Supersymmetric Flavour physics

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SUSY 10, Bonn, August 2010

in memory of Nicola Cabibbo
(10 Apr 1935 – 16 Aug 2010)
Evidence for an anomalous like-sign dimuon charge asymmetry
May 14, 2010
Fermilab Wine&Cheese seminar, talk by Guennadi Borrisov:

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Joe Lykken, a theorist at Fermilab, said, “So I would not say that this announcement is the equivalent of seeing the face of God, but it might turn out to be the toe of God.”
Contents

Basics

$B_s - \bar{B}_s$ mixing and new physics

Global analysis of $B_s - \bar{B}_s$ mixing and $B_d - \bar{B}_d$ mixing

SUSY

Conclusions
Flavour physics studies transitions between fermions of different generations.

**Standard Model:** misalignment of $3 \times 3$ Yukawa matrices in flavour space parametrised by the

Cabibbo-Kobayashi-Maskawa (CKM) matrix $V$

in the quark sector

Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix $U$

in the lepton sector

CKM matrix $V$ and PMNS matrix $U$ occur only in the couplings of $W$ bosons.
Expand the CKM matrix $V$ in $V_{us} \simeq \lambda = 0.2246$:

$$
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix} \simeq
\begin{pmatrix}
1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3 \left(1 + \frac{\lambda^2}{2}\right) (\bar{\rho} - i\bar{\eta}) \\
-\lambda - iA^2 \lambda^5 \bar{\eta} & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
A\lambda^3 (1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 - iA\lambda^4 \bar{\eta} & 1
\end{pmatrix}
$$

with the Wolfenstein parameters $\lambda$, $A$, $\bar{\rho}$, $\bar{\eta}$

CP violation $\Leftrightarrow \bar{\eta} \neq 0$
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\end{pmatrix}
$$

with the Wolfenstein parameters $\lambda$, $A$, $\bar{\rho}$, $\bar{\eta}$

$CP$ violation $\iff \bar{\eta} \neq 0$

Unitarity triangle:

Exact definition:

$$
\bar{\rho} + i\bar{\eta} \; = \; -\frac{V_{ub}^*}{V_{cd}} \frac{V_{ud}}{V_{cd}} = \left| \frac{V_{ub}^*}{V_{cd}} \right| e^{i\gamma}
$$
Suppression factors in Flavour-changing neutral current (FCNC) processes:

- weak loop, small CKM elements,
- often also GIM factor \( (m_c^2 - m_u^2)/M_W^2 \) or helicity suppression \( m_b/M_W \).

\[\Rightarrow\] **FCNC processes** are extremely sensitive to **new physics**.
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- often also GIM factor \((m_c^2 - m_u^2)/M_W^2\) or helicity suppression \(m_b/M_W\).

⇒ FCNC processes are extremely sensitive to new physics.

Examples of FCNC processes:

- \(B_s - \bar{B}_s\) mixing
- Penguin diagram
New-physics analysers:

- Global fit to UT: overconstrain \((\rho, \eta)\), probes FCNC processes \(K - \bar{K}\), \(B_d - \bar{B}_d\) and \(B_s - \bar{B}_s\) mixing.
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- Global fit to \(B_s - \bar{B}_s\) mixing: mass difference \(\Delta m_s\), width difference \(\Delta \Gamma_s\), CP asymmetries in \(B_s \rightarrow J/\psi \phi\) and \((\bar{B}_s) \rightarrow X \ell \nu_\ell\).
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- Penguin decays: \(B \to X_s \gamma\), \(B \to X_s \ell^+ \ell^-\), \(B \to K \pi\), \(B_d \to \phi K_S\), \(B_s \to \mu^+ \mu^-\), \(K \to \pi \nu \bar{\nu}\).
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- **Global fit to UT**: overconstrain \((\rho, \eta)\), probes FCNC processes \(K \to \bar{K}\), \(B_d \to \bar{B}_d\) and \(B_s \to \bar{B}_s\) mixing.
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- **Penguin decays**: \(B \to X_s \gamma\), \(B \to X_s \ell^+ \ell^-\), \(B \to K \pi\), \(B_d \to \phi K_S\), \(B_s \to \mu^+ \mu^-\), \(K \to \pi \nu \bar{\nu}\).
- **CKM-suppressed or helicity-suppressed tree-level decays**: \(B^+ \to \tau^+ \nu\), \(B \to \pi \ell \nu\), \(B \to D \tau \nu\), probe charged Higgses and right-handed W-couplings.
Global fit in the SM from CKMfitter:

