MASS MEASUREMENTS AT THE LHC

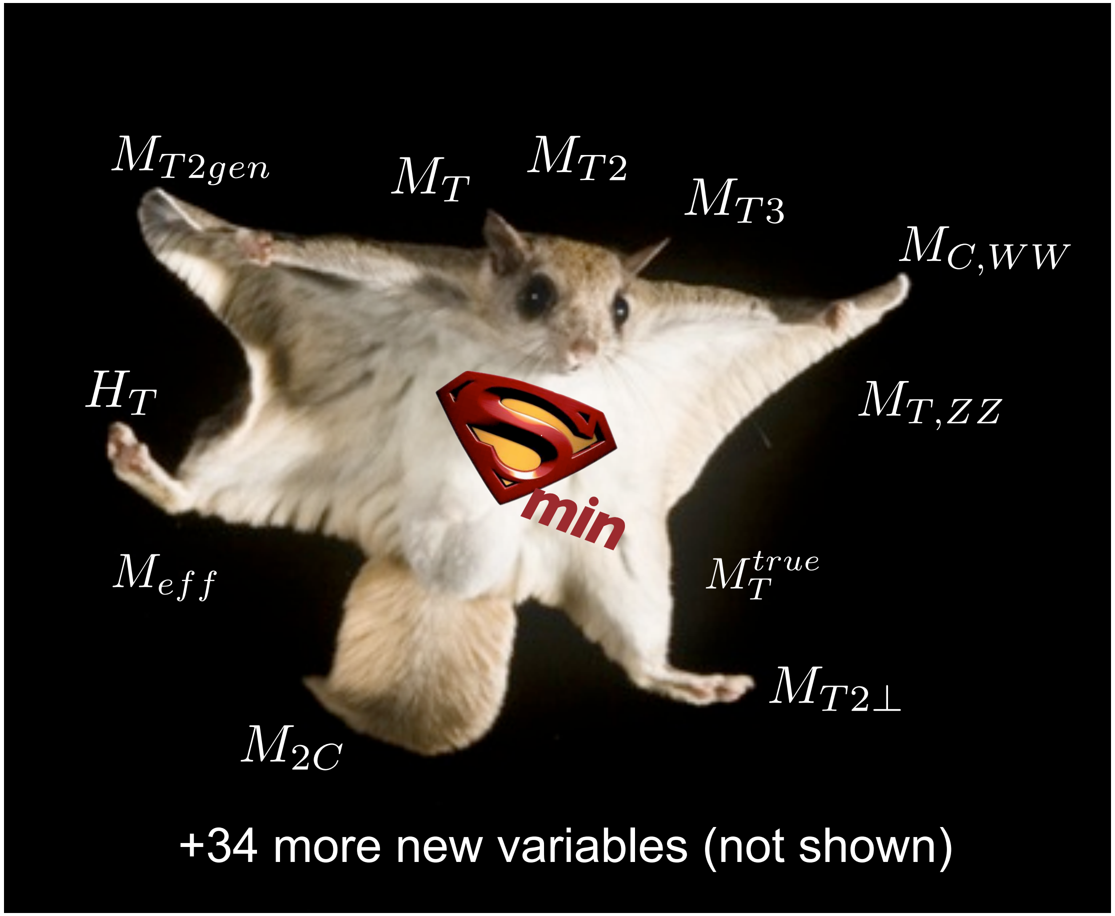
Konstantin Matchev

UNIVERSITY of FLORIDA

Work in progress with:
A. Barr, T. Khoo, P. Konar, K. Kong, C. Lester, M. Park
[Floxbridge collaboration]

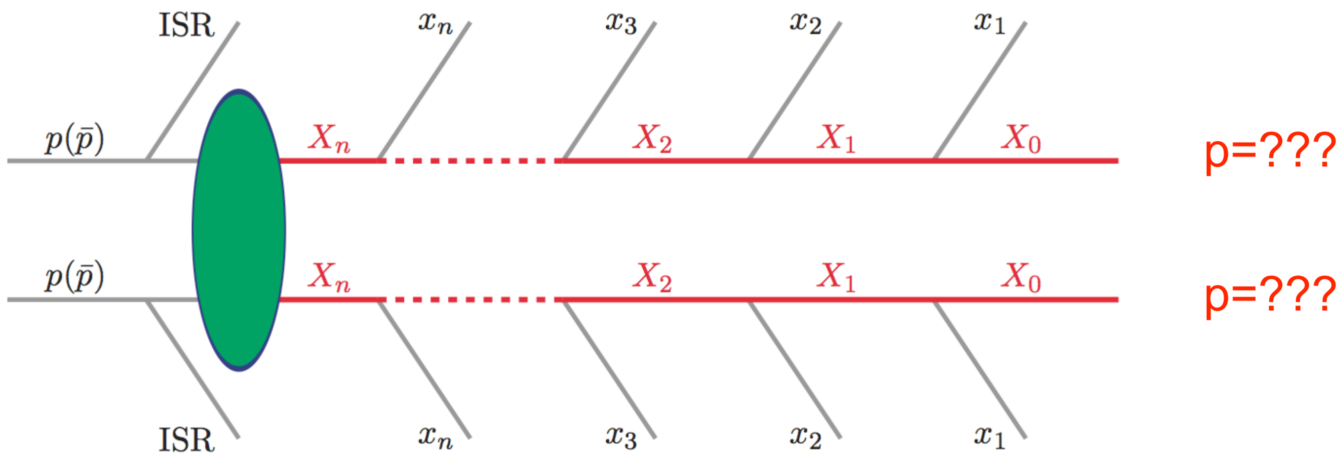
SUSY 2010, Bonn
August 24, 2010

M(other)-theory or M  -theory



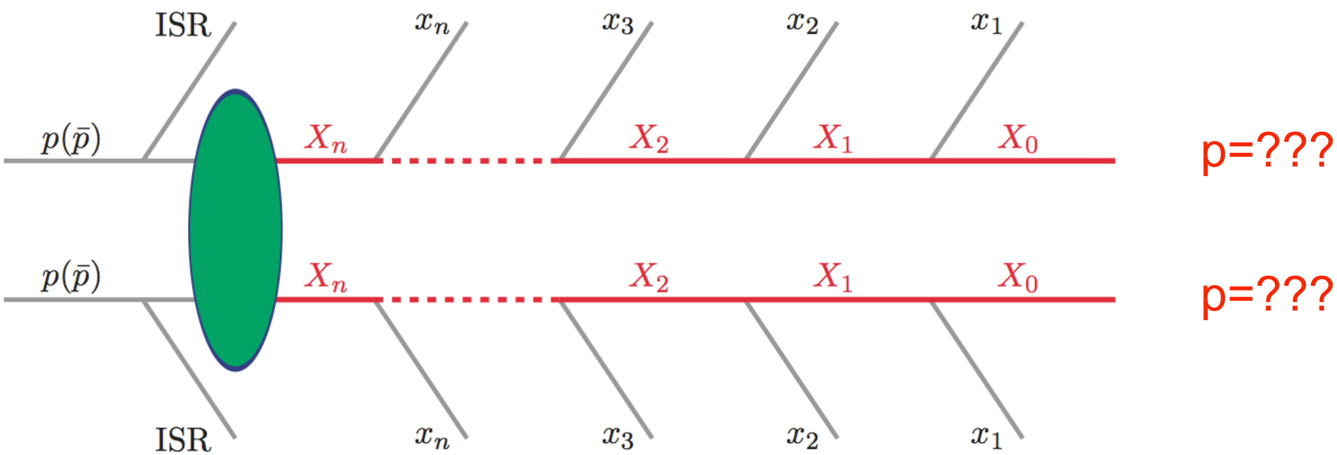
- W. Lamb (1955): "*The finder of a new elementary particle used to be rewarded by a Nobel Prize, but such a discovery now ought to be punished by a $10,000 fine*"

What to do about the LSP momenta?



- Nothing (no need to introduce the X_0 mass in the game)
  - invariant mass endpoint methods (need n>2 on either side)
  - invariant mass cusp method (requires an s-channel resonance)
  - M_{CT} methods (need only n>0 on each side)
- Use some prescription to fix them somehow
  - M_{T2} methods (need only n>0 on each side)
- Compute them exactly
  - "polynomial" methods (if n>2 on each side or if n>3 on one side)
- Some (hybrid) combination of the above

What to do about the LSP momenta?

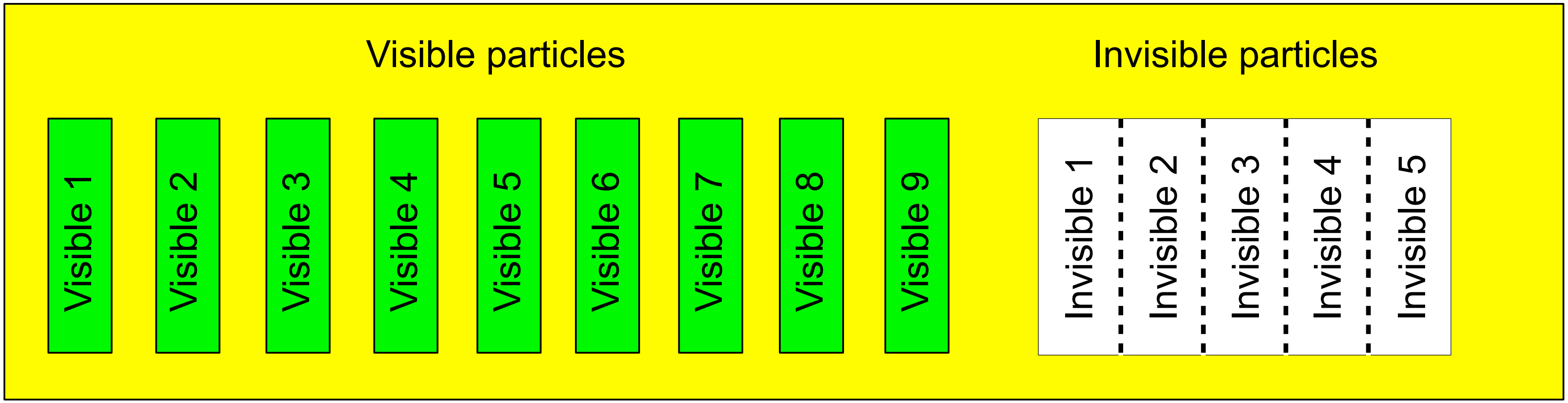


- Nothing (no need to introduce the X_0 mass in the game)
  - invariant mass endpoint methods (need n>2 on either side)
  - invariant mass cusp method (requires an s-channel resonance)
  - M_{CT} methods (need only n>0 on each side)
- Use some prescription to fix them somehow
  - M_{T2} methods (need only n>0 on each side)

