MASS MEASUREMENTS AT THE LHC

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Work in progress with:
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[ Flossbridge collaboration ]

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W. Lamb (1955): “The finder of a new elementary particle used to be rewarded by a Nobel Prize, but such a discovery now ought to be punished by a $10,000 fine”
What to do about the LSP momenta?

- Nothing (no need to introduce the $X_0$ mass in the game)
  - invariant mass endpoint methods (need $n>2$ on either side)
  - invariant mass cusp method (requires an s-channel resonance)
  - $M_{CT}$ methods (need only $n>0$ on each side)
- Use some prescription to fix them somehow
  - $M_{T2}$ methods (need only $n>0$ on each side)
- Compute them exactly
  - “polynomial” methods (if $n>2$ on each side or if $n>3$ on one side)
- Some (hybrid) combination of the above

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Anatomy of an event

- **Visible particles**
  - known: number, masses, momenta

- **Invisible particles**
  - unknown: number, masses, momenta
  - known: total $P_T$

Invariant mass variables are constructed by:

- (optional) partitioning and lumping within each partition
- (optional) transversifying w.r.t. the beam axis (NB! two different ways)
- (optional) transversifying w.r.t. the upstream $P_T$
- (mandatory) fixing invisible momenta by minimizing CME:

\[ \sqrt{s_{min}} \]
I. Basic definition of $\sqrt{s_{\text{min}}}$

Konar, Kong, KM 2008

- No partitioning; lump the visibles; separate the invisibles

\[
\begin{align*}
(E, \vec{P}_T, P_z) & \equiv P^\mu = \sum_{i=1}^{N_{\text{vis}}} P_i^\mu \\
\sqrt{s_{\text{min}}(M_{\text{inv}})} & = \sqrt{E^2 - P_z^2} + \sqrt{M_{\text{inv}}^2 + P_T^2}; \\
M_{\text{inv}} & = \sum_{i=1}^{N_{\text{inv}}} \tilde{M}_i
\end{align*}
\]

- What is the minimum possible value of the CM energy?
Applications of $\sqrt{s_{\text{min}}}$

- Single semi-invisibly decaying particle
  - SM Higgs to tt-bar
  - endpoint at the parent mass

- A pair of semi-invisibly decaying particles
  - direct tt-bar production
  - peak at the total parent mass
• Objection: “you should not include objects from ISR and UE”
  – Solution: very good, then don’t:

- Repeat the constrained minimization and find:

\[ \sqrt{s_{\text{min}}}^{(\text{sub})}(M_{\text{inv}}) = \sqrt{\left(\sqrt{E^2 - P_z^2} + \sqrt{M_{\text{inv}}^2 + P_T^2}\right)^2 - P_{T(\text{up})}^2} \]
Applications of subsystem \( \sqrt{s_{\text{min}}} \)

Konar, Kong, KM, Park 2010

- **tt-bar events**
  - identify the WW threshold from the 2 lepton subsystem

- **GMSB SUSY events**
  - identify the \( N_1N_1 \) threshold from the 2 photon subsystem
II. Partitioning and lumping together

Assuming N parents of equal mass, partition as

\[ M\( (1) \) = M\( (2) \) = M\( (3) \) \]

Now the invisible momenta are chosen to minimize the mass of any parent:

\[
M_N(\tilde{M}(1), \tilde{M}(2), \tilde{M}(3)) \equiv \min_{M(1) = M(2) = M(3)} \left\{ M(a) \right\}
\]
III. Transversification (once)

Barr, Khoo, Konar, Kong, Lester, KM, Park 2010

- So far $S_{\text{min}}$ and $M_N$ are genuine (1+3)-dim. quantities
- One often uses (1+2)-dim. “transverse” quantities
  - the transverse projection is not unique! there are two (inequivalent) ways to do it:
    - Type “T”: mass preserving
    - Type “t”: velocity preserving
  - the operations of transversification and partitioning do not commute! Depending on the particular order, there are different types of variables, e.g.:
    - $M_{2T}$ (the original Cambridge variable $M_{T2}$)
    - $M_{2t}$
    - $M_{T2}$
    - $M_{t2}$
Transversification alternatives

\( P^\mu \equiv (E, \vec{P}_T, P_z) \quad P^2 = M^2 \)

- \( \vec{p}_T = \vec{P}_T \)
- \( e_T = \sqrt{M^2 + \vec{P}_T^2} \)
- \( p^2 = M^2 = P^2 \)

- preserves the mass
- no \( P_z \) dependence
- test mass remains

- \( \tilde{e}_t = \sqrt{\tilde{M}^2 + Q_T^2} \)