Statistical method: Rfit, a Frequentist approach.
Global fit in the SM from UTfit:

Statistical method: Bayesian.
$B_s - \overline{B}_s$ mixing and new physics

Schrödinger equation for $B_s \sim \bar{b}s$ and $\overline{B}_s \sim b\bar{s}$:

$$i \frac{d}{dt} \begin{pmatrix} |B_s(t)\rangle \\ |\overline{B}_s(t)\rangle \end{pmatrix} = \left( M - i \frac{\Gamma}{2} \right) \begin{pmatrix} |B_s(t)\rangle \\ |\overline{B}_s(t)\rangle \end{pmatrix}$$

Here $|B_s(t)\rangle$ is a linear superposition of $|B_s\rangle$ and $|\overline{B}_s\rangle$ with $|B_s(0)\rangle = |B_s\rangle$.

Mass and decay matrices $M = M^\dagger$ and $\Gamma = \Gamma^\dagger$. 
B_s – B_{\bar{s}} mixing and new physics

Schrödinger equation for \( B_s \sim b_s \) and \( B_{\bar{s}} \sim b_{\bar{s}} \):

\[
\frac{i}{\hbar} \frac{d}{dt} \begin{pmatrix} |B_s(t)\rangle \\ |B_{\bar{s}}(t)\rangle \end{pmatrix} = \left( M - \frac{i}{2} \Gamma \right) \begin{pmatrix} |B_s(t)\rangle \\ |B_{\bar{s}}(t)\rangle \end{pmatrix}
\]

Here \( |B_s(t)\rangle \) is a linear superposition of \( |B_s\rangle \) and \( |B_{\bar{s}}\rangle \) with \( |B_s(0)\rangle = |B_s\rangle \).

Mass and decay matrices \( M = M^\dagger \) and \( \Gamma = \Gamma^\dagger \).

3 physical quantities in \( B_s – B_{\bar{s}} \) mixing:

\[
|M_{12}^s|, \quad |\Gamma_{12}^s|, \quad \phi_s \equiv \arg \left( -\frac{M_{12}^s}{\Gamma_{12}^s} \right)
\]
Two mass eigenstates with masses $M_H$, $M_L$ and widths $\Gamma_H$, $\Gamma_L$.

Mass and width differences:

$$
\Delta m_s = M_H - M_L \simeq 2|M_{12}^s|,
\Delta \Gamma_s = \Gamma_L - \Gamma_H \simeq 2|\Gamma_{12}^s| \cos \phi_s
$$
Standard Model:

$M_{12}^s$ from dispersive part of box, only internal $t$ relevant;

New physics can barely affect $\Gamma_{12}^s$, which stems from tree-level decays.
$M_{12}^s$ is very sensitive to virtual effects of new heavy particles.
Generic new physics

The phase $\phi_s = \arg(-M_{12}/\Gamma_{12})$ is negligibly small in the Standard Model:

$$\phi_s^{\text{SM}} = 0.2^\circ.$$ 

Define the complex parameter $\Delta_s$ through

$$M_{12}^s \equiv M_{12}^{\text{SM},s} \cdot \Delta_s, \quad \Delta_s \equiv |\Delta_s| e^{i\phi_s^\Delta}.$$ 

In the Standard Model $\Delta_s = 1$. Use $\phi_s = \phi_s^{\text{SM}} + \phi_s^\Delta \approx \phi_s^\Delta$. 
The phase \( \phi_s = \arg(-M_{12}/\Gamma_{12}) \) is negligibly small in the Standard Model:
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\]
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The CDF measurement
\[
\Delta m_s = (17.77 \pm 0.10 \pm 0.07) \text{ ps}^{-1}
\]
implies
\[
|\Delta_s| = 0.92 \pm 0.14_{(\text{th})} \pm 0.01_{(\text{exp})}
\]
Flavour-specific decay: $B_s \to f$ is allowed, while $\bar{B}_s \to f$ is forbidden

