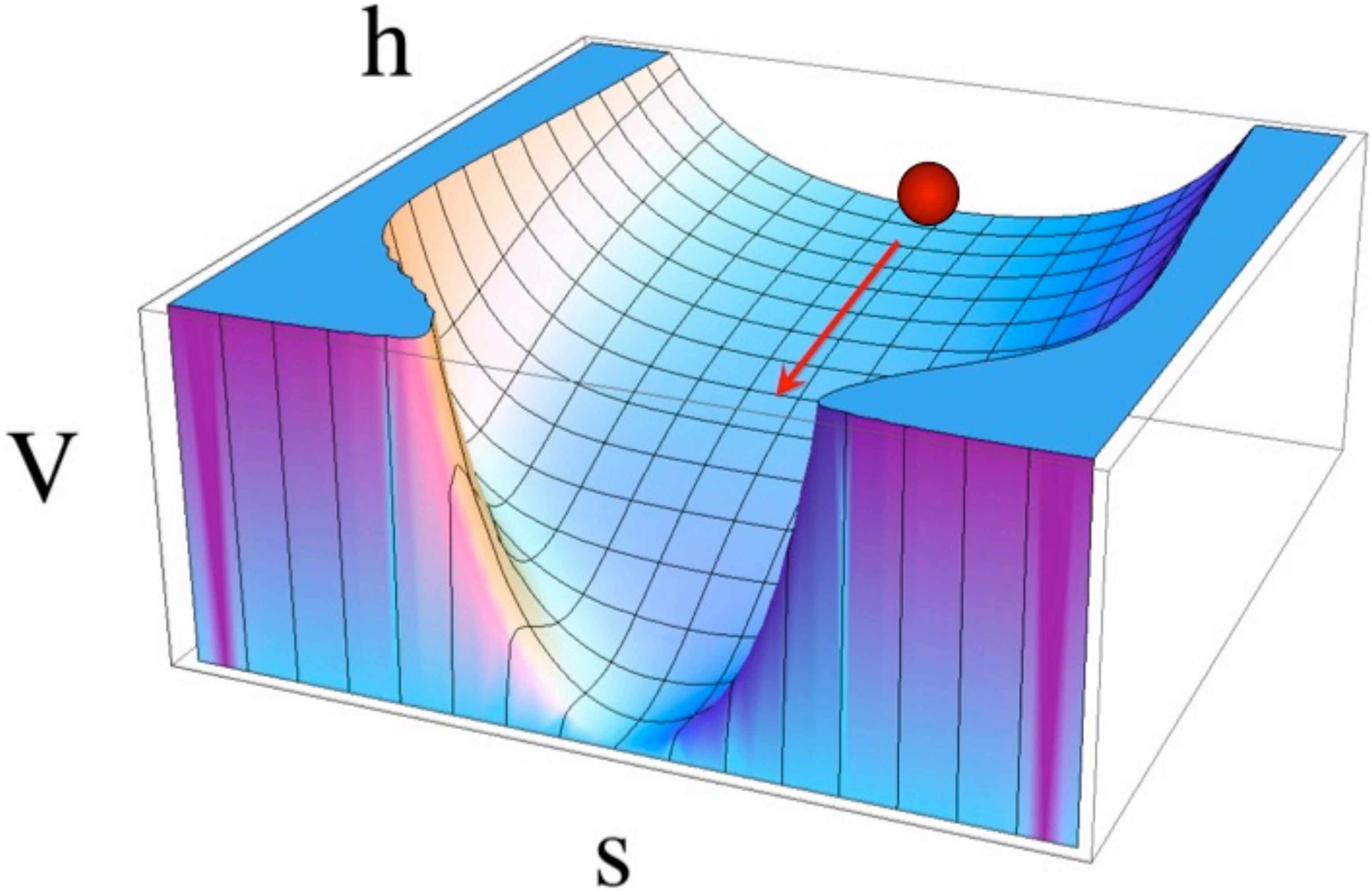
