Gauge Theories as Curved Spacetimes

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Gauge/Gravity Duality

- A remarkable connection, conjectured over a dozen years ago, relates a quantum field theory in d flat dimensions to a gravitational theory in d+1 dimensions

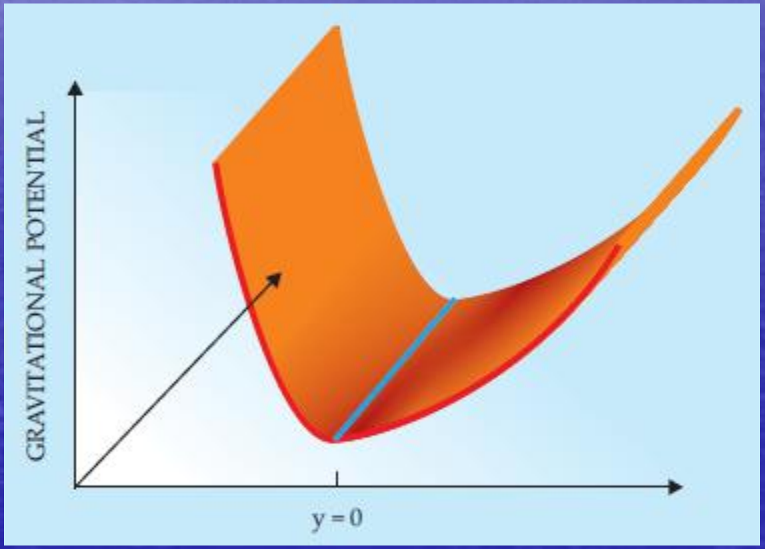
$$S = \frac{1}{16\pi G}\int d^d x dy \sqrt{-g}(R + \ldots)$$

which describes a dynamic space-time fluctuating around a special curved background

$$ds^2 = e^{2A(y)}\left(-dx_0^2 + dx_1^2 + \cdots + dx_{d-1}^2\right) + dy^2$$

- For an introduction, see the January 2009 Physics Today article. IK, J. Maladacena

- The gravitational energy of a massive particle is proportional to $e^{A(y)}$. It rises exponentially at large y, which is the `extra dimension.'

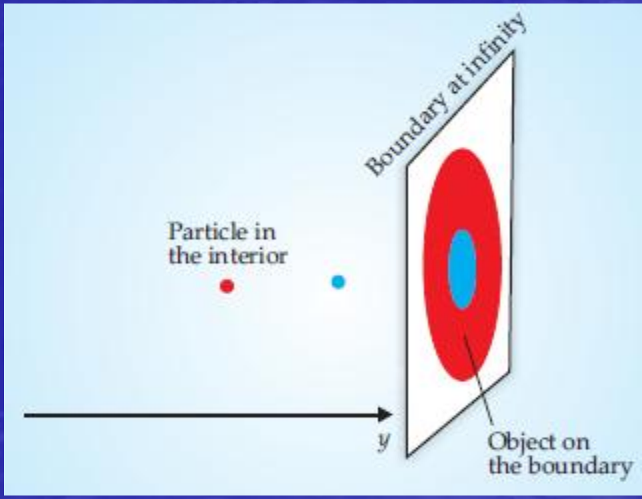


- Obviously, a low energy quantum particle behaves is if it were moving in the d dimensions near y=0.

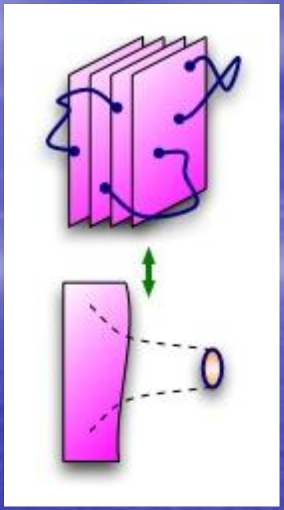
- It is far less obvious that even motion along the extra dimension y may be reproduced by a d-dimensional QFT.
- A particular background with enhanced symmetry is A(y)=y. This is the Anti-de Sitter space of constant negative curvature.
- The symmetry under $x_\mu \to \lambda x_\mu,\ y \to y - \ln\lambda$ corresponds to the scale invariance of the dual QFT. The full symmetry group of AdS_{d+1} space is SO(2,d) corresponding to the conformal invariance in d dimensions.