- Compute them exactly
  - "polynomial" methods (if n>2 on each side or if n>3 on one side)
- Some (hybrid) combination of the above

Anatomy of an event

- Visible particles
  - known: number, masses, momenta
- Invisible particles
  - unknown: number, masses, momenta
  - known: total P_T



- Invariant mass variables are constructed by:
  - (optional) partitioning and lumping within each partition
  - (optional) transversifying w.r.t. the beam axis (NB! two different ways)
  - (optional) transversifying w.r.t. the upstream P_T
  - (mandatory) fixing invisible momenta by minimizing CME: $\boxed{\sqrt{s}_{min}}$

I. Basic definition of $\boxed{\sqrt{s}_{min}}$

Konar, Kong, KM 2008

- No partitioning; lump the visibles; separate the invisibles

Visible particles	Invisible particles
Visible 1, Visible 2, Visible 3, Visible 4, Visible 5, Visible 6, Visible 7, Visible 8, Visible 9	Invisible 1, Invisible 2, Invisible 3, Invisible 4, Invisible 5

$$(E, \vec{P}_T, P_z) \equiv P^\mu = \sum_{i=1}^{N_{vis}} P_i^\mu$$

Q_1^μ .... Q_5^μ

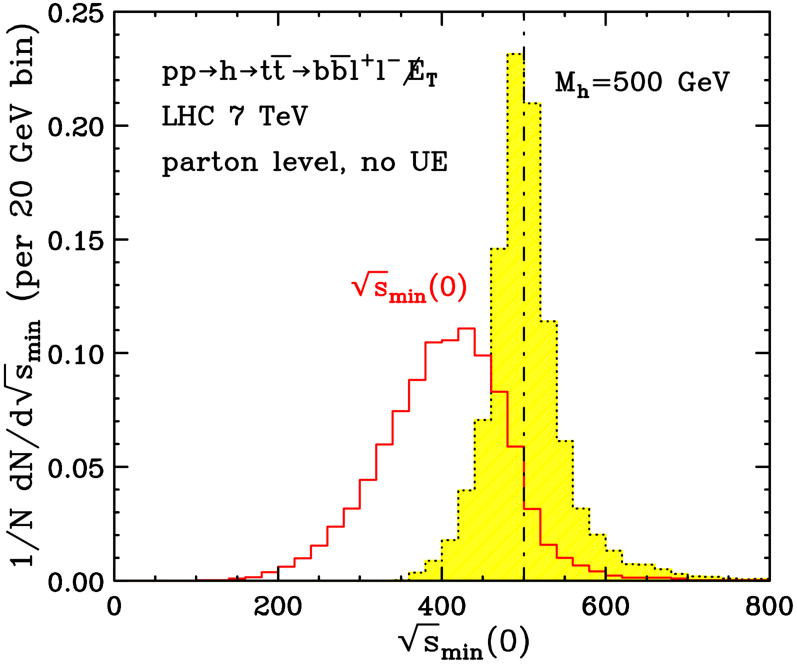
$\tilde{M}_1$ .... $\tilde{M}_5$

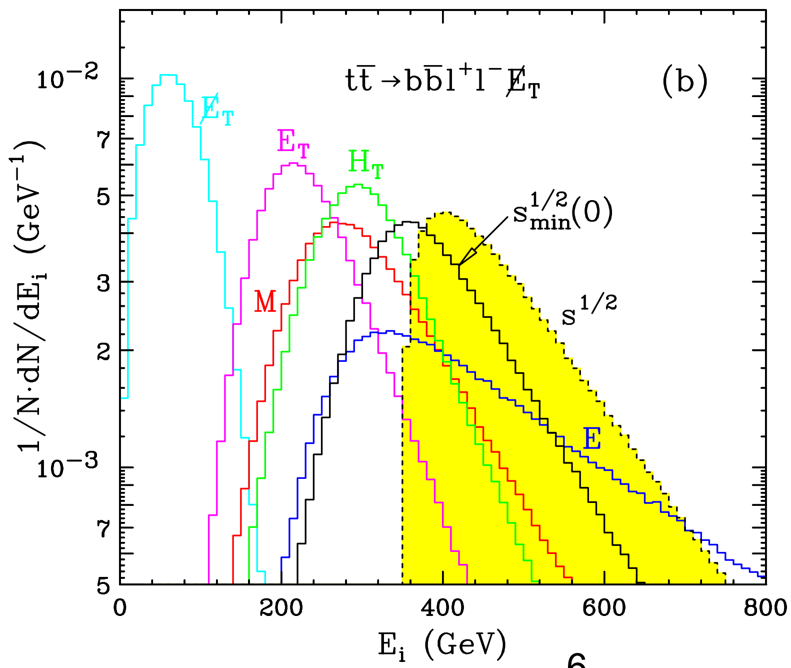
- What is the minimum possible value of the CM energy?

$$\sqrt{s}_{min}(M_{inv}) = \sqrt{E^2 - P_z^2} + \sqrt{M_{inv}^2 + \not{P}_T^2}; \qquad M_{inv} = \sum_{i=1}^{N_{inv}} \tilde{M}_i$$

Applications of $\boxed{\sqrt{s}_{min}}$

- Single semi-invisibly decaying particle
  - SM Higgs to tt-bar
  - endpoint at the parent mass
- A pair of semi-invisibly decaying particles
  - direct tt-bar production
  - peak at the total parent mass

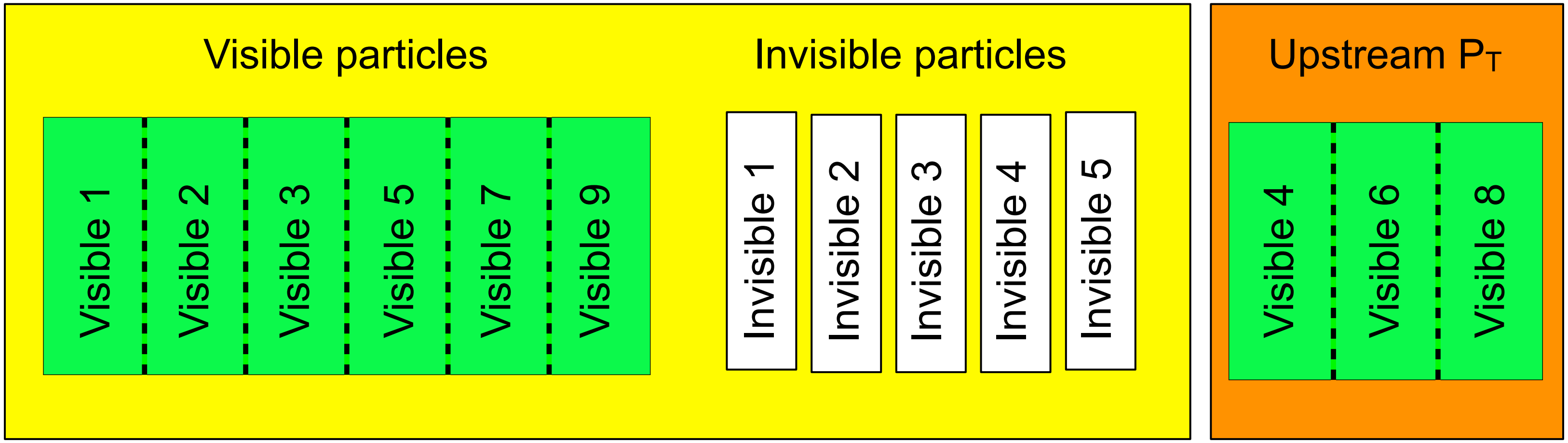




Subsystem $\boxed{\sqrt{s}_{min}}$

Konar, Kong, KM, Park 2010

- Objection: “you should not include objects from ISR and UE”
  - Solution: very good, then don’t:



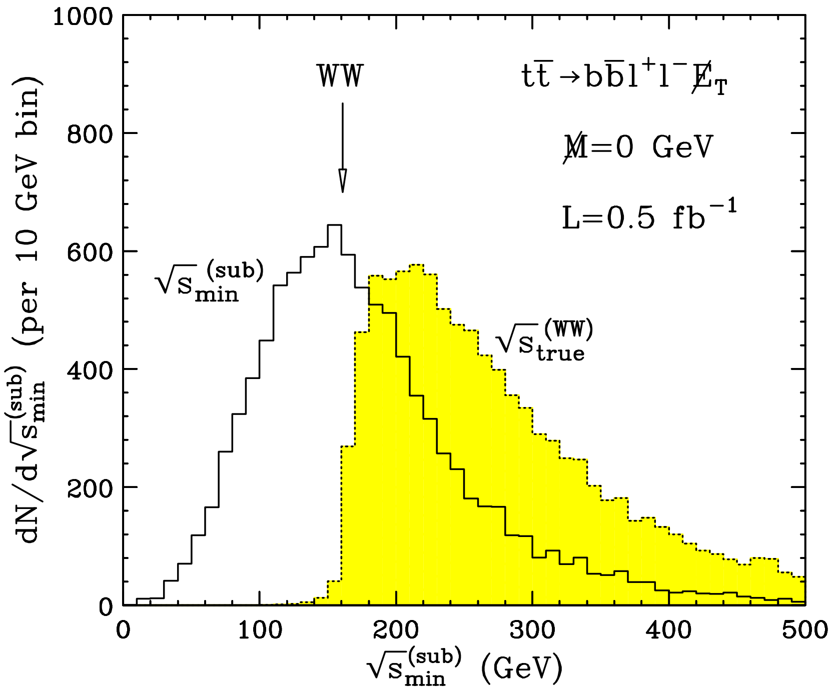
- Repeat the constrained minimization and find:

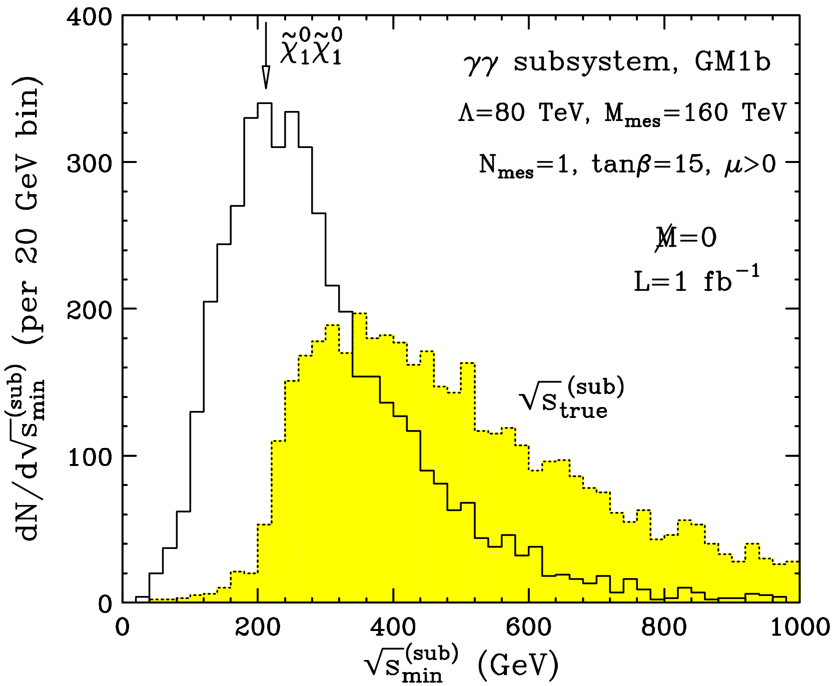
$$\boxed{\sqrt{s}_{min}^{(sub)}(M_{inv}) = \sqrt{\left(\sqrt{E^2 - P_z^2} + \sqrt{M_{inv}^2 + \not{P}_T^2}\right)^2 - P_{T(up)}^2}}$$

Applications of subsystem $\boxed{\sqrt{s}_{min}}$

Konar, Kong, KM, Park 2010

- tt-bar events
  - identify the WW threshold from the 2 lepton subsystem
- GMSB SUSY events
  - identify the N_1N_1 threshold from the 2 photon subsystem

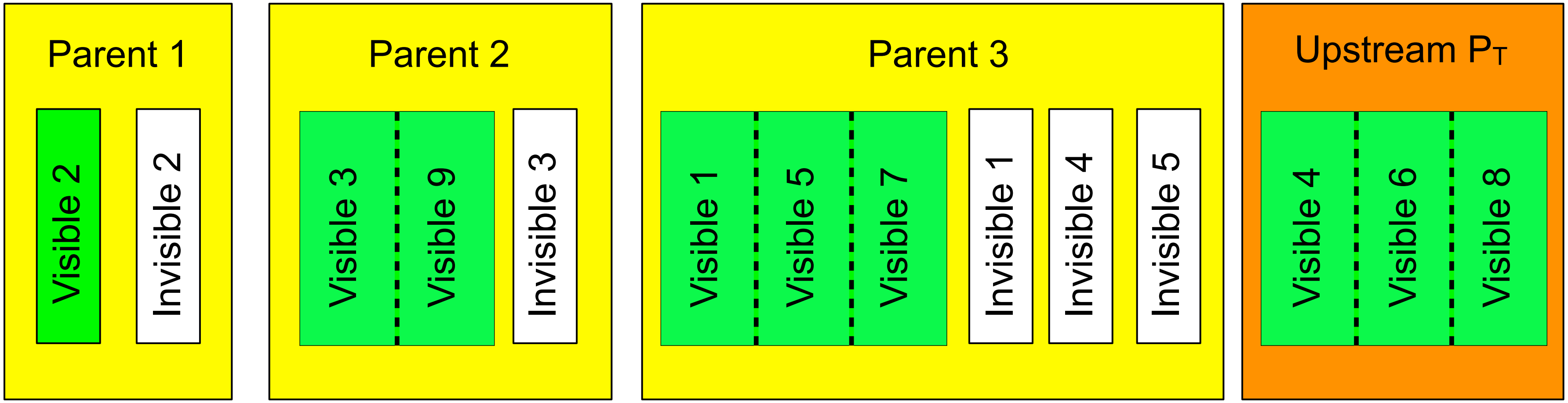




II. Partitioning and lumping together

Barr,Khoo,Konar,Kong,Lester,KM,Park 2010

- Assuming N parents of equal mass, partition as



$$\boxed{M_{(1)}} = \boxed{M_{(2)}} = \boxed{M_{(3)}}$$

- Now the invisible momenta are chosen to minimize the mass of any parent:

$$\boxed{M_N(\tilde{M}_{(1)}, \tilde{M}_{(2)}, \tilde{M}_{(3)}) \equiv \min_{M_{(1)} = M_{(2)} = M_{(3)}} \{M_{(a)}\}}$$

III. Transversification (once)

Barr,Khoo,Konar,Kong,Lester,KM,Park 2010

- So far S_{min} and M_N are genuine (1+3)-dim. quantities
- One often uses (1+2)-dim. “transverse” quantities
  - the transverse projection is not unique! there are two (inequivalent) ways to do it:
    - Type “T”: mass preserving
    - Type “t”: velocity preserving
  - the operations of transversification and partitioning do not commute! Depending on the particular order, there are different types of variables, e.g.:
    - M_{2T} (the original Cambridge variable M_{T2})
    - M_{2t}
    - M_{T2}
    - M_{t2}

Transversification alternatives

$$P^\mu \equiv (E, \vec{P}_T, P_z) \qquad P^2 = M^2$$

T

t

$$\vec{p}_T = \vec{P}_T$$

$$e_T = \sqrt{M^2 + \vec{P}_T^2}$$

$$p^2 = M^2 = P^2$$

- preserves the mass
- no P_z dependence
- test mass remains

$$\boxed{\tilde{e}_t = \sqrt{\tilde{M}^2 + Q_T^2}}$$

$$\vec{p}_t = \vec{P}_T$$

$$e_t = E sin\theta = E \frac{P_T}{\sqrt{P_T^2 + P_z^2}}$$

$$v^2 = 1 - \frac{M^2}{E^2} = V^2$$

- preserves the velocity
- test mass disappears during minimization (Q_z->infinity)

$$\boxed{\tilde{e}_t = Q_T \sqrt{1 + \frac{\tilde{M}^2}{Q_T^2 + Q_z^2}}}$$

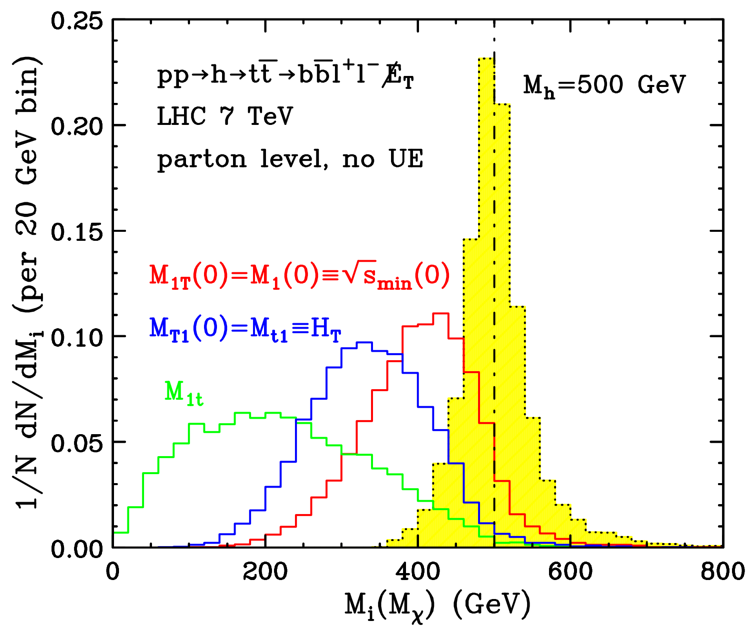
The recipe

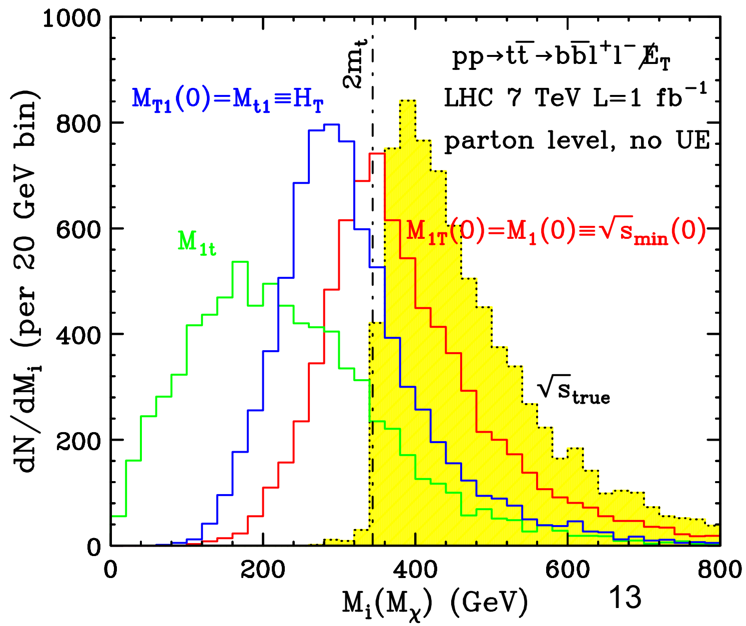
- Partition the observed particles into N parent sets plus a separate set for Upstream objects
- Do one then the other
  - Lump the energies and momenta of the visible particles within each set
  - Transversify all energies and momenta
- Fix the unknown momenta of the invisible particles by minimizing the largest parent transverse mass
- Record the minimum value of the largest parent mass
- For any value of N, there are 4 different cases:
  - M_{NT}, M_{Nt}, M_{TN}, M_{tN}

Mathematical identities for N=1

Barr,Khoo,Konar,Kong,Lester,KM,Park 2010

- $M_{1T}=M_1=\boxed{\sqrt{s}_{min}}$ for any invisible test mass
  - the "T" transverse mass is a (1+3)-dim. quantity!
- $H_T=M_{t1}$ (neither depends on an invisible test mass)
  - reveals the physical meaning of H_T in the $\boxed{\sqrt{s}_{min}}$ sense
- $H_T=M_{T1}(0)$, but not for general invisible test mass