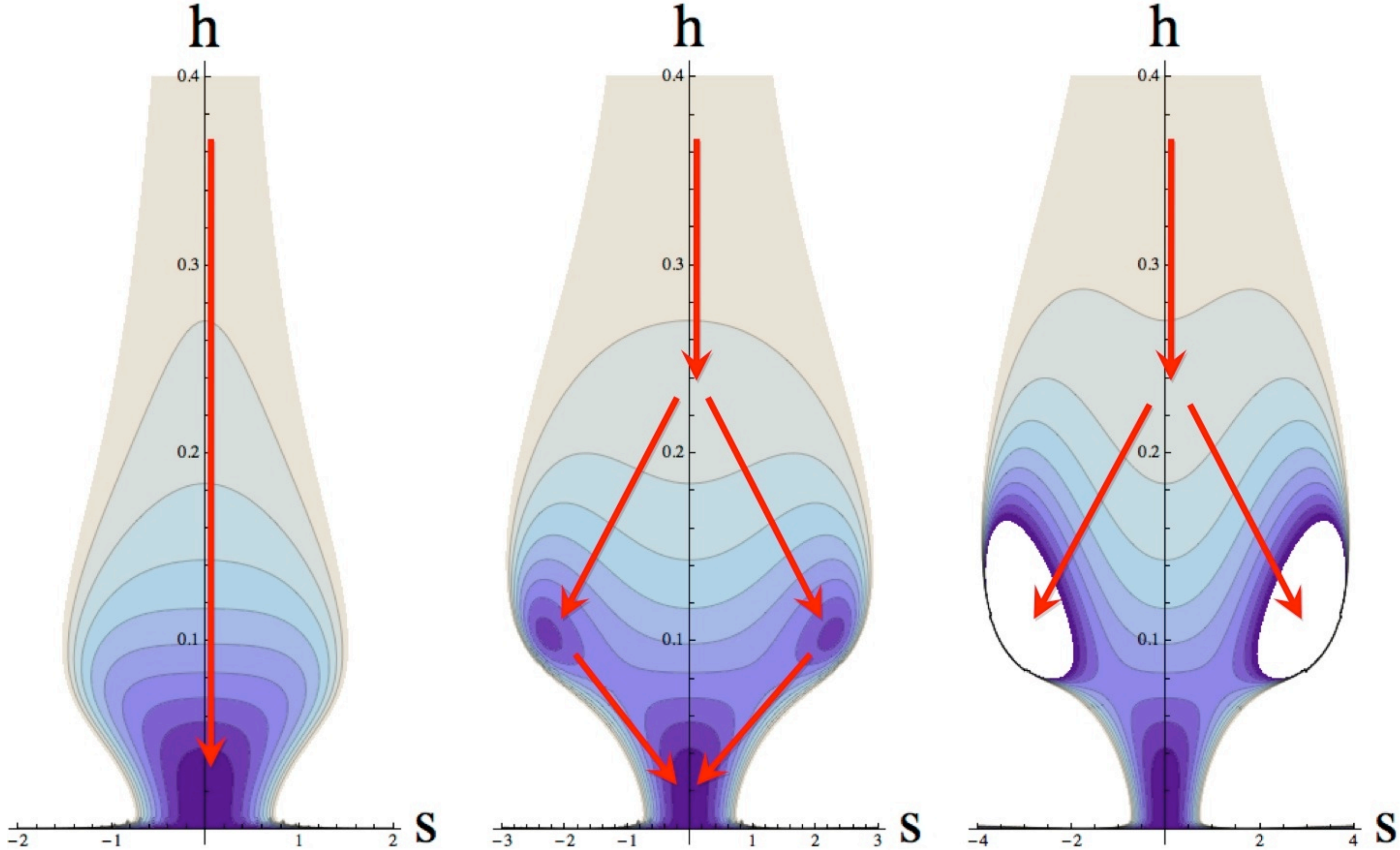
Potential before stabilization