Origin of the Extra Dimension

- For a quantum state of length scale a in a CFT, the energy ~ 1/a.
- By performing scale transformations one can obtain states of all possible sizes.
- In the dual gravity theory the extra coordinate y labels the size of the state. The size is ~ e^{-y}.
- Objects fall to smaller y due to the gravitational potential. In the CFT this corresponds to the state expanding to lower its energy.



D-Branes vs. Geometry



- A stack of N Dirichlet 3-branes realizes $\mathcal{N}$=4 supersymmetric SU(N) gauge theory in 4 dimensions. It also creates a curved background of 10-d theory of type IIB closed superstrings

$$ds^2 = \left(1 + \frac{L^4}{r^4}\right)^{-1/2} \left(-(dx^0)^2 + (dx^i)^2\right) + \left(1 + \frac{L^4}{r^4}\right)^{1/2} \left(dr^2 + r^2 d\Omega_5^2\right)$$

which for small r approaches $AdS_5 \times \mathbf{S}^5$

whose radius is related to the coupling by

$$L^4 = g_{\rm YM}^2 N \alpha'^2$$

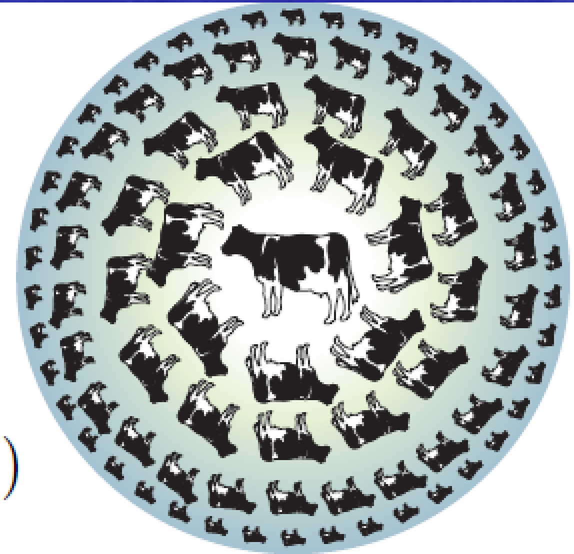
The AdS/CFT Duality

Maldacena; Gubser, IK, Polyakov; Witten

- Relates conformal gauge theory in 4 dimensions to string theory on 5-d Anti-de Sitter space times a 5-d compact space. For the $\mathcal{N}$=4 SYM theory this compact space is a 5-d sphere.
- The geometrical symmetry of the AdS_5 space realizes the conformal symmetry of the gauge theory.
- The AdS_{d+1} space is a hyperboloid

$$(X^0)^2 + (X^d)^2 - \sum_{i=1}^{d} (X^i)^2 = L^2$$

- Its metric is $ds^2 = \frac{L^2}{z^2}(dz^2 - (dx^0)^2 + \sum_{i=1}^{d-1}(dx^i)^2)$



- When a gauge theory is strongly coupled, the radius of curvature of the dual AdS_5 and of the 5-d compact space becomes large: $\frac{L^2}{\alpha'} \sim \sqrt{g^2_{\rm YM} N}$

- String theory in such a weakly curved background can be studied in the effective (super)-gravity approximation, which allows for a host of explicit calculations. Corrections to it proceed in powers of $\frac{\alpha'}{L^2} \sim \lambda^{-1/2}$

- Feynman graphs instead develop a weak coupling expansion in powers of λ. At weak coupling the dual string theory becomes difficult.