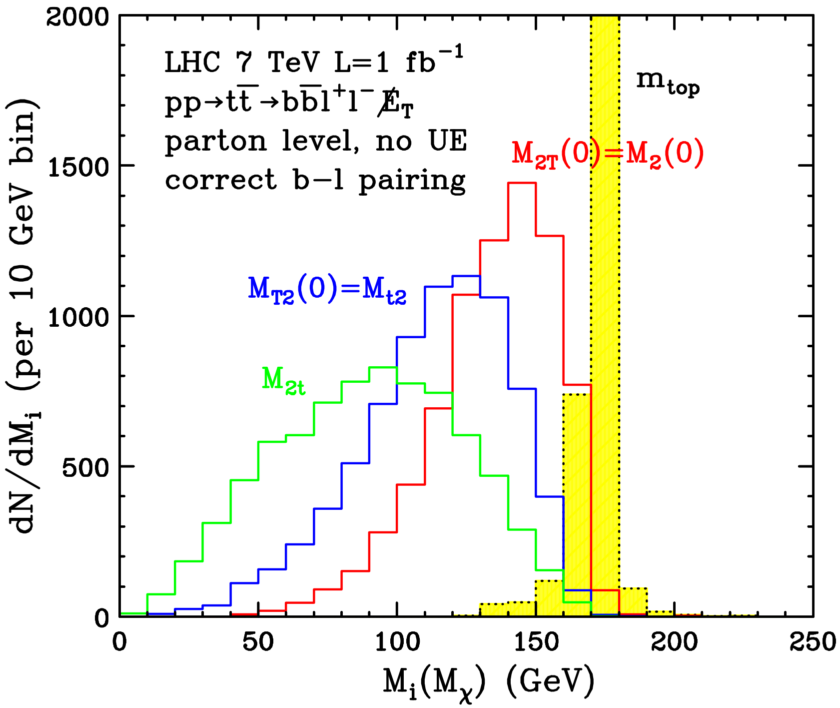


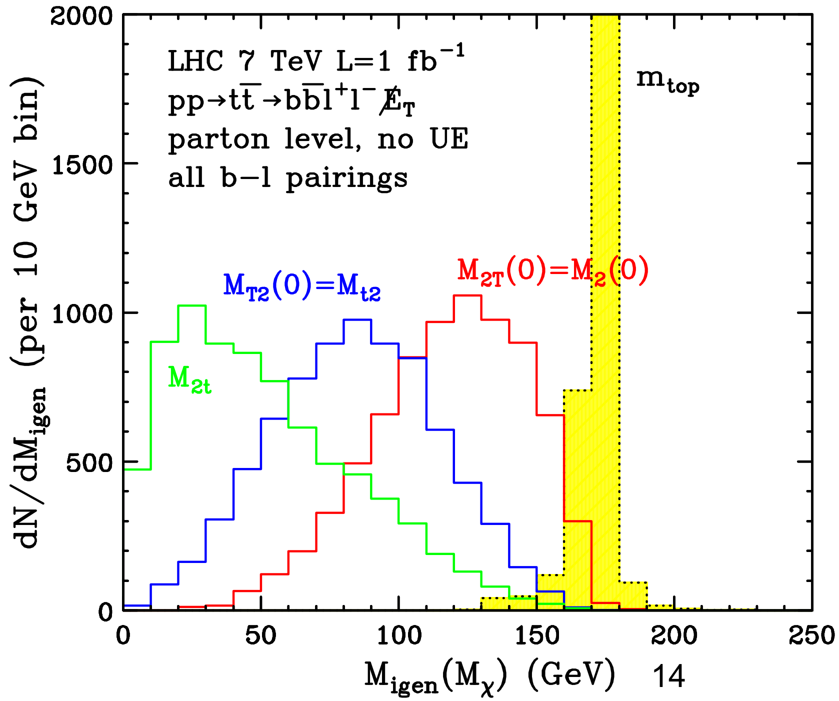


Mathematical identities for N=2

Barr,Khoo,Konar,Kong,Lester,KM,Park 2010

- $M_{2T}=M_2$ for any invisible test mass
  - the "2T" type Cambridge variable is a (1+3)-dim. quantity!
- $M_{T2}(0)=M_{t2}$ for massless visibles
- Notice the different shapes near the upper endpoint

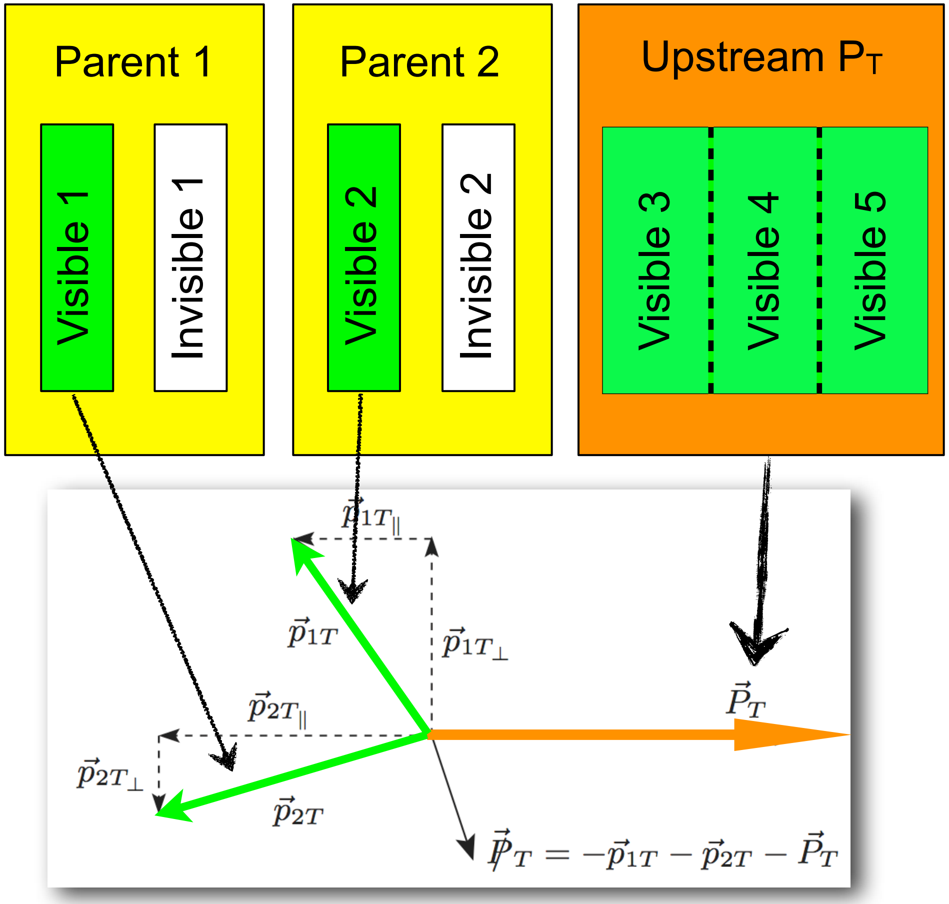




Transversification (twice)

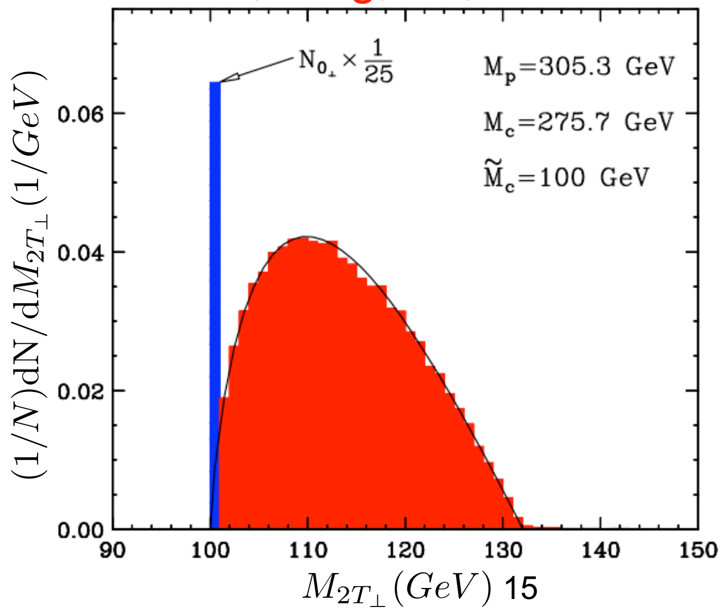
KM,Park 2009

- Having projected on the transverse plane, one can additionally project on the direction of Upstream P_T:

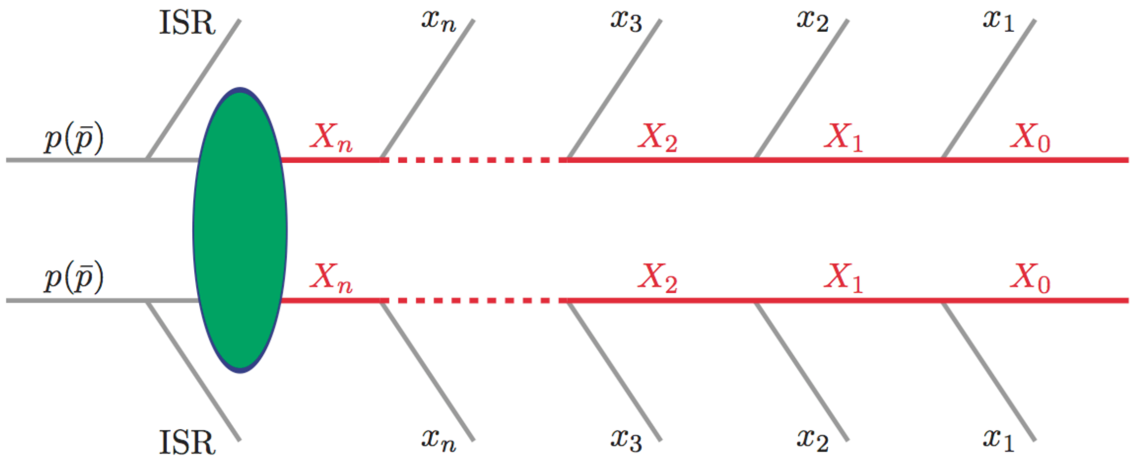


- The endpoints of "perp" distributions are stable against P_T variations

Konar,Kong,KM,Park 2009



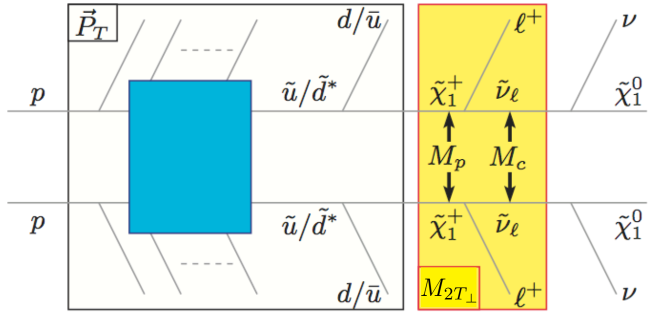
The two interesting limiting cases



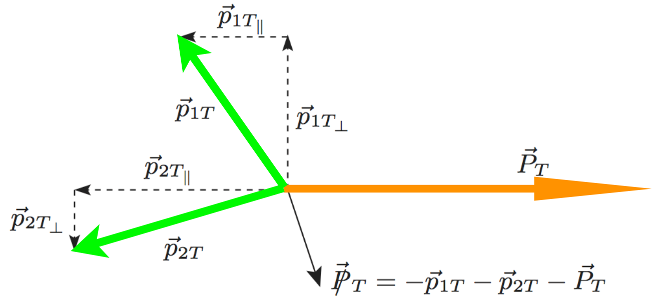
- Very simple events (n=0 and n=1)
  - good news: no combinatorial problem
  - bad news: insufficient information, difficult to extract dark matter properties (mass, spin etc.)
  - generally more SM background
- Very complex events (n=infinity [4])
  - multijet events with 10-15 jets per event
  - very severe combinatorial problem