CP asymmetry in flavour-specific decays (semileptonic CP asymmetry):

$$a_{fs}^s = \frac{\Gamma(\bar{B}_s(t) \to f) - \Gamma(B_s(t) \to \bar{f})}{\Gamma(\bar{B}_s(t) \to f) + \Gamma(B_s(t) \to \bar{f})}$$

with e.g. $f = X\ell^+\nu_\ell$ and $\bar{f} = X\ell^-\bar{\nu}_\ell$. Untagged rate:

$$a_{fs,unt}^s \equiv \frac{\int_0^\infty dt \left[ \Gamma(\bar{B}_s \to \mu^+X) - \Gamma(\bar{B}_s \to \mu^-X) \right]}{\int_0^\infty dt \left[ \Gamma(\bar{B}_s \to \mu^+X) + \Gamma(\bar{B}_s \to \mu^-X) \right]} \approx \frac{a_{fs}^s}{2}$$
Relation to $M_{12}^s$:

$$a_{fs}^s = \frac{\left| \Gamma_{12}^s \right|}{\left| M_{12}^s \right|} \sin \phi_s = \frac{\left| \Gamma_{12}^s \right|}{\left| M_{12}^{SM,s} \right|} \cdot \frac{\sin \phi_s}{\left| \Delta_s \right|} = (4.97 \pm 0.94) \cdot 10^{-3} \cdot \frac{\sin \phi_s}{\left| \Delta_s \right|}$$

A. Lenz, UN, 2006
Dilepton events:

Compare the number $N_{++}$ of decays $(B_s(t), \bar{B}_s(t)) \rightarrow (f, f)$ with the number $N_{--}$ of decays to $(\bar{f}, \bar{f})$.

Then $a_{fs}^s = \frac{N_{++} - N_{--}}{N_{++} + N_{--}}$.

At the Tevatron all $b$-flavoured hadrons are produced. Still only those events contribute to $(N_{++} - N_{--})/(N_{++} + N_{--})$, in which one of the $b$ hadronises as a $B_d$ or $B_s$ and undergoes mixing.
May 15, 2010: $\text{DØ}$ presents

$$a_{fs} = (-9.57 \pm 2.51 \pm 1.46) \cdot 10^{-3}$$

for a mixture of $B_d$ and $B_s$ mesons with

$$a_{fs} = (0.506 \pm 0.043)a_{fs}^d + (0.494 \pm 0.043)a_{fs}^s$$

The result is $3.2\sigma$ away from $a_{fs}^{\text{SM}} = (-0.23^{+0.05}_{-0.06}) \cdot 10^{-3}$.

Averaging with an older CDF measurement yields

$$a_{fs} = (-8.5 \pm 2.8) \cdot 10^{-3},$$

which is $3.0\sigma$ away from $a_{fs}^{\text{SM}}$. 
\[ a_{fs}^s = (4.97 \pm 0.94) \cdot 10^{-3} \cdot \frac{\sin \phi_s}{|\Delta_s|} \]

If there is no new physics in \( a_{fs}^d \), the Tevatron measurement of \( a_{fs} = (-8.5 \pm 2.8) \cdot 10^{-3} \) roughly implies \( a_{fs}^s = (-17 \pm 6) \cdot 10^{-3} \). With \( |\Delta_s| \geq 0.78 \) find

\[ \sin \phi_s \leq -2.2 \pm 0.7. \]
Closer look: Allow for new physics in $B_d - \overline{B}_d$ mixing as well:

$$\frac{M_{12}^d}{M_{12}^{SM,d}} \equiv \Delta_d = |\Delta_d|e^{i\phi_d^\Delta}$$

Measurement by B factories: $a_{fs}^d = (-4.7 \pm 4.6) \cdot 10^{-3}$

However: $a_{fs}^d$ can be better determined indirectly through

$$a_{fs}^d = \frac{|\Gamma_{12}^d|}{M_{12}^{SM,d}} \frac{\sin(\phi_d^{SM} + \phi_d^\Delta)}{|\Delta_d|}$$

with $\phi_d^{SM} = (-5 \pm 2)^\circ$

using the measurements of $\Delta m_d = 2|M_{12}^d|$ and of $2\beta + \phi_d^\Delta = (21 \pm 1)^\circ$ from $A_{CP}^{mix}(B_d \to J/\psi K_S)$.
Closer look: Allow for new physics in $B_d - \overline{B}_d$ mixing as well:

\[
\frac{M_{12}^d}{M_{12}^{SM,d}} \equiv \Delta_d = |\Delta_d| e^{i\phi_d^A}
\]