AdS/CFT `Dictionary'

- Gauge invariant operators in the CFT_4 are in one-to-one correspondence with fields (or extended objects) in AdS_5
- Operator scaling dimensions are an important set of quantities

$$\langle \mathcal{O}_{\Delta_1}(x_1)\mathcal{O}_{\Delta_2}(x_2)\rangle = \frac{\delta_{\Delta_1,\Delta_2}}{|x_1 - x_2|^{2\Delta_1}}$$

- The dimension of a `short' operator is related to mass of the corresponding field in AdS space: $\Delta_\pm = 2 \pm \sqrt{4 + m^2 L^2}$
- For example, $\mathcal{O} = \mathrm{tr} F_{\mu\nu} F^{\mu\nu} + \ldots$ is dual to the massless dilaton.

- SUGRA fields of m~1/L are often dual to operator protected by supersymmetry. Their dimensions are independent of λ.
- The unprotected operators, e.g.

$$\mathcal{O}_{\mathrm{Konishi}} = \mathrm{tr}\left(\phi^I\right)^2$$

  are dual to massive string states which are much heavier: $m \sim 1/\sqrt{\alpha'}$

  AdS/CFT predicts that at strong coupling their dimensions grow as $\lambda^{1/4}$.
- This growth of most operator dimensions at strong coupling is a striking prediction of AdS/CFT.

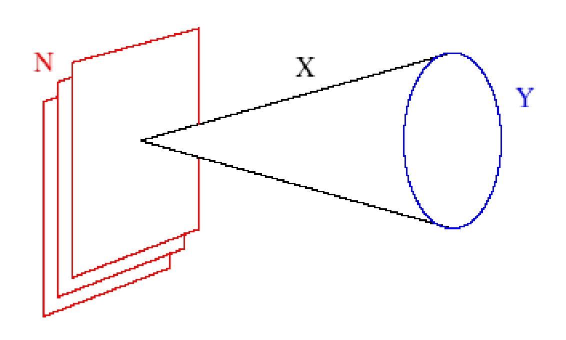
Tests

- These arguments provide a solid motivation for the AdS/CFT correspondence, but its proof has not yet been found.
- It has become a time-honored tradition to simply assume that the correspondence holds. This consistently yields plausible predictions.
- To test the duality one needs to solve for some non-BPS gauge theory quantity as a function of λ and compare the strong coupling expansion with string theory in AdS. There is a growing set of successful tests. Erickson, Semenoff, Zarembo; Pestun; Beisert, Eden, Staudacher; …

Conebrane Dualities

- To reduce the number of supersymmetries in AdS/CFT, we may place the stack of N D3-branes at the tip of a 6-d Ricci-flat cone X whose base is a 5-d Einstein space Y:

$$ds_X^2 = dr^2 + r^2 ds_Y^2$$



- Taking the near-horizon limit of the background created by the N D3-branes, we find the space AdS_5 x Y, with N units of RR 5-form flux, whose radius is given by

$$L^4 = \frac{\sqrt{\pi}\kappa N}{2\,\mathrm{Vol}(Y)} = 4\pi g_s N \alpha'^2 \frac{\pi^3}{\mathrm{Vol}(Y)}$$

- This type IIB background is conjectured to be dual to the IR limit of the gauge theory on N D3-branes at the tip of the cone X.

D3-branes on the Conifold

- The conifold is a Calabi-Yau 3-fold cone X described by the constraint $\sum_{a=1}^{4} z_a^2 = 0$ on 4 complex variables.
- Its base Y is a coset $T^{1,1}$ which has symmetry $SO(4) \sim SU(2)_A \times SU(2)_B$ that rotates the z's, and also $U(1)_R$: $z_a \to e^{i\theta} z_a$
- The Sasaki-Einstein metric on $T^{1,1}$ is

$$\mathrm{d}s^2_{T^{1,1}} = \frac{1}{9}\left(\mathrm{d}\psi + \sum_{i=1}^{2} \cos\theta_i \mathrm{d}\phi_i\right)^2 + \frac{1}{6}\sum_{i=1}^{2}\left(\mathrm{d}\theta_i^2 + \sin^2\theta_i \mathrm{d}\phi_i^2\right)$$

  where $\theta_i \in [0,\pi], \phi_i \in [0,2\pi], \psi \in [0,4\pi]$
- The topology of $T^{1,1}$ is $S^2 \times S^3$.