Potential after stabilization





If the term S^4 is small, the field falls to one of the two AdS minima which look like white eyes of an alien, and the universe collapses.

The term S^4 must be large enough to achieve stability, as in the left part of the figure. Then we have inflation in the NMSSM.

Inflation in CSS-type models with minimal coupling to gravity

Different form of the same Kahler potential

$$\mathcal{K} = -3\log\Big[1 - \frac{1}{3}S\bar{S} + \frac{1}{12}\left(1 + \frac{3}{2}\chi\right)(\Phi - \bar{\Phi})^2 - \frac{1}{12}\left(1 - \frac{3}{2}\chi\right)(\Phi + \bar{\Phi})^2 + \ldots\Big].$$

For $\chi = 2/3$ the Kahler potential does not depend on the real part of the field Φ. In particular, for

$$\mathcal{K} = -3\log\left[1 + \frac{1}{6}(\Phi - \bar{\Phi})^2 - \frac{1}{3}S\bar{S} + \zeta(S\bar{S})^2/3\right]$$

and $W = S\,F(\Phi)$

the potential as a function of the real part of Φ at S = 0 is

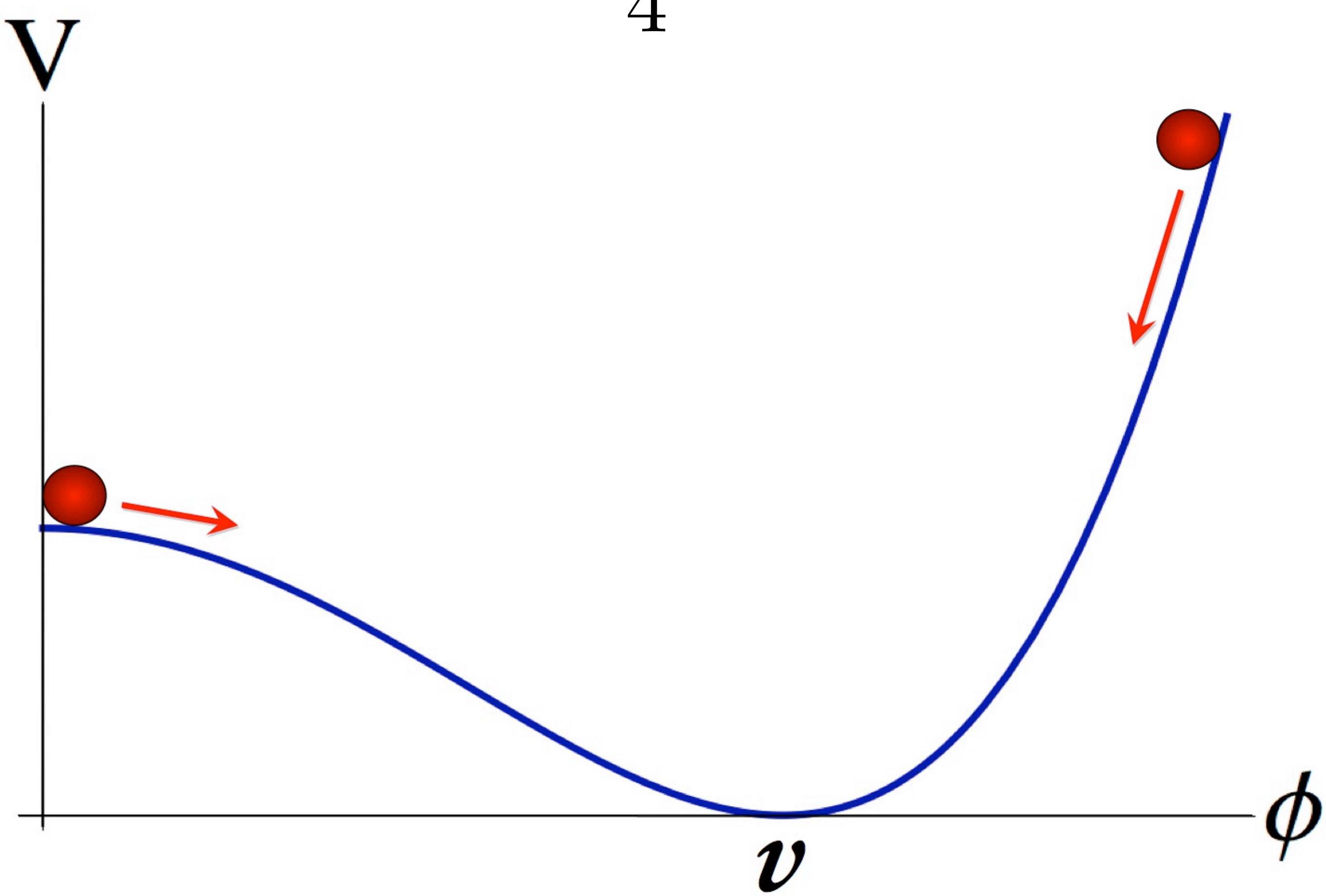
$$V = |F(\Phi)|^2$$

Thus we have a **FUNCTIONAL FREEDOM** in choosing inflationary potential

Example: $W = -\lambda S(\Phi^2 - v^2/2)$

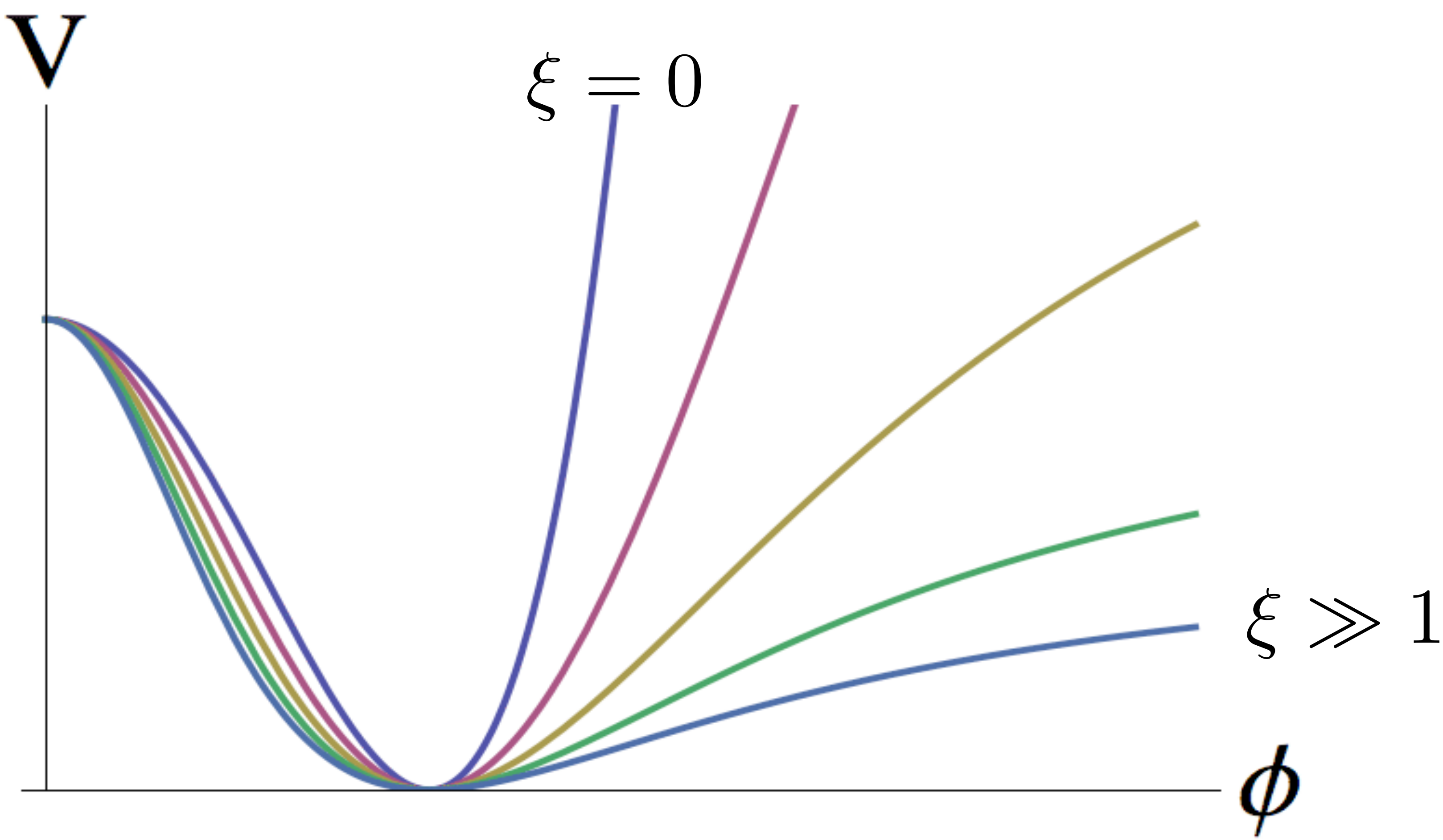
During inflation $S = 0,\ \mathrm{Im}\,\Phi = 0,\ \mathrm{Re}\,\Phi = \sqrt{2}\,\phi$