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$\Rightarrow$ requires fit to unitarity triangle to find $\beta$
Other connection between $B_d$ and $B_s$ mixing:
The global fit to the unitarity triangle involves $\frac{\Delta m_d}{\Delta m_s}$ from which hadronic uncertainties cancel to a large extent.
Global analysis of $B_s - \overline{B}_s$ mixing and $B_d - \overline{B}_d$ mixing

Based on work with A. Lenz and the CKMfitter Group (J. Charles, S. Descotes-Genon, A. Jantsch, C. Kaufhold, H. Lacker, S. Monteil, V. Niess)  

[arXiv:1008.1593]

**Rfit method:** No statistical meaning is assigned to systematic errors and theoretical uncertainties.

We have performed a simultaneous fit to the Wolfensteinn parameters and to the new physics parameters $\Delta_s$ and $\Delta_d$ in three scenarios.
Scenario I: arbitrary complex parameters $\Delta_s$ and $\Delta_d$

Scenario II: new physics is minimally flavour violating (MFV) (meaning that all flavour violation stems from the Yukawa sector) and $y_b$ is small:
one real parameter $\Delta = \Delta_s = \Delta_d$

Scenario III: MFV with a large $y_b$: one complex parameter $\Delta = \Delta_s = \Delta_d$
Scenario I: arbitrary complex parameters $\Delta_s$ and $\Delta_d$

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one real parameter $\Delta = \Delta_s = \Delta_d$

Scenario III: MFV with a large $y_b$: one complex parameter $\Delta = \Delta_s = \Delta_d$

Examples:
Scenario I covers the MSSM with generic flavour structure of the soft terms and small $\tan \beta$.
Scenario II covers the MSSM with MFV and small $\tan \beta$.
Scenario III covers certain two-Higgs models (but not the MFV-MSSM).
Results in scenario I:

SM point $\Delta_d = 1$ disfavoured by $\geq 2.5\sigma$.

$\phi^\Delta_d < 0$ helps to explain DØ dimuon asymmetry.
Reason for the tension with the SM: $B(B^+ \rightarrow \tau^+ \nu_\tau)$

SM prediction ($\text{CL}=2\sigma$): 

$$B(B^+ \rightarrow \tau^+ \nu_\tau) = \left(0.763^{+0.214}_{-0.097}\right) \cdot 10^{-4}$$

Average of several measurements by BaBar and Belle:

$$B^{\text{exp}}(B^+ \rightarrow \tau^+ \nu_\tau) = (1.68 \pm 0.31) \cdot 10^{-4}$$
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$$B^{\text{exp}}(B^+ \rightarrow \tau^+ \nu_\tau) = (1.68 \pm 0.31) \cdot 10^{-4}$$

$$B^{\text{SM}}(B^+ \rightarrow \tau^+ \nu_\tau) = \frac{G_F^2 m_{B^+} m_\tau^2}{8\pi} \left( 1 - \frac{m_\tau^2}{m_{B^+}^2} \right)^2 |V_{ub}|^2 f_{B^+}^2 f_{\tau B^+} f_{\tau B^+}.$$ 

But with e.g. $f_B = 210$ MeV and $|V_{ub}| = 4.4 \cdot 10^{-3}$ find $B^{\text{SM}}(B^+ \rightarrow \tau^+ \nu_\tau) = 1.51 \cdot 10^{-4}$. These parameters comply with the global fit to the UT only, if new physics changes the constraints from $A^{\text{mix}}_{CP}(B_d \rightarrow J/\psi K_S)$, $\Delta m_d$ or $\Delta m_d/\Delta m_s$. 
Global fit in the SM:
SM point $\Delta_s = 1$ disfavoured by $\geq 2.7\sigma$.

without 2010 CDF/DØ data on $B_s \rightarrow J/\psi\phi$
Global fit to UT hinting at $\phi_d^\Delta < 0$:
Other authors have seen a tension with the SM in the same direction stemming from $\epsilon_K$.