1-Dim M_{2T} method

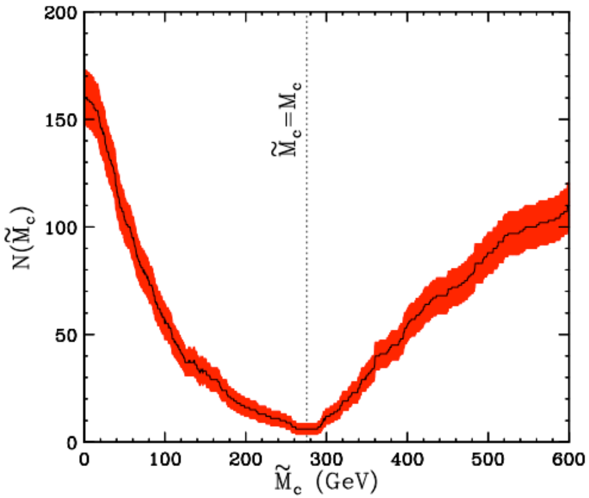
Konar,Kong,KM,Park 2009



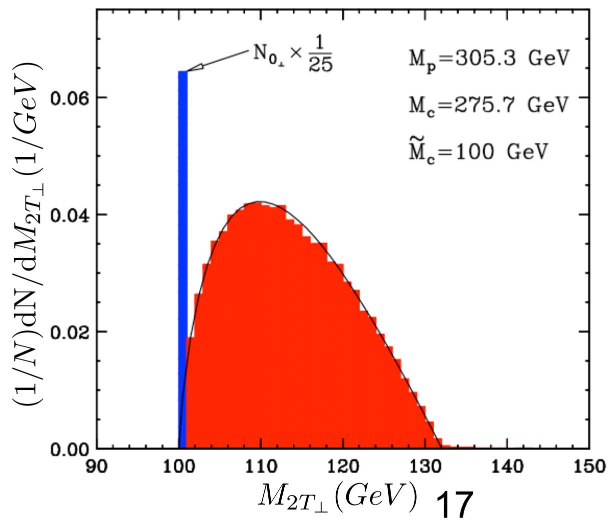
- Basic idea: vary the LSP test mass and count how many events have M_{2T} above the doubly transverse M_{2T} endpoint



$$N(\tilde{M}_c) = \sum_{all\ events} H(M_{2T}(\tilde{M}_c) - M_{2T\perp}^{max}(\tilde{M}_c))$$



Transverse M_{2T}

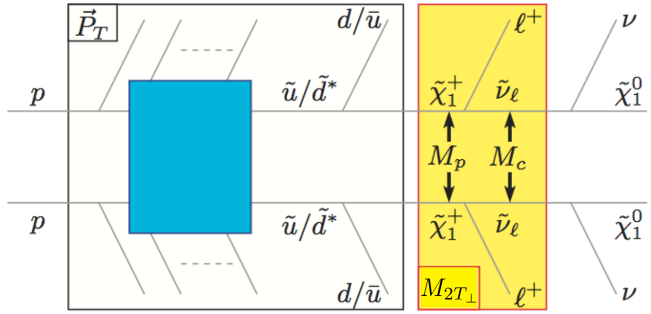


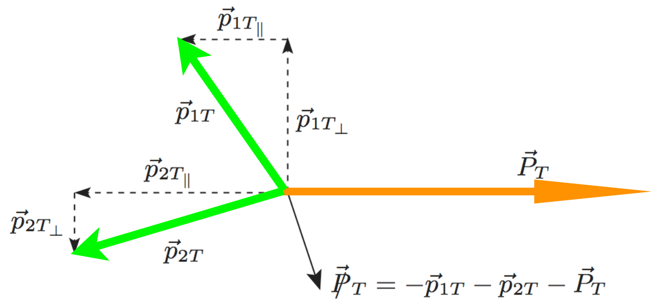
1-Dim M_{2T} method

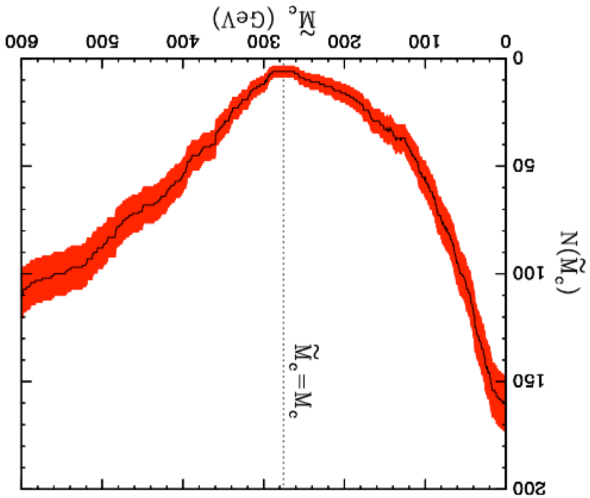
Konar, Kong, KM, Park 2009

- Basic idea: vary the LSP test mass and count how many events have M_{2T} above the doubly transverse M_{2T} endpoint

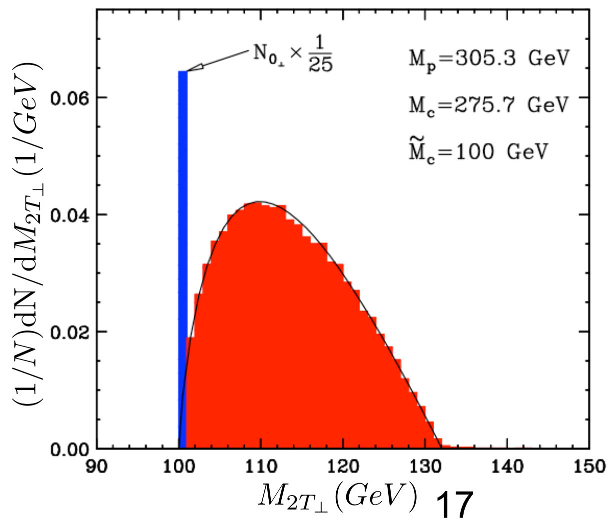
$$\boxed{N(\tilde{M}_c) = \sum_{all\ events} H(M_{2T}(\tilde{M}_c) - M_{2T\perp}^{max}(\tilde{M}_c))}$$







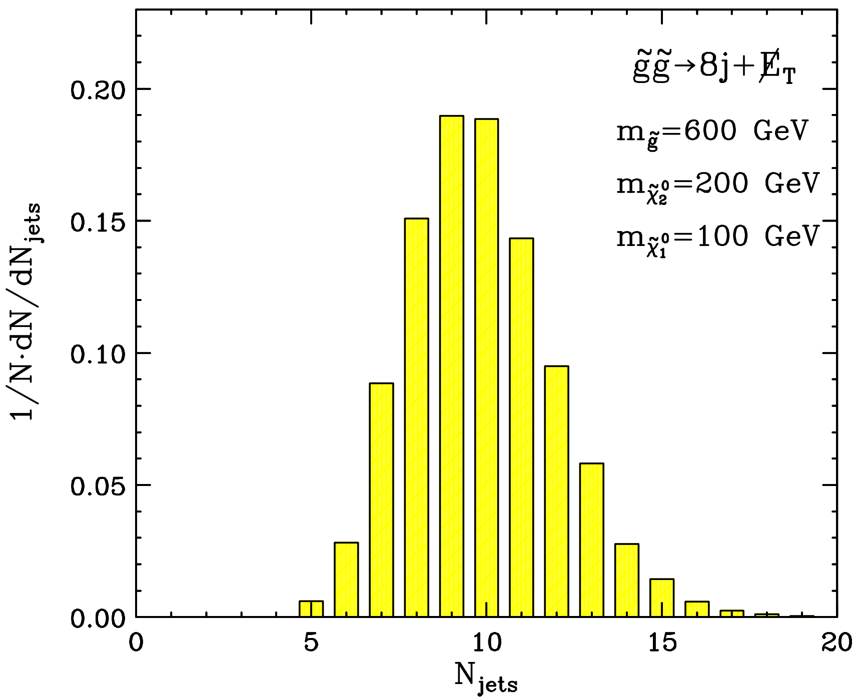
Transverse M_{2T}



The other extreme: very complicated events

- Let's now do n=infinity
- Imagine something very complicated like
  - gluino pair production
  - gluino decays to N_2 and 2 jets
  - N_2 decays to N_1 and 2 jets
- Typical jet multiplicity is about 10
- Exclusive reconstruction seems hopeless