$$V(\phi) = \frac{\lambda^2}{4}(\phi^2 - v^2)^2$$



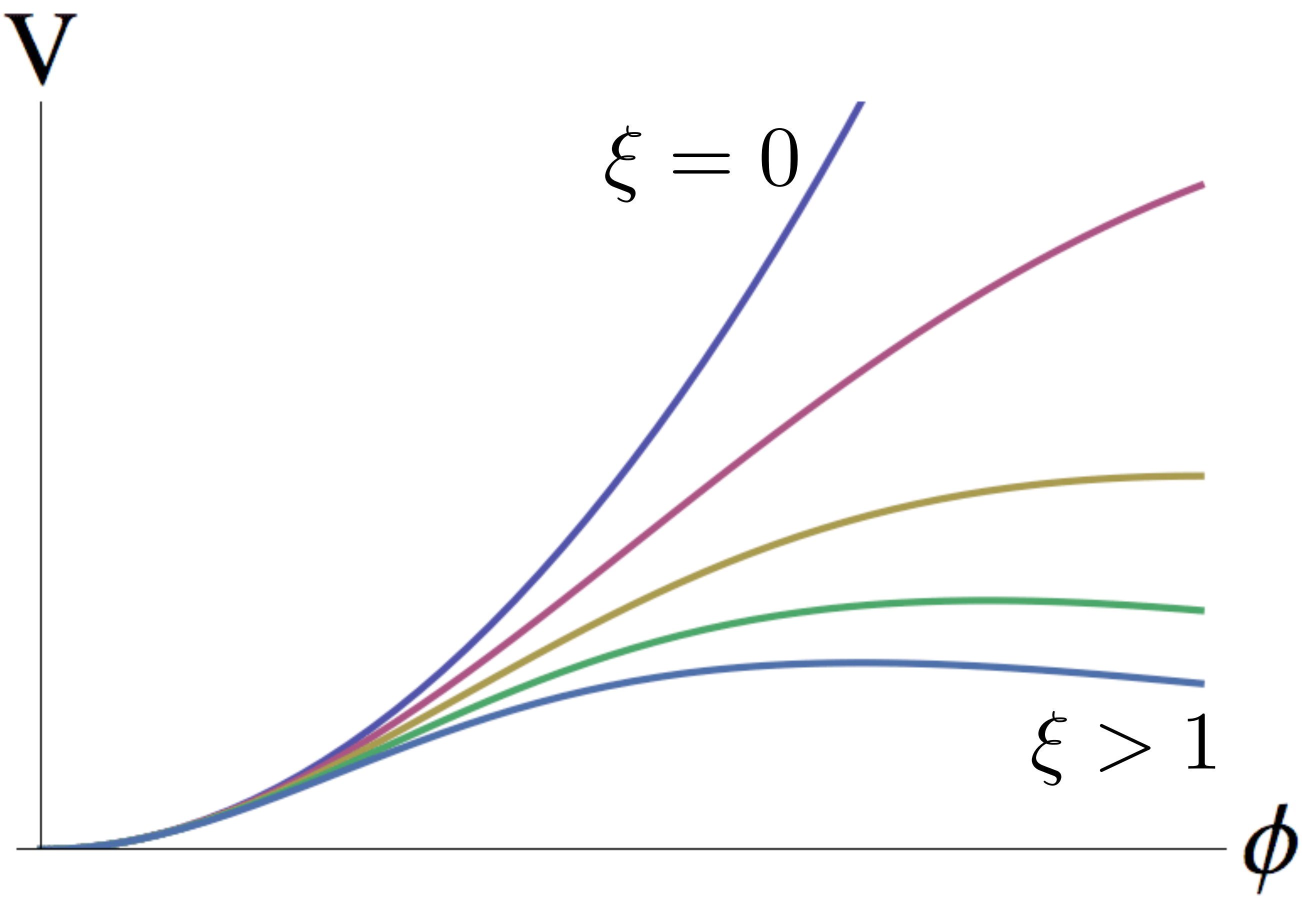
Nonminimal coupling $\frac{\xi}{2}\phi^2 R$