- To `solve' the conifold constraint det Z = 0 we introduce another set of convenient coordinates:

$$Z = \begin{pmatrix} z^3 + iz^4 & z^1 - iz^2 \\ z^1 + iz^2 & -z^3 + iz^4 \end{pmatrix} = \begin{pmatrix} w_1 & w_3 \\ w_4 & w_2 \end{pmatrix} = \begin{pmatrix} a_1 b_1 & a_1 b_2 \\ a_2 b_1 & a_2 b_2 \end{pmatrix}$$

- The action of global symmetries is

$$\begin{aligned} SU(2)\times SU(2)\ \text{symmetry} &: \quad \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \to L \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \qquad \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \to R \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \\ \text{R-symmetry} &: \quad (a_i, b_j) \to e^{i\frac{\alpha}{2}}(a_i, b_j)\ , \end{aligned}$$

- There is a redundancy under

$$a_i \to \lambda\, a_i \quad , \quad b_j \to \frac{1}{\lambda}\, b_j \quad (\lambda \in \mathbf{C})$$

It may be fixed by identifying $a \sim e^{i\alpha} a, b \sim e^{-i\alpha} b$ and imposing $|a_1|^2 + |a_2|^2 - |b_1|^2 - |b_2|^2 = 0$

- In the field theory this is implemented by a U(1) gauge symmetry and its D-flatness condition.
- In the IR gauge theory on D3-branes at the apex of the conifold, the coordinates a_1, a_2, b_1, b_2 are replaced by chiral superfields. For a single D3-brane it is necessary to introduce gauge group U(1) x U(1). The A's have charges (1,-1); the B's (-1,1). The `sum' U(1) is free.
- The moduli space of this gauge theory is the conifold.

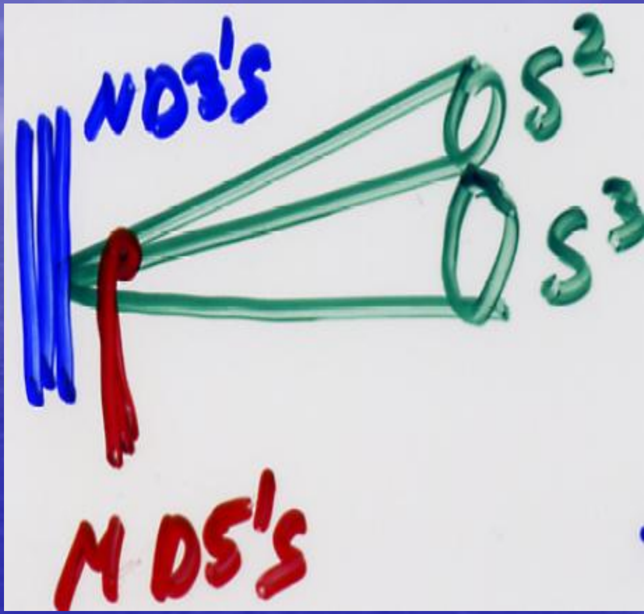
- The $\mathcal{N}$=1 SCFT on N D3-branes at the apex of the conifold has gauge group SU(N)xSU(N) coupled to bifundamental chiral superfields A_1, A_2, in $(\overline{\mathbf{N}}, \mathbf{N})$, and B_1, B_2 in $(\mathbf{N}, \overline{\mathbf{N}})$. IK, Witten
- The R-charge of each field is ½. This insures $U(1)_R$ anomaly cancellation.
- The unique $SU(2)_A xSU(2)_B$ invariant, exactly marginal quartic superpotential is added:

$$W = \epsilon^{ij}\epsilon^{kl}\, \mathrm{tr}\, A_i B_k A_j B_l$$

- This theory also has a `baryonic' U(1) symmetry under which $A_k \to e^{ia} A_k$; $B_l \to e^{-ia} B_l$. It starts out as a U(1) gauge symmetry on D3-branes, but its gauge coupling flows to zero in the IR.