Lunghi, Soni; Buras, Guadagnoli

In our fit the tension with $\epsilon_K$ is mild, because we use a more conservative error on the hadronic parameter $\hat{B}_K = 0.724 \pm 0.004 \pm 0.067$ and because the Rfit method is more conservative.
p-values:
Calculate $\chi^2 / N_{dof}$ with and without a hypothesis to find:

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_d = 1$</td>
<td>2.5 $\sigma$</td>
</tr>
<tr>
<td>$\Delta_s = 1$</td>
<td>2.7 $\sigma$</td>
</tr>
<tr>
<td>$\Delta_d = \Delta_s = 1$</td>
<td>3.4 $\sigma$</td>
</tr>
<tr>
<td>$\Delta_d = \Delta_s$</td>
<td>2.1 $\sigma$</td>
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</tbody>
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Is the result driven by the DØ dimuon asymmetry? One can remove $a_{fs}$ as an input and instead predict it from the global fit:

$$a_{fs} = \left(-4.2^{+2.7}_{-2.6}\right) \cdot 10^{-3} \quad \text{at } 2\sigma.$$
Is the result driven by the DØ dimuon asymmetry? One can remove $a_{fs}$ as an input and instead predict it from the global fit:

$$a_{fs} = \left(-4.2^{+2.7}_{-2.6}\right) \cdot 10^{-3} \text{ at } 2\sigma.$$ 

This is just $1.5\sigma$ away from the DØ/CDF average

$$a_{fs} = (-8.5 \pm 2.8) \cdot 10^{-3}.$$
The fit in scenario II (real $\Delta_s = \Delta_d$) is not better than the SM fit and gives $\Delta = 0.907^{+0.091}_{-0.067}$.

Scenario III (complex $\Delta_s = \Delta_d$) fits the data quite well irrespective of whether $B(B^+ \rightarrow \tau^+ \nu_\tau)$ is included or not.

<table>
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<th>p-value</th>
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<td>$\Delta = 1$</td>
<td>3.1 $\sigma$</td>
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The **MSSM** has many new sources of flavour violation, all in the supersymmetry-breaking sector.

No problem to get big effects in $B_s - \overline{B}_s$ mixing, but rather to suppress the big effects elsewhere.
Squark mass matrix

Diagonalise the Yukawa matrices $\mathbf{Y}^u_{jk}$ and $\mathbf{Y}^d_{jk}$

$\Rightarrow$ quark mass matrices are diagonal, super-CKM basis
Diagonalise the Yukawa matrices $Y^d_{jk}$ and $Y^u_{jk}$

\[ \Rightarrow \text{ quark mass matrices are diagonal, super-CKM basis} \]

E.g. Down-squark mass matrix:

\[
M^2_{\tilde{d}} = \left( \begin{array}{cccccccc}
(M_{\tilde{d}}^1)^2 & \Delta^\dagger & \Delta^\dagger & \Delta^\dagger & \Delta^\dagger & \Delta^\dagger & \Delta^\dagger & \Delta^\dagger \\
\Delta^\dagger & (M_{\tilde{d}}^2)^2 & \Delta^\dagger & \Delta^\dagger & \Delta^\dagger & \Delta^\dagger & \Delta^\dagger & \Delta^\dagger \\
\Delta^\dagger & \Delta^\dagger & (M_{\tilde{d}}^3)^2 & \Delta^\dagger & \Delta^\dagger & \Delta^\dagger & \Delta^\dagger & \Delta^\dagger \\
\Delta^\dagger & \Delta^\dagger & \Delta^\dagger & (M_{\tilde{d}}^1\text{R})^2 & \Delta^\dagger & \Delta^\dagger & \Delta^\dagger & \Delta^\dagger \\
\Delta^\dagger & \Delta^\dagger & \Delta^\dagger & \Delta^\dagger & (M_{\tilde{d}}^2\text{R})^2 & \Delta^\dagger & \Delta^\dagger & \Delta^\dagger \\
\Delta^\dagger & \Delta^\dagger & \Delta^\dagger & \Delta^\dagger & \Delta^\dagger & (M_{\tilde{d}}^3\text{R})^2 & \Delta^\dagger & \Delta^\dagger \\
\Delta^\dagger & \Delta^\dagger & \Delta^\dagger & \Delta^\dagger & \Delta^\dagger & \Delta^\dagger & (M_{\tilde{d}}^1\text{L})^2 & \Delta^\dagger \\
\Delta^\dagger & \Delta^\dagger & \Delta^\dagger & \Delta^\dagger & \Delta^\dagger & \Delta^\dagger & \Delta^\dagger & (M_{\tilde{d}}^2\text{L})^2
\end{array} \right)
\]
Diagonalise the Yukawa matrices $Y^u_{jk}$ and $Y^d_{jk}$