- \( \vec{p}_t = \vec{P}_T \)
- \( e_t = E \sin \theta = E \frac{P_T}{\sqrt{P_T^2 + P_z^2}} \)
- \( v^2 = 1 - \frac{M^2}{E^2} = V^2 \)

- preserves the velocity
- test mass disappears during minimization \((Q_z \rightarrow \infty)\)

\( \tilde{e}_t = Q_T \sqrt{1 + \frac{\tilde{M}^2}{Q_T^2 + Q_z^2}} \)
The recipe

• Partition the observed particles into $N$ parent sets plus a separate set for Upstream objects
• Do one then the other
  – Lump the energies and momenta of the visible particles within each set
  – Transversify all energies and momenta
• Fix the unknown momenta of the invisible particles by minimizing the largest parent transverse mass
• Record the minimum value of the largest parent mass
• For any value of $N$, there are 4 different cases:
  – $M_{NT}$, $M_{Nt}$, $M_{TN}$, $M_{tN}$
Mathematical identities for N=1

- $M_{1T} = M_1 = \sqrt{s_{min}}$ for any invisible test mass
  - the “T” transverse mass is a (1+3)-dim. quantity!
- $H_T = M_{t1}$ (neither depends on an invisible test mass)
  - reveals the physical meaning of $H_T$ in the $\sqrt{s_{min}}$ sense
- $H_T = M_{T1}(0)$, but not for general invisible test mass

Barr,Khoo,Konar,Kong,Lester,KM,Park 2010

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Mathematical identities for $N=2$

- $M_{2T} = M_2$ for any invisible test mass
  - the “2T” type Cambridge variable is a $(1+3)$-dim. quantity!
- $M_{T2}(0) = M_{t2}$ for massless visitibles
- Notice the different shapes near the upper endpoint
Transversification (twice)

KM, Park 2009

• Having projected on the transverse plane, one can additionally project on the direction of Upstream $P_T$:

• The endpoints of “perp” distributions are stable against $P_T$ variations

Konar, Kong, KM, Park 2009
The two interesting limiting cases

• Very simple events (n=0 and n=1)
  – good news: no combinatorial problem
  – bad news: insufficient information, difficult to extract dark matter properties (mass, spin etc.)
  – generally more SM background

• Very complex events (n=\infty [4])
  – multijet events with 10-15 jets per event
  – very severe combinatorial problem
1-Dim $M_{2T}$ method

Konar, Kong, KM, Park 2009

- Basic idea: vary the LSP test mass and count how many events have $M_{2T}$ above the doubly transverse $M_{2T}$ endpoint

$$N(\tilde{M}_c) = \sum_{\text{all events}} H(M_{2T}(\tilde{M}_c) - M_{2T}^{\text{max}}(\tilde{M}_c))$$
1-Dim $M_{2T}$ method

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$$N(\tilde{M}_c) = \sum_{\text{all events}} H(M_{2T}(\tilde{M}_c) - M_{2T}^{\text{max}}(\tilde{M}_c))$$

Transverse $M_{2T}$

$\vec{p}_{1T} = -\vec{p}_{1T} - \vec{p}_{2T} - \vec{p}_T$

$\vec{p}_{1T}$

$\vec{p}_{2T}$

$\vec{p}_{2T}$

$\vec{p}_{1T}$

$\vec{p}_T$

$\vec{p}_T$

$\vec{p}_T$

$\vec{p}_T$
The other extreme: very complicated events

- Let’s now do $n=\infty$
- Imagine something very complicated like
  - gluino pair production
  - gluino decays to $N_2$ and 2 jets
  - $N_2$ decays to $N_1$ and 2 jets
- Typical jet multiplicity is about 10
- Exclusive reconstruction seems hopeless

\[ \tilde{g} \rightarrow jj\tilde{\chi}_2 \rightarrow jjjjj\tilde{\chi}_1 \]

Unit-normalized distribution of jet multiplicity in gluino pair production events, with each gluino forced to undergo a two-stage cascade decay to the LSP. The resulting gluino signature is 4 jets plus missing energy.
Application of $S_{\min}$ to complex events

- One can measure SUSY masses in terms of the LSP mass. The peak of $S_{\min}$ marks (the sum of) the masses of all particles produced in the hard scattering

\[
\left( S_{\min}^{1/2} \right)_{\text{thr}} \approx \left( S_{\min}^{1/2} (2m_\chi) \right)_{\text{peak}}
\]