Confinement and χSB

- To break conformal invariance, change the gauge theory: add to the N D3-branes M D5-branes wrapped over the S^2 at the tip of the conifold.



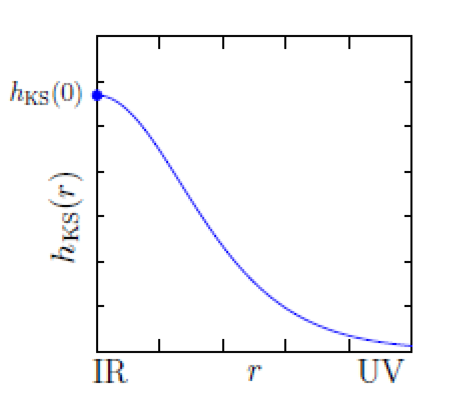
- The 10-d geometry dual to the gauge theory on these branes is the warped deformed conifold (IK, Strassler)

$$ds_{10}^2 = h^{-1/2}(y)\big(-(dx^0)^2 + (dx^i)^2\big) + h^{1/2}(y)ds_6^2$$

- ds_6^2 is the metric of the deformed conifold, a simple Calabi-Yau space defined by the following constraint on 4 complex variables:

$$\sum_{i=1}^{4} z_i^2 = \varepsilon^2$$

- The warp factor is finite at the `tip of the cigar' as required for the color confinement:



- The string tension, is proportional to h(0) $^{-1/2}$
- Dimensional transmutation in the IR. The dynamically generated confinement scale is

$$\sim \varepsilon^{2/3}$$

- The pattern of R-symmetry breaking is the same as in the SU(M) SYM theory: Z_{2M} -> Z_2

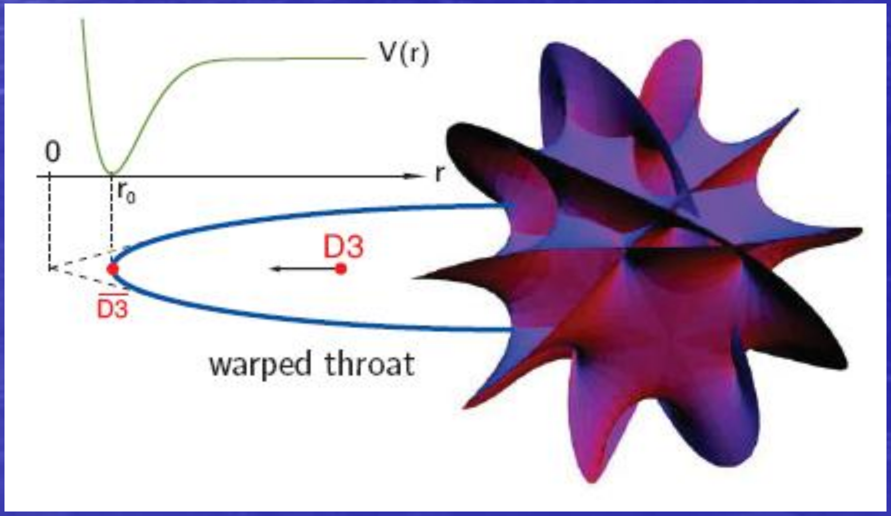
- All of this provides us with an exact solution of a class of 4-d large N confining supersymmetric gauge theories.
- This should be a good playground for studying strongly coupled gauge theory: a `hyperbolic cow' approximation to $\mathcal{N}=1$ supersymmetric gluodynamics.
- Some results on glueball spectra are available. Krasnitz; Caceres, Hernandez; Dymarsky, Melnikov; Berg, Haack, Muck; Benna, Dymarsky, IK, Soloviev; …
- Possible applications of these models to new physics include Randall-Sundrum warped extra dimension models, KKLT moduli stabilization in flux compactifications, as well as warped throat D-brane cosmology (KKLMMT).