$$\tilde{g} \to jj\tilde{\chi}_2^0 \to jjjj\tilde{\chi}_1^0$$

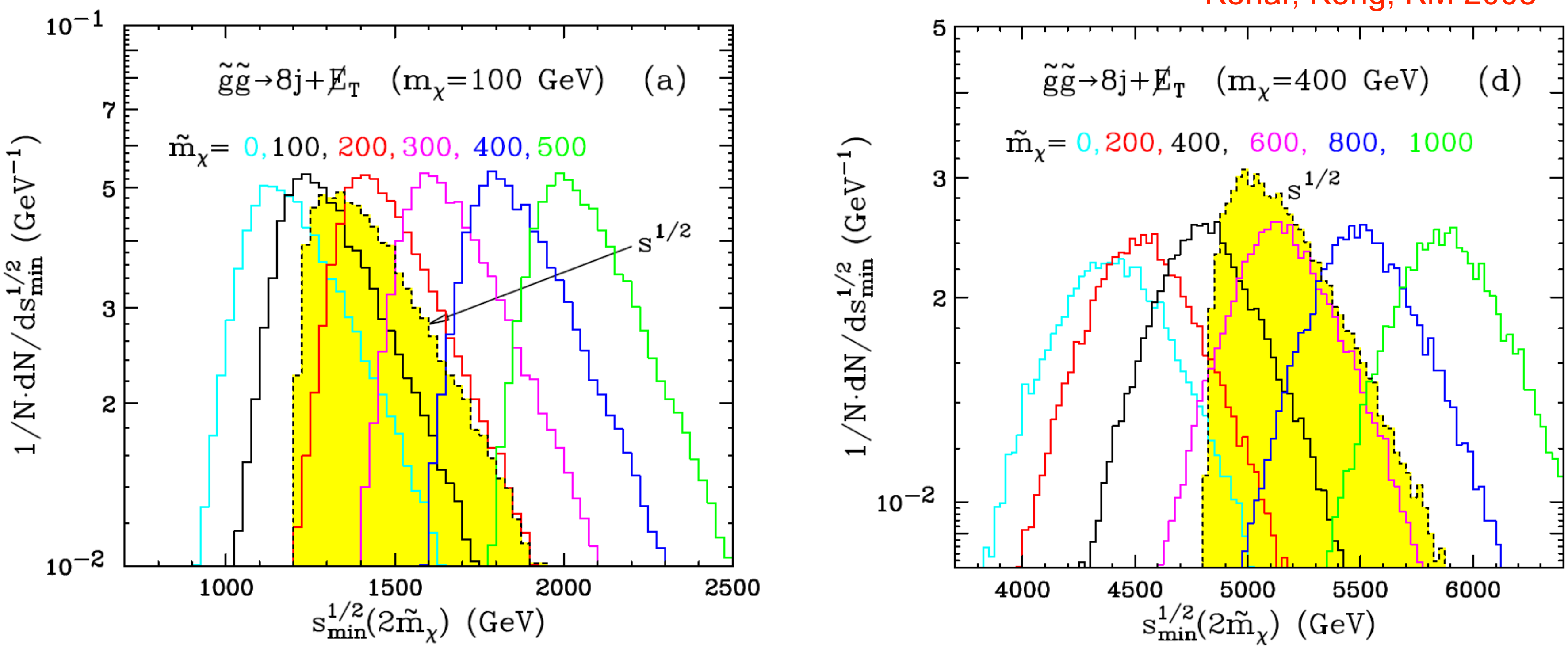


Application of S_{min} to complex events

- One can measure SUSY masses in terms of the LSP mass. The peak of S_{min} marks (the sum of) the masses of all particles produced in the hard scattering

$$\boxed{\left(\hat{s}^{1/2}\right)_{thr} \approx \left(\hat{s}^{1/2}_{min}(2m_\chi)\right)_{peak}}$$

Konar, Kong, KM 2008



Summary

- By partitioning, lumping, transversifying and minimizing, one can obtain a whole series of invariant mass variables

$$N=1$$
$$M_1, M_{T1}, M_{t1}, M_{1T}, M_{1t}$$
$$M_{1T\perp}, M_{1t\perp}, M_{T1\perp}, M_{t1\perp}, M_{T\perp 1}, M_{t\perp 1}$$
$$M_{1T\parallel}, M_{1t\parallel}, M_{T1\parallel}, M_{t1\parallel}, M_{T\parallel 1}, M_{t\parallel 1}$$

$$N=2$$
$$M_2, M_{T2}, M_{t2}, M_{2T}, M_{2t}$$
$$M_{2T\perp}, M_{2t\perp}, M_{T2\perp}, M_{t2\perp}, M_{T\perp 2}, M_{t\perp 2}$$
$$M_{2T\parallel}, M_{2t\parallel}, M_{T2\parallel}, M_{t2\parallel}, M_{T\parallel 2}, M_{t\parallel 2}$$

- Which are most suitable, depends on the case at hand
- Some of these are old friends in disguise (H_T, M_{T2}, M_T, ...)
- Watch for the exact meaning of the “transverse” index
- It helps to think of these variables as resulting from minimizing the total CM energy (LHC energy is “expensive”)
- Homework: figure out the meaning of the remaining variables on the second slide.

BACKUPS

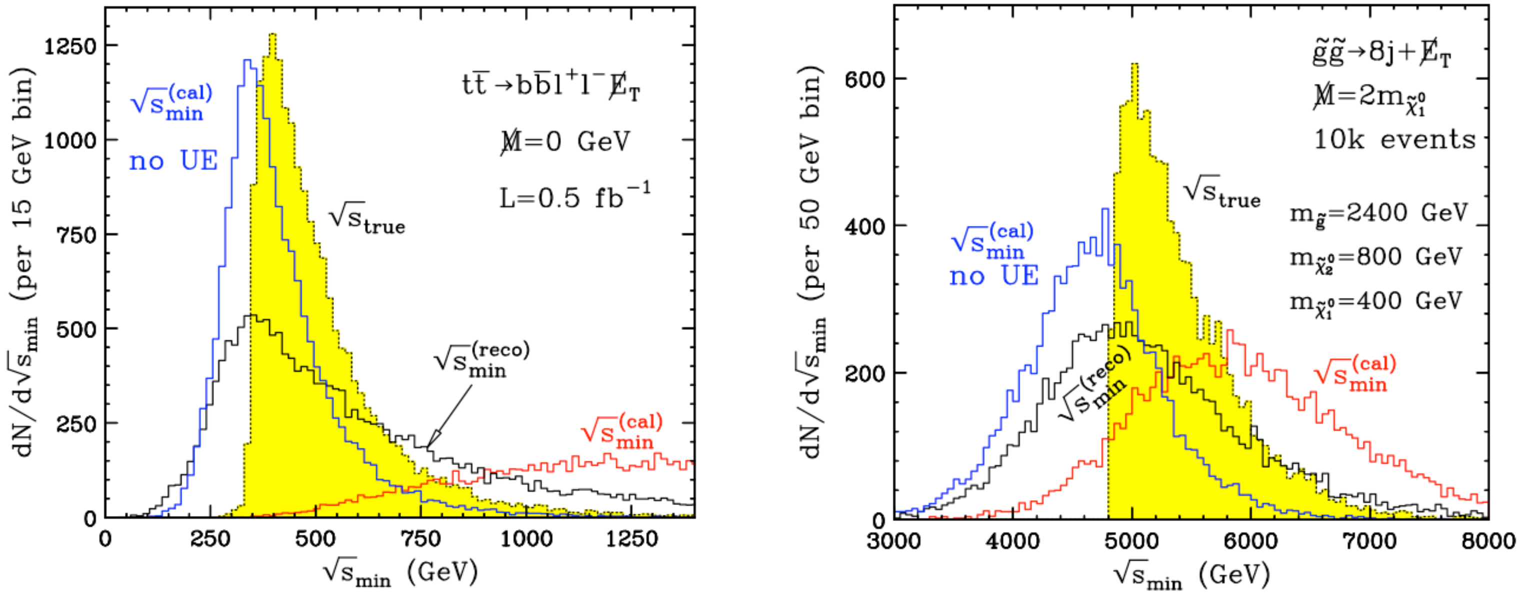
Effect of ISR and UE

- ISR and UE destroy all these nice correlations…
  - Calculate ISR from first principles and undo the effect
    - What about UE?

Papaefsthatiou, Webber 2009

  - Use only reconstructed objects (e.g. jets instead of cal towers)
    - MHT in place of MET

Konar, Kong, KM, Park 2010

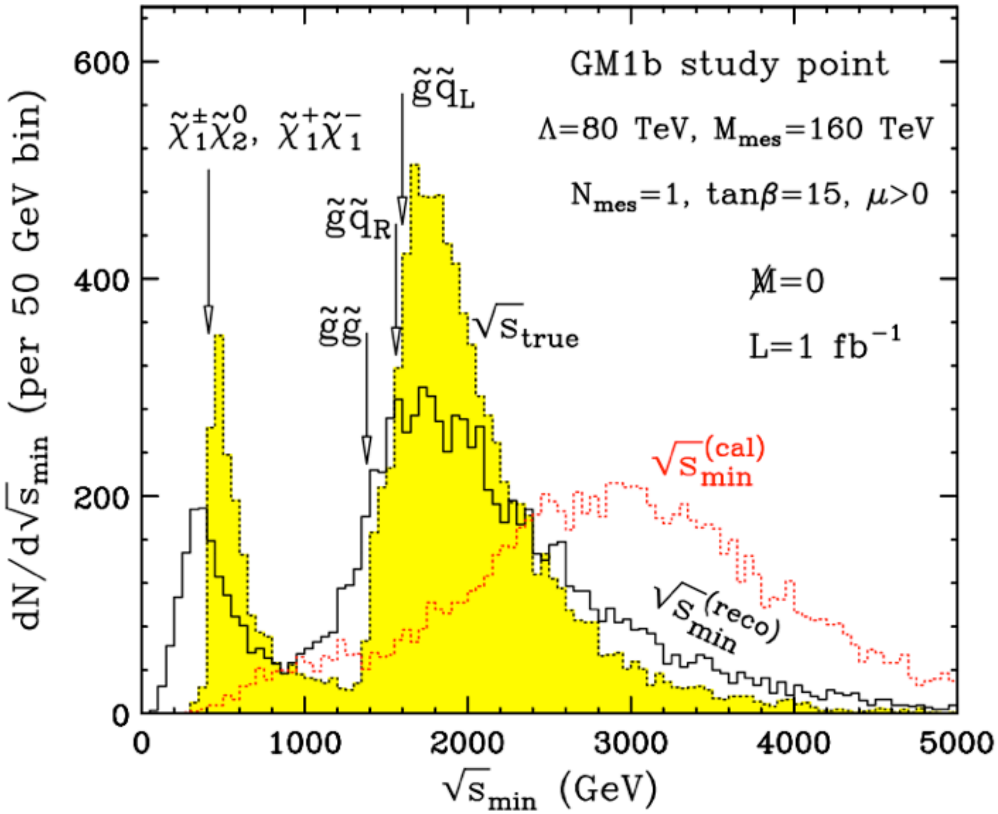


Inclusive SUSY production

- The peaks in S_{min} mark the thresholds for the individual production subprocesses