$\Rightarrow$ quark mass matrices are diagonal, super-CKM basis

E.g. Down-squark mass matrix:

$$M^2_{\tilde{d}} = \begin{pmatrix}
(M_{\tilde{d}}^1)^2 & \Delta_{12}^{\tilde{d} LL} & \Delta_{13}^{\tilde{d} LL} & \Delta_{11}^{\tilde{d} LR} & \Delta_{12}^{\tilde{d} LR} \\
\Delta_{12}^{\tilde{d} LL} & (M_{\tilde{d}}^2)^2 & \Delta_{23}^{\tilde{d} LL} & \Delta_{12}^{\tilde{d} RL} & \Delta_{23}^{\tilde{d} RL} \\
\Delta_{13}^{\tilde{d} LL} & \Delta_{23}^{\tilde{d} LL} & (M_{\tilde{d}}^3)^2 & \Delta_{13}^{\tilde{d} RL} & \Delta_{33}^{\tilde{d} RL} \\
\Delta_{11}^{\tilde{d} LR} & \Delta_{12}^{\tilde{d} RL} & \Delta_{13}^{\tilde{d} RL} & (M_{\tilde{d}}^1)^2 & \Delta_{12}^{\tilde{d} RR} \\
\Delta_{12}^{\tilde{d} LR} & \Delta_{13}^{\tilde{d} LR} & \Delta_{23}^{\tilde{d} LR} & \Delta_{13}^{\tilde{d} RR} & (M_{\tilde{d}}^2)^2 \\
\Delta_{13}^{\tilde{d} LR} & \Delta_{23}^{\tilde{d} LR} & \Delta_{33}^{\tilde{d} LR} & \Delta_{13}^{\tilde{d} RR} & \Delta_{23}^{\tilde{d} RR} \\
\Delta_{13}^{\tilde{d} LR} & \Delta_{23}^{\tilde{d} LR} & \Delta_{33}^{\tilde{d} LR} & \Delta_{13}^{\tilde{d} RR} & (M_{\tilde{d}}^2)^2
\end{pmatrix}$$

Not diagonal! $\Rightarrow$ new FCNC transitions.
Model-independent analyses constrain

\[
\delta^{q}_{ij} = \frac{\Delta^{q}_{ij}}{\frac{1}{6} \sum_{s} \left[ M^{2}_{\tilde{q}} \right]_{ss}}
\]

with \( XY = LL, LR, RR \) and \( q = u, d \)

using data on FCNC (and also charged-current) processes.

⇒ see next talk by A. Dedes
Model-independent analyses constrain

\[
\delta_{ij}^{qxy} = \frac{\Delta_{ij}^{\tilde{q}xy}}{\frac{1}{6} \sum_s \left[ M_{\tilde{q}}^2 \right]_{ss}}
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with \( XY = LL, LR, RR \) and \( q = u, d \)

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\[ \Rightarrow \text{ see next talk by A. Dedes} \]

Remarks:

- For \( M_{\tilde{g}} \gtrsim 1.5 M_{\tilde{q}} \) the gluino contribution is small and chargino/neutralino contributions are important.

parallel talk by M. Davidkov, 27-1, FR 14:17
Model-independent analyses constrain

\[
\delta_{ij}^{q \, X Y} = \frac{\Delta_{ij}^{\tilde{q} \, X Y}}{\frac{1}{6} \sum_{s} \left[ M_{\tilde{q}}^{2} \right]_{ss}}
\]

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  chargino/neutralino contributions are important.
  parallel talk by M. Davidkov, 27-1, FR 14:17

- To derive meaningful bounds on \( \delta_{ij}^{q \, LR} \) chirally enhanced
  higher-order contributions must be taken into account.

  A. Crivellin, UN, 2009
Are there natural ways to motivate sizable new flavour violation in $B_s - B_s$ mixing and $B_d - B_d$ mixing while simultaneous suppressing flavour violation elsewhere?
Flavour violation from trilinear terms

Origin of the **SUSY flavour problem**: Misalignment of squark mass matrices with Yukawa matrices.

Unorthodox solution: Set $Y^u_{ij}$ and $Y^d_{ij}$ to zero, except for $(i,j) = (3, 3)$.

$\Rightarrow$ No flavour violation from $Y^u_{ij}, Y^d_{ij}$ and $V_{\text{CKM}} = 1$. 