Connection with Cosmic Strings

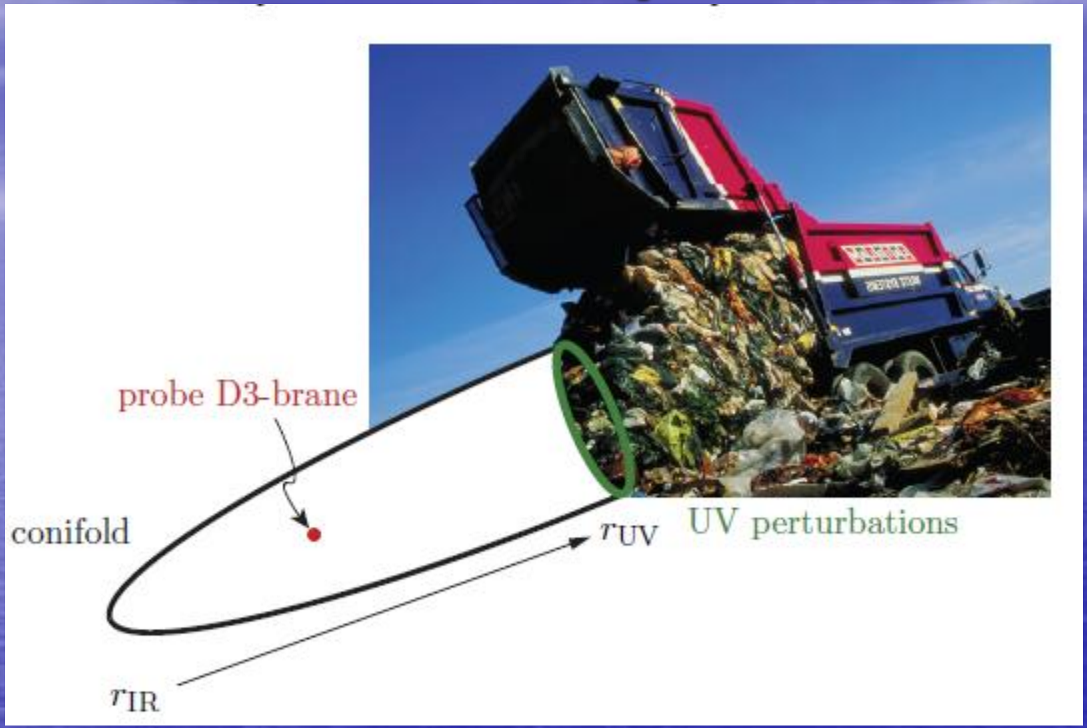
- A fundamental string at the bottom of the warped deformed conifold is dual to a confining string. A D-string is dual to a certain solitonic string due to spontaneously broken baryonic U(1) symmetry in the throat. There is also a rich spectrum of (p,q) strings.
- Upon embedding of the warped throat into a flux compactification, these objects can be used to model cosmic strings. Copeland, Myers, Polchinski; ...
- Detectable via gravity wave bursts? Damour, Vilenkin
- This throat is not the `standard model throat' but another, `inflationary throat,' dual to a hidden sector gauge theory with confining scale $\sim 10^{14}$ GeV.