Konar, Kong, KM 2008
Summary

• By partitioning, lumping, transversifying and minimizing, one can obtain a whole series of invariant mass variables

\[ N = 1 \]
\[
M_1, M_{T1}, M_{t1}, M_{1T}, M_{1t}
\]
\[
M_{1T\perp}, M_{1t\perp}, M_{T1\perp}, M_{t1\perp}, M_{T\perp1}, M_{t\perp1}
\]
\[
M_{1T\parallel}, M_{1t\parallel}, M_{T1\parallel}, M_{t1\parallel}, M_{T\parallel1}, M_{t\parallel1}
\]

\[ N = 2 \]
\[
M_2, M_{T2}, M_{t2}, M_{2T}, M_{2t}
\]
\[
M_{2T\perp}, M_{2t\perp}, M_{T2\perp}, M_{t2\perp}, M_{T\perp2}, M_{t\perp2}
\]
\[
M_{2T\parallel}, M_{2t\parallel}, M_{T2\parallel}, M_{t2\parallel}, M_{T\parallel2}, M_{t\parallel2}
\]

• Which are most suitable, depends on the case at hand
• Some of these are old friends in disguise (\(H_T, M_{T2}, M_T, \ldots\))
• Watch for the exact meaning of the “transverse” index
• It helps to think of these variables as resulting from minimizing the total CM energy (LHC energy is “expensive”)
• Homework: figure out the meaning of the remaining variables on the second slide.
BACKUPS
Effect of ISR and UE

- ISR and UE destroy all these nice correlations...
  - Calculate ISR from first principles and undo the effect
    - What about UE? [Papaefstathiou, Webber 2009]
    - Use only reconstructed objects (e.g. jets instead of cal towers)
    - MHT in place of MET

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Inclusive SUSY production

- The peaks in $S_{\text{min}}$ mark the thresholds for the individual production subprocesses
- Example: GMSB point GM1b
  - $\Lambda=80$ TeV, $M=160$ TeV, $N=1$, $\tan\beta=15$, $\mu>0$

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Another $M_{T2}$ method

- Basic idea: study $M_{T2}$ distribution
  - of the two leptons only
  - for any two test masses
  - for two different values of $P_T$

New (our) method

Old (kink) method

$P_T=500$ GeV
$P_T=200$ GeV
Advantage of the new method

- The left branch endpoints are systematically underestimated.
- We can pick both measurements to be on the right branch, which is measured much better.

**Diagram:***

- Method I.
- Method II.

**Figure:***

- KM, Moortgat, Pape, Park 2009
- MSUGRA LM6
- LHC14, 100 fb⁻¹
- $P_T = 420 \pm 50$ GeV

**Table:***

- $\tilde{M}_P(\tilde{M}_c, P_T)$ (GeV)
- $\delta \tilde{M}_P$ (GeV)
- $\tilde{M}_c$ (GeV)
1-Dim $M_{CT}$ method

$KM, Park 2009$

$M_{CT\parallel} = \sqrt{m_1^2 + m_2^2 + 2(e_{1T\parallel} e_{2T\parallel} + \vec{p}_{1T\parallel} \cdot \vec{p}_{2T\parallel})}$

$M_{CT\perp} = \sqrt{m_1^2 + m_2^2 + 2(e_{1T\perp} e_{2T\perp} + \vec{p}_{1T\perp} \cdot \vec{p}_{2T\perp})}$

$e_i T_{\parallel} \equiv \sqrt{m_i^2 + |\vec{p}_i T_{\parallel}|^2}$,  \hspace{0.5cm} $e_i T_{\perp} \equiv \sqrt{m_i^2 + |\vec{p}_i T_{\perp}|^2}$.

**Transverse $M_{CT}$**

![Transverse $M_{CT}$ distribution](image)

$N_0 \times \frac{1}{50}$  \hspace{1cm} $M_p = 305.3$ GeV  \hspace{1cm} $M_c = 275.7$ GeV

**Longitudinal $M_{CT}$**

![Longitudinal $M_{CT}$ distribution](image)

$N_0 \times \frac{1}{10}$  \hspace{1cm} $M_p = 305.3$ GeV  \hspace{1cm} $M_c = 275.7$ GeV

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1-Dim $M_{CT}$ method

KM, Park 2009

\[ M_p = \frac{P_T M_{CT\parallel}^{max}(P_T) M_{CT\perp}^{max}}{(M_{CT\parallel}^{max}(P_T))^2 - (M_{CT\perp}^{max})^2}, \]

\[ M_c = \sqrt{M_p \left( M_p - M_{CT\perp}^{max} \right)}. \]

Transverse $M_{CT}$

\[ M_p = 305.3 \ \text{GeV} \]
\[ M_c = 275.7 \ \text{GeV} \]

Longitudinal $M_{CT}$

\[ M_p = 305.3 \ \text{GeV} \]
\[ M_c = 275.7 \ \text{GeV} \]

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**Generalized $M_{T2}$ method**

- Basic idea: test whether the two missing particles are the same
  - Neutrinos?
  - Multi-component dark matter?

**ISR invariance method**

**Ridge method**

**Gradient method**

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Barr, Gripaios, Lester 2009

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