Konar, Kong, KM, Park 2010

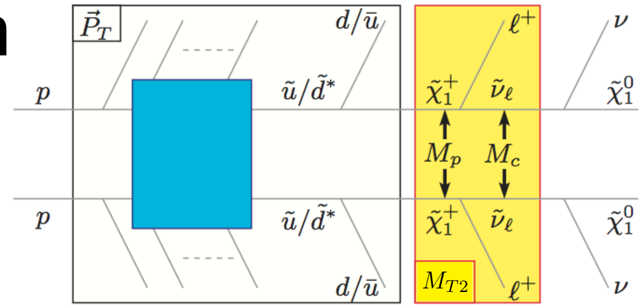
- Example: GMSB point GM1b
  - Lambda=80 TeV, M=160 TeV, N=1, tanb=15, mu>0



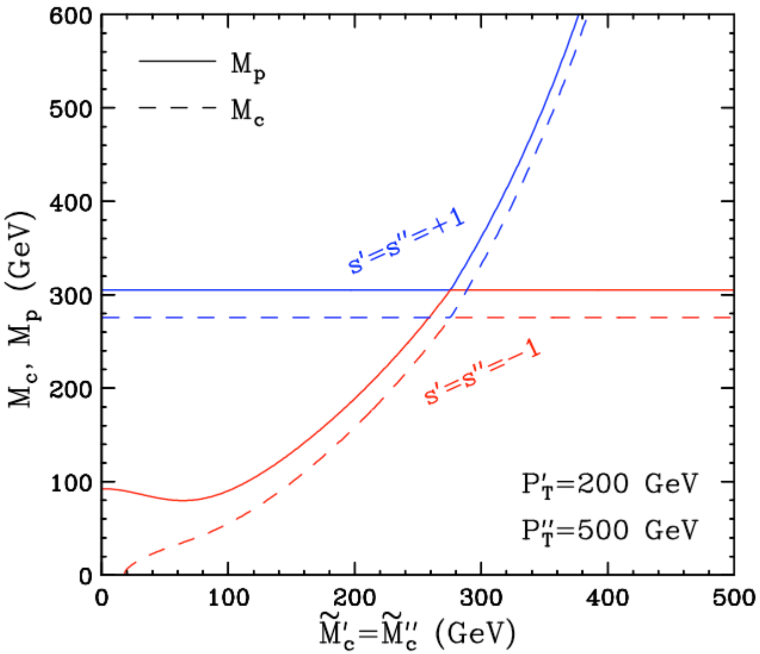
Another M_{T2} method

KM,Moortgat,Pape,Park 2009

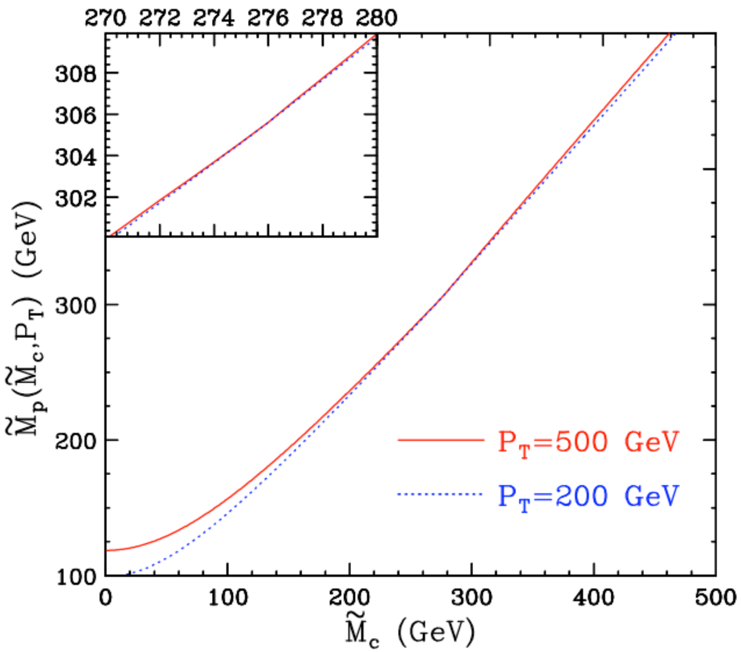
- Basic idea: study M_{T2} distribution
  - of the two leptons only
  - for any two test masses
  - for two different values of P_T



New (our) method



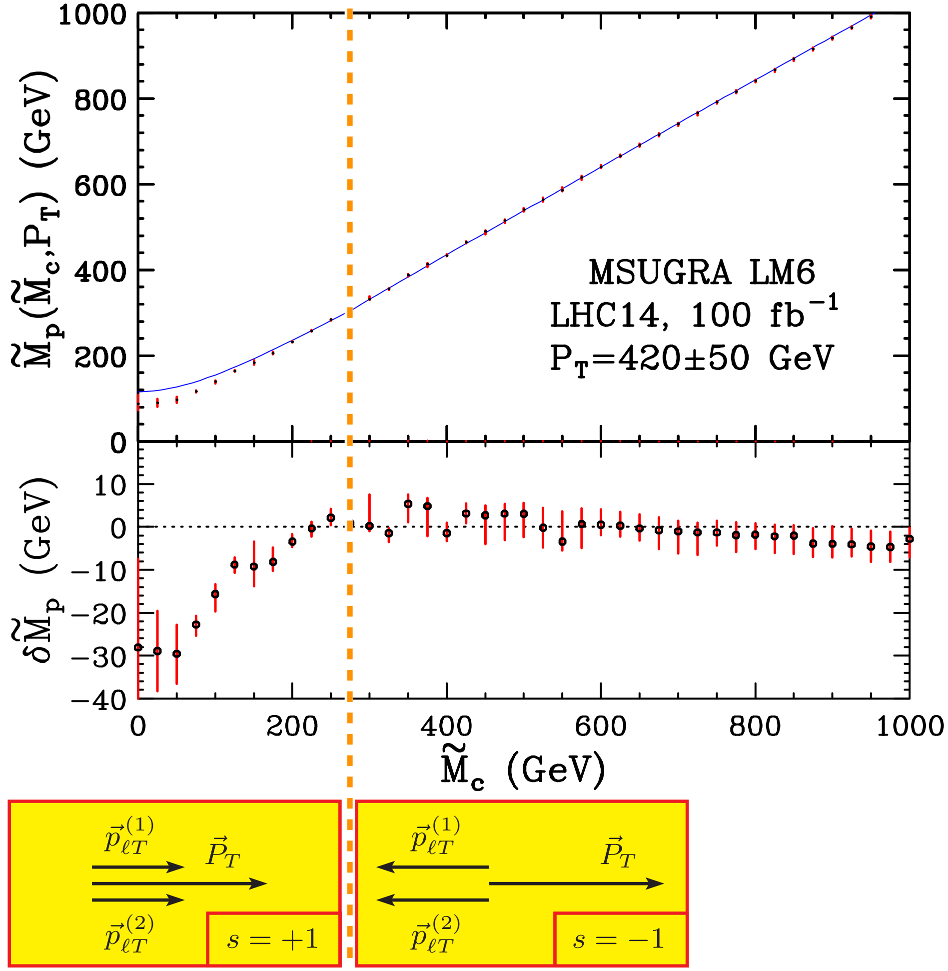
Old (kink) method



Advantage of the new method

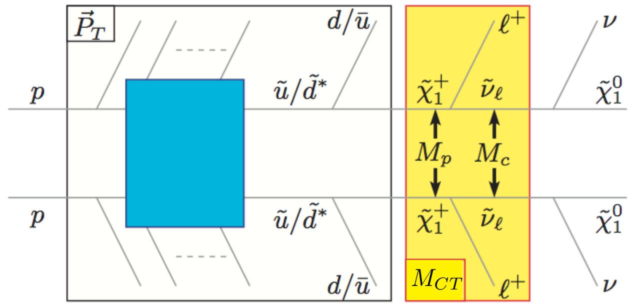
KM,Moortgat,Pape,Park 2009

- The left branch endpoints are systematically underestimated
- We can pick both measurements to be on the right branch, which is measured much better



1-Dim M_{CT} method

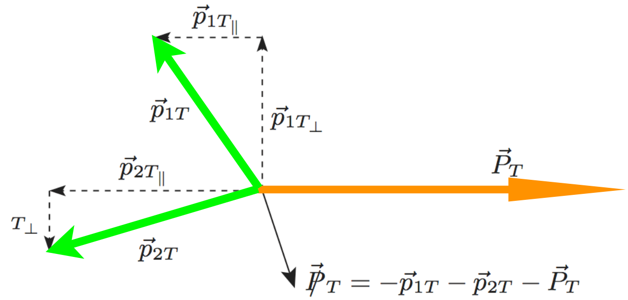
KM,Park 2009



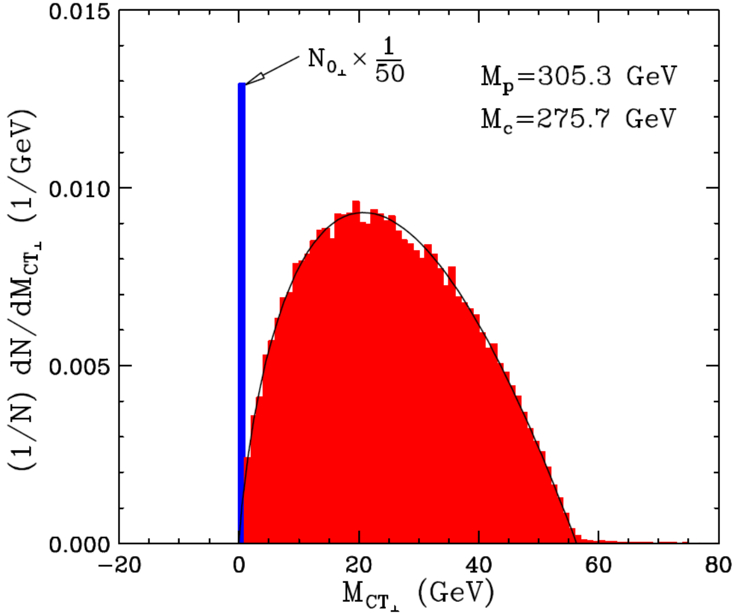
$$M_{CT_{||}} = \sqrt{m_1^2 + m_2^2 + 2\left(e_{1T_{||}} e_{2T_{||}} + \vec{p}_{1T_{||}} \cdot \vec{p}_{2T_{||}}\right)}$$

$$M_{CT_{\perp}} = \sqrt{m_1^2 + m_2^2 + 2\left(e_{1T_{\perp}} e_{2T_{\perp}} + \vec{p}_{1T_{\perp}} \cdot \vec{p}_{2T_{\perp}}\right)}$$