D-brane Inflation

- Finding models with very flat inflaton potentials has proven to be difficult. Recent string theory constructions use moving D-branes. Dvali, Tye, …
- In the KKLT/KKLMMT model, the warped deformed conifold is embedded into a string compactification. An anti-D3-brane is added at the bottom to break SUSY and generate a potential. A D3-brane rolls in the throat. Its radial coordinate plays the role of an inflaton. Kachru, Kallosh, Linde, Maldacena, McAllister, Trivedi



Effects of Compactification



- Gluing the throat into a Calabi-Yau manifold modifes the inflaton potential. Large r perturbations of the throat geometry correspond to adding operators to the gauge theory action. Baumann, Dymarsky, Kachru, IK, McAllister

- The structure of the full inflaton potential is

$$V(\phi) \;=\; V_{D3/\overline{D3}}(\phi) \;+\; H^2\phi^2 \;+\; \Delta V(\phi)$$

$$\phi \equiv r\sqrt{\tfrac{3}{2}T_3}$$

- The power of a linear perturbation to the potential is determined by the dimension of the dual operator

$$\Delta V = -c\, M_{\rm UV}^{-\Delta}\,|F_X|^2\,\phi^{\Delta}$$

- In the warped conifold throat, the lowest dimension is 3/2. Balancing the two terms can lead to an inflection point.

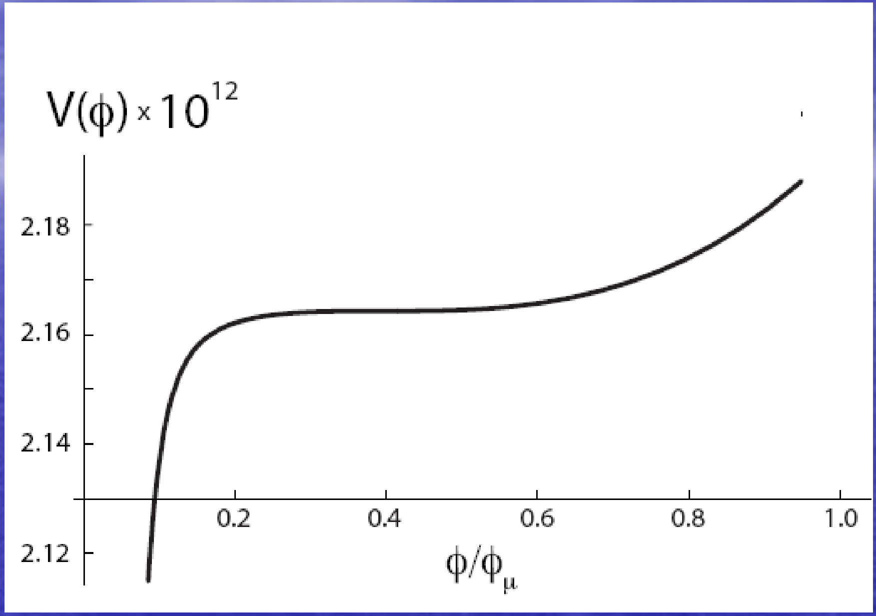
- In our recent work we showed that imaginary anti-self dual 3-form field strength perturbations $\star_6 \Lambda = -i\Lambda$ $\mathrm{d}\Lambda = 0$ may create important corrections to D3-brane potential at quadratic order through the equations

$$\nabla^2 \Phi_- = \frac{g_s}{96}|\Lambda|^2 + \mathcal{R}_4$$

$$V_{\mathrm{D3}} = T_3\left(e^{4A} - \alpha\right) \equiv T_3 \Phi_-$$

- For example, (2,1) forms Λ correspond to superpotential perturbations in the conifold gauge theory which, as expected, create potential at quadratic order.

- The effective potential for the inflaton can be fine-tuned to have an inflection point.