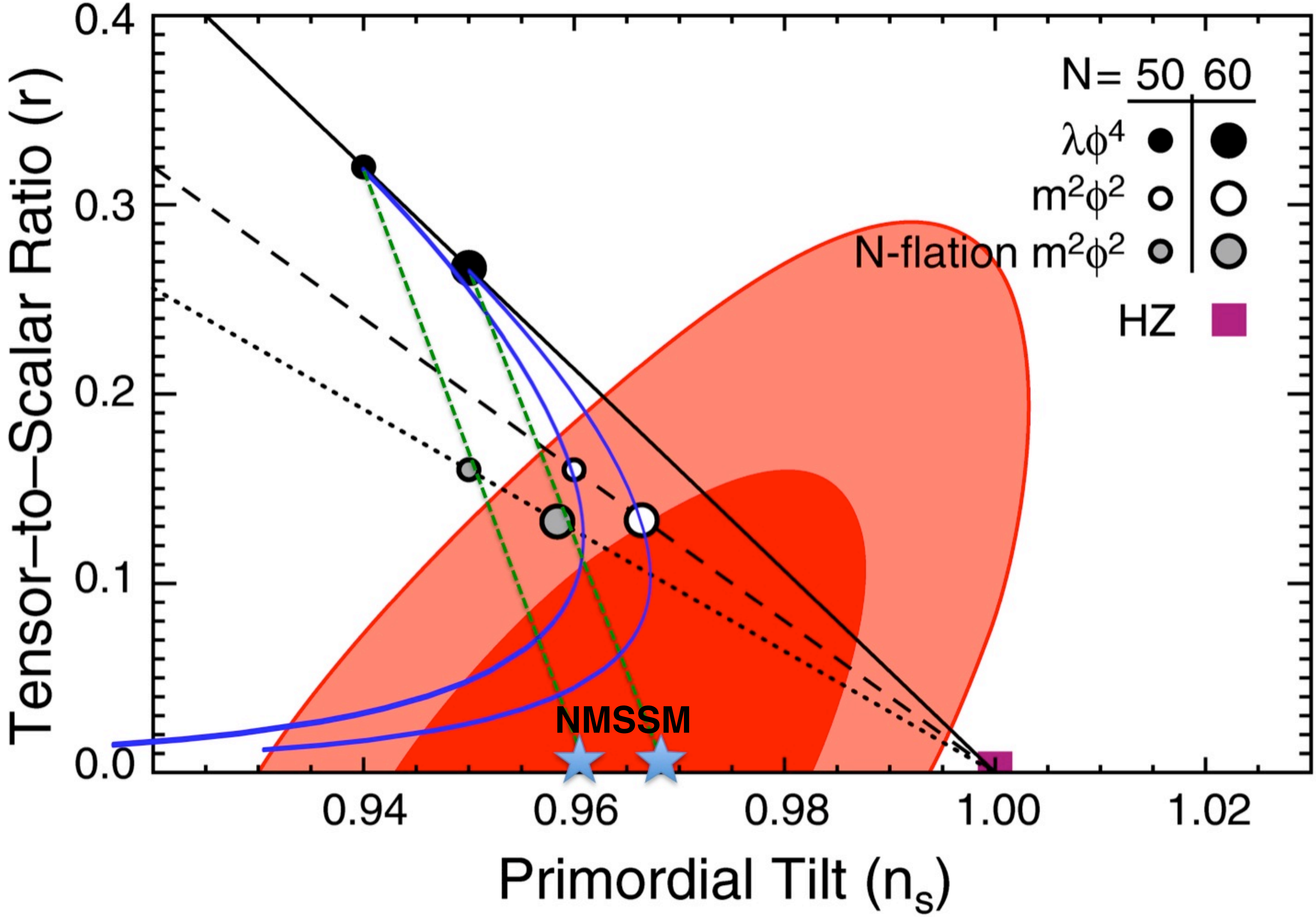
$$\xi = (\chi - 2/3)/4 \qquad V(\phi) = \frac{\lambda^2}{4}\frac{(\phi^2 - v^2)^2}{(1+\xi\phi^2)^2}$$



$$W = m\,S\,\Phi$$

$$V(\phi) = \frac{m^2\phi^2}{2(1+\xi\phi^2)^2}$$





N= 50 60
λφ4
m2φ2
N-flation m2φ2
HZ
NMSSM
0.0
0.1
0.2
0.3
0.4
0.94
0.96
0.98
1.00
1.02
Tensor–to–Scalar Ratio (r)
Primordial Tilt (ns)

Conclusions

1) Scale-free NMSSM can be easily embedded into a canonical superconformal supergravity (CSS). This embedding is amazingly simple: Canonical kinetic terms, potential as in the global NMSSM. A special way of breaking of this symmetry by gravitational interactions and hidden sector allows to address the μ problem and domain wall problem, and implement inflation in the NMSSM.

2) Softly broken CSS provides functional freedom of choice for the inflationary potential, which is important for a proper interpretation of new and upcoming observational data.