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$V_{\text{CKM}} \neq 1$ is then generated radiatively, through finite squark-gluino loops. ⇒ SUSY-breaking is the origin of flavour.
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Radiative flavour violation: S. Weinberg 1972

flavour from soft SUSY terms:

- W. Buchmuller, D. Wyler 1983
- T. Banks 1988
- J. Ferrandis, N. Haba 2004
Today: Strong constraints from FCNCs probed at B factories.
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But: Radiative flavour violation in the MSSM is still viable, albeit only with $A^d_{ij}$ and $A^u_{ij}$ entering

\[
\begin{align*}
M^d_{ij}^{LR} &= A^d_{ij}v_d + \delta_{i3}\delta_{j3}y_b\mu v_u, \\
M^u_{ij}^{LR} &= A^u_{ij}v_u + \delta_{i3}\delta_{j3}y_t\mu v_d.
\end{align*}
\]

Andreas Crivellin, UN, PRD 79 (2009) 035018
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Andreas Crivellin, UN, PRD 79 (2009) 035018
Darkest corner of the MSSM: The phases of $A^q_{ij}$ and $\mu$ generate too large EDMs. If light quark masses are generated radiatively through soft SUSY-breaking terms, this “supersymmetric CP problem” is substantially alleviated:

- The phases of $A^q_{ij}$ and $m_q$ are aligned, i.e. zero.
- The phase of $\mu$ (essentially) does not enter the EDMs at the one-loop level, because the Yukawa couplings of the first two generations are zero.

Borzumati, Farrar, Polonsky, Thomas 1998, 1999
Flavour and SUSY GUTs

Linking quarks to neutrinos: Flavour mixing:
quarks: Cabibbo-Kobayashi-Maskawa (CKM) matrix
leptons: Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix

Consider $\text{SU}(5)$ multiplets:

$$
\begin{align*}
\bar{5}_1 &= \begin{pmatrix}
d^c_R \\
d^c_R \\
d^c_R \\
e_L \\
-\nu_e 
\end{pmatrix}, \\
\bar{5}_2 &= \begin{pmatrix}
s^c_R \\
s^c_R \\
s^c_R \\
\mu_L \\
-\nu_\mu 
\end{pmatrix}, \\
\bar{5}_3 &= \begin{pmatrix}
b^c_R \\
b^c_R \\
b^c_R \\
\tau_L \\
-\nu_\tau 
\end{pmatrix}.
\end{align*}
$$

If the observed large atmospheric neutrino mixing angle stems from a rotation of $\bar{5}_2$ and $\bar{5}_3$, it will induce a large $\tilde{b}_R - \tilde{s}_R$-mixing (Moroi; Chang,Masiero,Murayama).

$\Rightarrow$ new $b_R-s_R$ transitions from gluino–squark loops possible.
Key ingredients: Some weak basis with

$$Y_d = V_{\text{CKM}}^* \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix} U_{\text{PMNS}}$$

and right-handed down squark mass matrix:

$$m_{\tilde{d}}^2(M_Z) = \text{diag} \left( m_{\tilde{d}}^2, m_{\tilde{d}}^2, m_{\tilde{d}}^2 - \Delta_{\tilde{d}} \right).$$

with a calculable real parameter $\Delta_{\tilde{d}}$, typically generated by top-Yukawa RG effects.
Rotating $Y_d$ to diagonal form puts the large atmospheric neutrino mixing angle into $m_d^2$:

$$U_{\text{PMNS}}^\dagger m_d^2 U_{\text{PMNS}} = \begin{pmatrix} m_d^2 & 0 & 0 \\ 0 & m_d^2 - \frac{1}{2} \Delta_\tilde{d} & -\frac{1}{2} \Delta_\tilde{d} e^{i\xi} \\ 0 & -\frac{1}{2} \Delta_\tilde{d} e^{-i\xi} & m_d^2 - \frac{1}{2} \Delta_\tilde{d} \end{pmatrix}$$

The CP phase $\xi$ affects $B_s - \overline{B}_s$ mixing!
Realistic GUTs involve further dimension-5 Yukawa terms to fix the Yukawa unification in the first two generations. One can use these terms to shuffle a part of the effect from $b_R \rightarrow s_R$ into $b_R \rightarrow d_R$ transitions. This “leakage” is strongly constrained by $K - \bar{K}$ mixing.