- Similar inflection points may be achieved in models with explicit

  D7-branes in the throat.
  Baumann, Dymarsky, IK, McAllister, Steinhardt; Krause, Pajer

- Models of Inflection Point Inflation were also recently considered in string theory by Itzhaki and Kovetz; Linde and Westphal; and in MSSM inflation by Allahverdi, Enqvist, Garcia-Bellido and Mazumdar; …

Inflection Point Inflation

- Assume a potential $\mathbb{V} = V_0 + \lambda_1(\phi - \phi_0) + \frac{1}{3!}\lambda_3(\phi - \phi_0)^3$

$$\frac{V_0}{\lambda_3} \ll 1 \,, \qquad \frac{V_0}{\sqrt{\lambda_1 \lambda_3}} \gg 1$$

- The slow-roll parameters

$$\begin{aligned} \epsilon &\equiv \frac{1}{2}\left(\frac{\mathbb{V}_{,\phi}}{\mathbb{V}}\right)^2 \approx \frac{1}{2}\left(\frac{\lambda_1 + \frac{1}{2}\lambda_3(\phi - \phi_0)^2}{V_0}\right)^2 \,, \\ \eta &\equiv \frac{\mathbb{V}_{,\phi\phi}}{\mathbb{V}} \approx \frac{\lambda_3}{V_0}(\phi - \phi_0) \,. \end{aligned}$$

- The number of e-folds until the end of inflation is $N_e(\phi) = \int_{\phi_{\rm end}}^{\phi} \frac{{\rm d}\phi}{\sqrt{2\epsilon}}$

- The total number N_{tot} is ~ $\pi\sqrt{\frac{2V_0^2}{\lambda_1\lambda_3}}$

The Scalar Spectral Index

- The usual slow-roll formula is

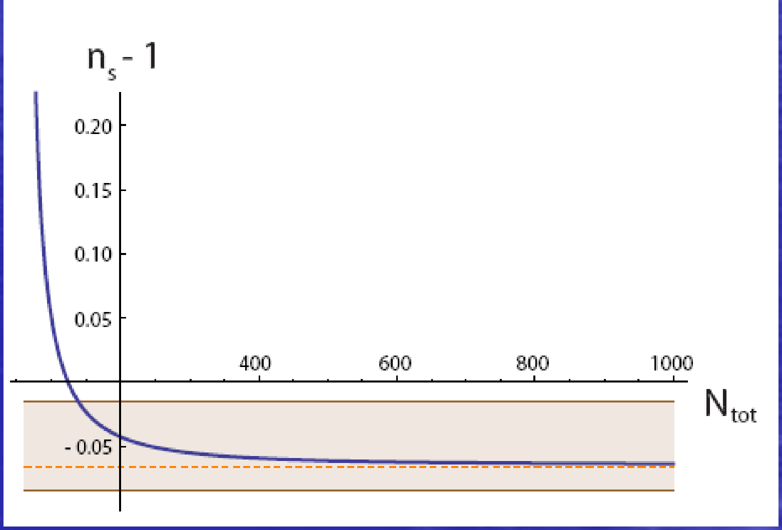
$$n_s - 1 = (2\eta - 6\epsilon)\big|_{\phi_{\text{CMB}}} \approx 2\eta(\phi_{\text{CMB}})$$

- For IPI, in terms of $N_{\text{CMB}} \sim 60$

$$n_s - 1 \approx -\frac{4\pi}{N_{\text{tot}}} \cot\left(\pi \frac{N_{\text{CMB}}}{N_{\text{tot}}}\right)$$

- For large N_{tot}, $n_s - 1 \approx -\frac{4}{N_{\text{CMB}}}$ which is around 0.933.
- The running of the spectral index is small for $N_{\text{tot}} \gg N_{\text{CMB}}$

$$\alpha_s \equiv \frac{dn_s}{d\ln k} \approx -\frac{4}{N_{\text{CMB}}^2} - \frac{4\pi^2}{3}\frac{1}{N_{\text{tot}}^2} + \mathcal{O}\left(\frac{N_{\text{CMB}}^2}{N_{\text{tot}}^4}\right)$$



Conclusions

- Throughout its history, string theory has been intertwined with the theory of strong interactions.
- The AdS/CFT correspondence makes this connection precise. It makes many dynamical statements about strongly coupled conformal gauge theories.
- Extensions of AdS/CFT provide a new geometrical understanding of confinement, chiral symmetry breaking and other strong coupling phenomena.
- Possible applications of the new methods include BSM physics and cosmological inflation.