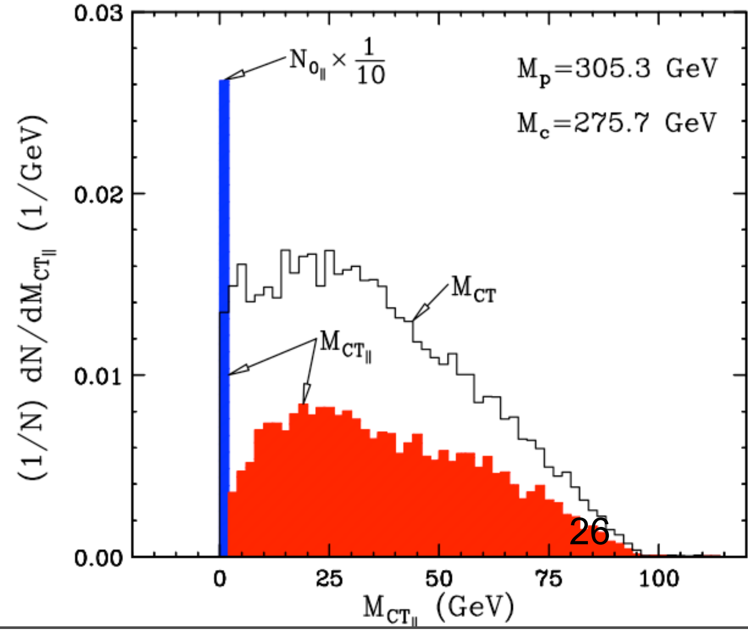
$$e_{iT_{\|}} \equiv \sqrt{m_i^2 + |\vec{p}_{iT_{\|}}|^2}, \quad e_{iT_{\perp}} \equiv \sqrt{m_i^2 + |\vec{p}_{iT_{\perp}}|^2}.$$



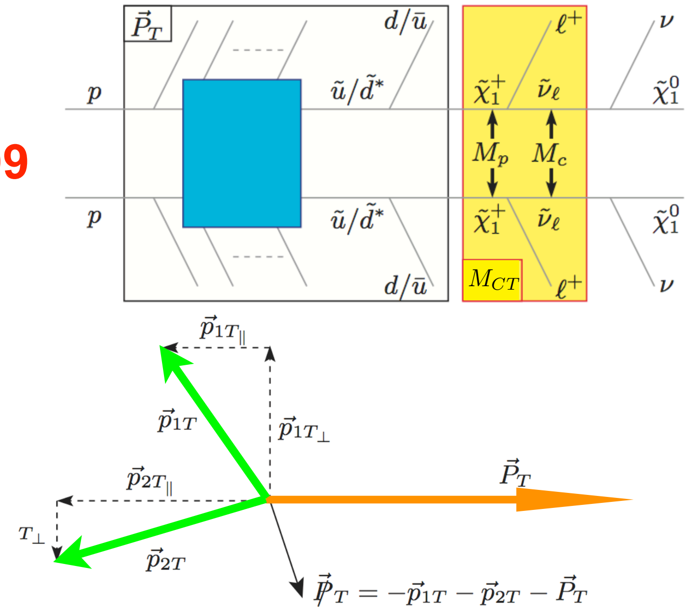
Transverse M_{CT}



Longitudinal M_{CT}



1-Dim M_{CT} method

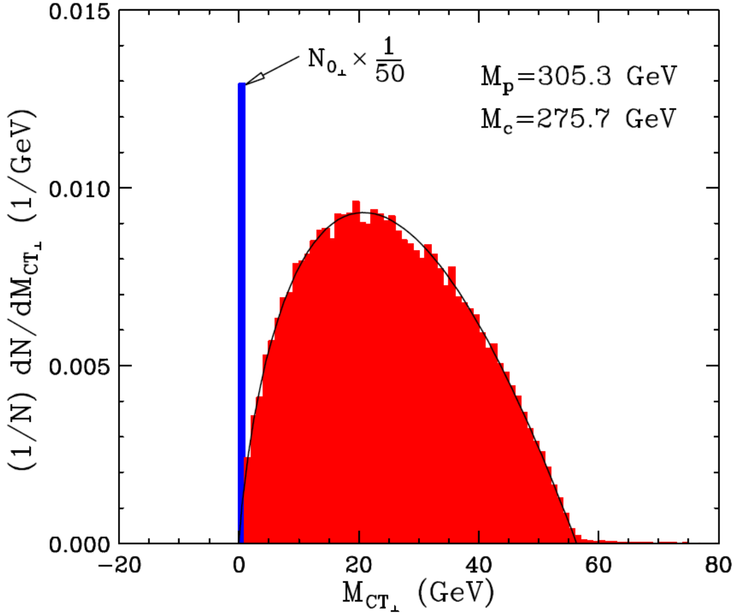


KM,Park 2009

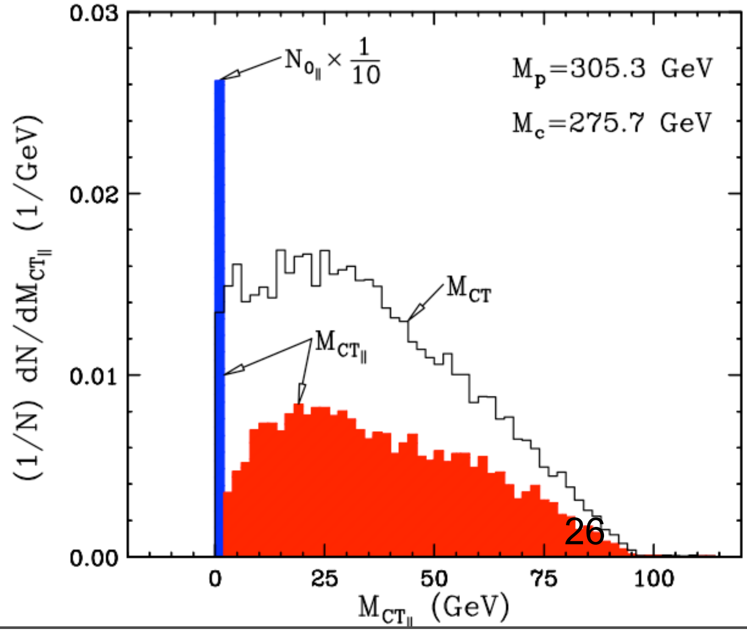
$$M_p = \frac{P_T\, M_{CT_\parallel}^{max}(P_T)\, M_{CT_\perp}^{max}}{(M_{CT_\parallel}^{max}(P_T))^2 - (M_{CT_\perp}^{max})^2},$$

$$M_c = \sqrt{M_p\left(M_p - M_{CT_\perp}^{max}\right)}.$$

Transverse M_{CT}



Longitudinal M_{CT}

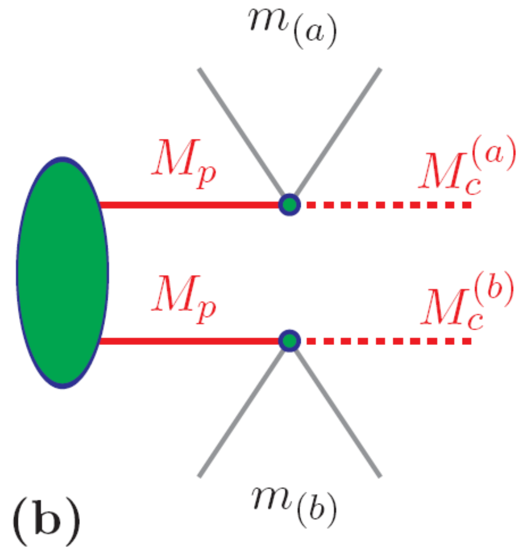


Generalized M_{T2} method

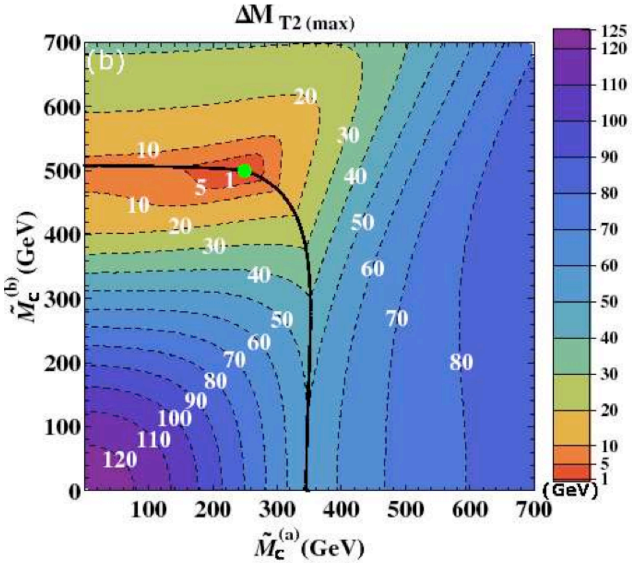
Barr, Gripaios, Lester 2009

Konar,Kong,KM,Park 2009

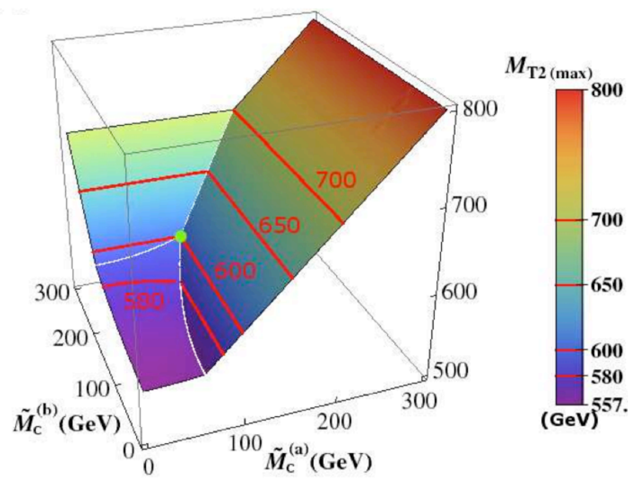
- Basic idea: test whether the two missing particles are the same
  - Neutrinos?
  - Multi-component dark matter?



ISR invariance method



Ridge method



Gradient method