Trine, Wiesenfeldt, Westhoff 2009
Realistic GUTs involve further dimension-5 Yukawa terms to fix the Yukawa unification in the first two generations. One can use these terms to shuffle a part of the effect from $b_R \rightarrow s_R$ into $b_R \rightarrow d_R$ transitions. This “leakage” is strongly constrained by $K - \bar{K}$ mixing.

Trine, Wiesenfeldt, Westhoff 2009

Similar constraints can be found from $\mu \rightarrow e\gamma$.

Borzumati, Yamashita 2009; Girrbach, Mertens, UN, Wiesenfeldt 2009
Chang-Masireo-Murayama model

We have considered $B_s - \bar{B}_s$ mixing, $b \rightarrow s\gamma$, $\tau \rightarrow \mu\gamma$, vacuum stability bounds, lower bounds on sparticle masses and the mass of the lightest Higgs boson. The analysis involves 7 parameters in addition to those of the Standard Model.

Generic results: Largest effect in $B_s - \bar{B}_s$ mixing tension with $M_h \geq 114$ GeV

J. Girrbach, S. Jäger, M. Knopf, W. Martens, UN, C. Scherrer, S. Wiesenfeldt
Contour plot for $M_{\tilde{q}} = 350$ GeV, $\arg \mu = 0$:

Black: negative soft masses
Green: excluded by $\tau \rightarrow \mu \gamma$ and $b \rightarrow s \gamma$
Blue: excluded by $\tau \rightarrow \mu \gamma$
Gray: excluded by $B_s - \bar{B}_s$ mixing
Yellow: allowed

Dashed lines: $10^4 \cdot Br(b \rightarrow s \gamma)$; dotted lines: $10^8 \cdot Br(\tau \rightarrow \mu \gamma)$. 
Parallel talks addressing topics touched in this talk:

- **MO** Pheno 23-2 17:37  David Straub
- **TU** Pheno 24-1 14:17  Jennifer Girrbach
- **TU** Pheno 24-1 15:25  Stefania Gori
- **TH** Model Building 26-1 14:17  Andreas Crivellin
- **FR** Model Building 27-1 14:17  Momchil Davidkov
- **FR** Model Building 27-1 14:34  Jisuke Kubo
- **FR** Pheno 27-2 17:37  Wolfgang Altmannshofer
Conclusions

- The DØ result for the dimuon asymmetry in $B_s$ decays supports the hints for $\phi_s < 0$ seen in $B_s \rightarrow J/\psi \phi$ data. The central value is easier to accommodate if both $a_{fs}^s$ and $a_{fs}^d$ receive negative contributions from new physics.
Conclusions

• The DØ result for the dimuon asymmetry in $B_s$ decays supports the hints for $\phi_s < 0$ seen in $B_s \to J/\psi \phi$ data. The central value is easier to accommodate if both $a_{fs}^s$ and $a_{fs}^d$ receive negative contributions from new physics.

• A global fit to the UT indeed shows a slight preference for a new CP phase $\phi_d^\Delta < 0$, driven by $B(B^+ \to \tau^+ \nu_{\tau})$ (and possibly $\epsilon_K$). In a simultaneously global fit to the UT and the $B_s - \bar{B}_s$ mixing complex a plausible picture of new CP-violating physics emerges.
Conclusions

- Large CP-violating contributions to $B_S - \bar{B}_S$ mixing are possible in supersymmetry without violating constraints from other FCNC processes. If confirmed the DØ/CDF results imply physics beyond the MFV-MSSM.
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- An attractive variant is the MSSM with vanishing Yukawa couplings for the first two generations and radiative flavour violation.
Conclusions

• Large CP-violating contributions to $B_s - \bar{B}_s$ mixing are possible in supersymmetry without violating constraints from other FCNC processes. If confirmed the DØ/CDF results imply physics beyond the MFV-MSSM.

• An attractive variant is the MSSM with vanishing Yukawa couplings for the first two generations and radiative flavour violation.

• Models of GUT flavour physics with $\tilde{b}_R - \tilde{s}_R$ mixing driven by the atmospheric neutrino mixing angle can explain the Tevatron data on $B_s - \bar{B}_s$ mixing without conflicting with $b \rightarrow s\gamma$ and $\tau \rightarrow \mu\gamma$. 