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# ***Physics in the MSSM with Complex Parameters***

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DESY

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Based mainly on work done in collaboration with Alison Fowler (IPPP Durham)

- Introduction
- Higgs phenomenology
- Renormalisation for complex parameters and unstable particles
- Interference effects and narrow-width approximation
- Conclusions

# *Introduction*

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## Prospects for SUSY searches at the LHC:

Global  $\chi^2$  fit in the CMSSM ( $m_{1/2}$ ,  $m_0$ ,  $A_0$  (GUT scale),  $\tan \beta$ ,  $\text{sign}(\mu)$  (weak scale)) and the NUHM1 ( $m_H^2$  as add. param.)

Fit includes (*MasterCode*, Markov-chain Monte Carlo sampling):

[*O. Buchmueller, R. Cavanaugh, A. De Roeck, J. Ellis, H. Flächer, S. Heinemeyer, G. Isidori, K. Olive, P. Paradisi, F. Ronga, G. W.* '08]

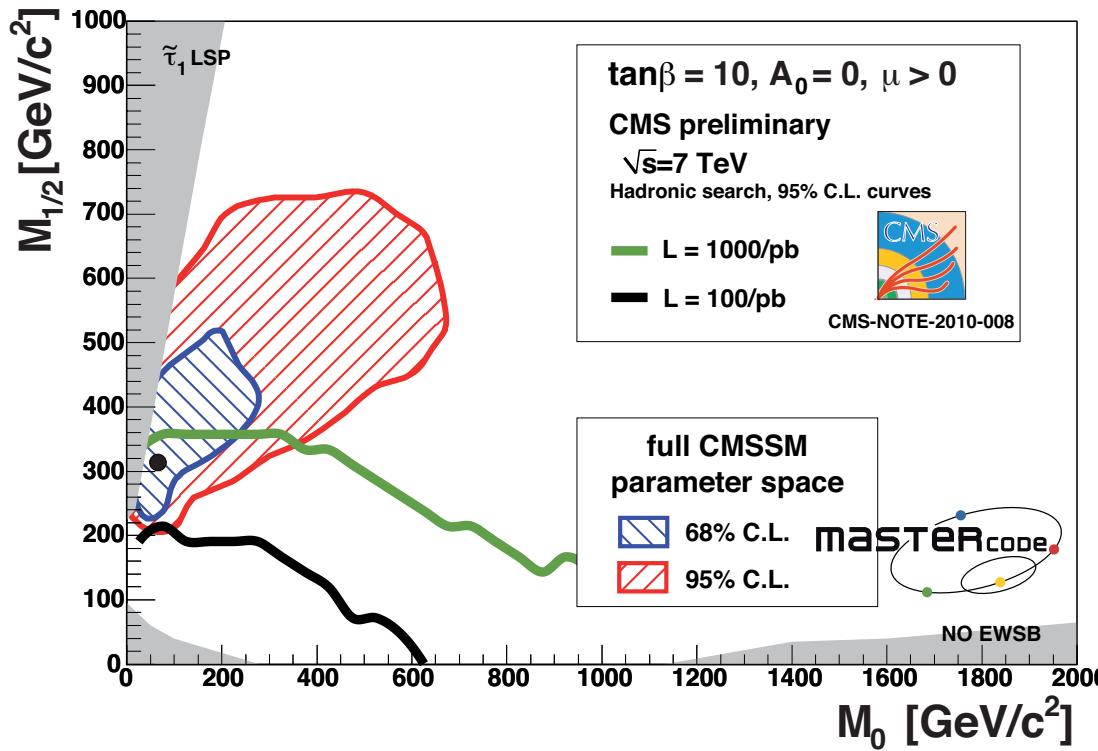
- Electroweak precision observables:  $M_W$ ,  $\sin^2 \theta_{\text{eff}}$ ,  $\Gamma_Z$ , ...
- + Cold dark matter (CDM) density (WMAP, ...),  
 $\Omega_{\text{CDM}} h^2 = 0.1099 \pm 0.0062$
- +  $(g - 2)_\mu$
- + BPO:  $\text{BR}(b \rightarrow s\gamma)$ ,  $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$ ,  $\text{BR}(B \rightarrow \tau\nu)$ , ...
- + Kaon decay data:  $\text{BR}(K \rightarrow \mu\nu)$ , ...

# *Predictions for the SUSY scale from precision data: CMSSM*

Comparison: preferred region in the  $m_0 - m_{1/2}$  plane vs. CMS

95% C.L. reach for 0.1, 1  $\text{fb}^{-1}$  at 7 TeV

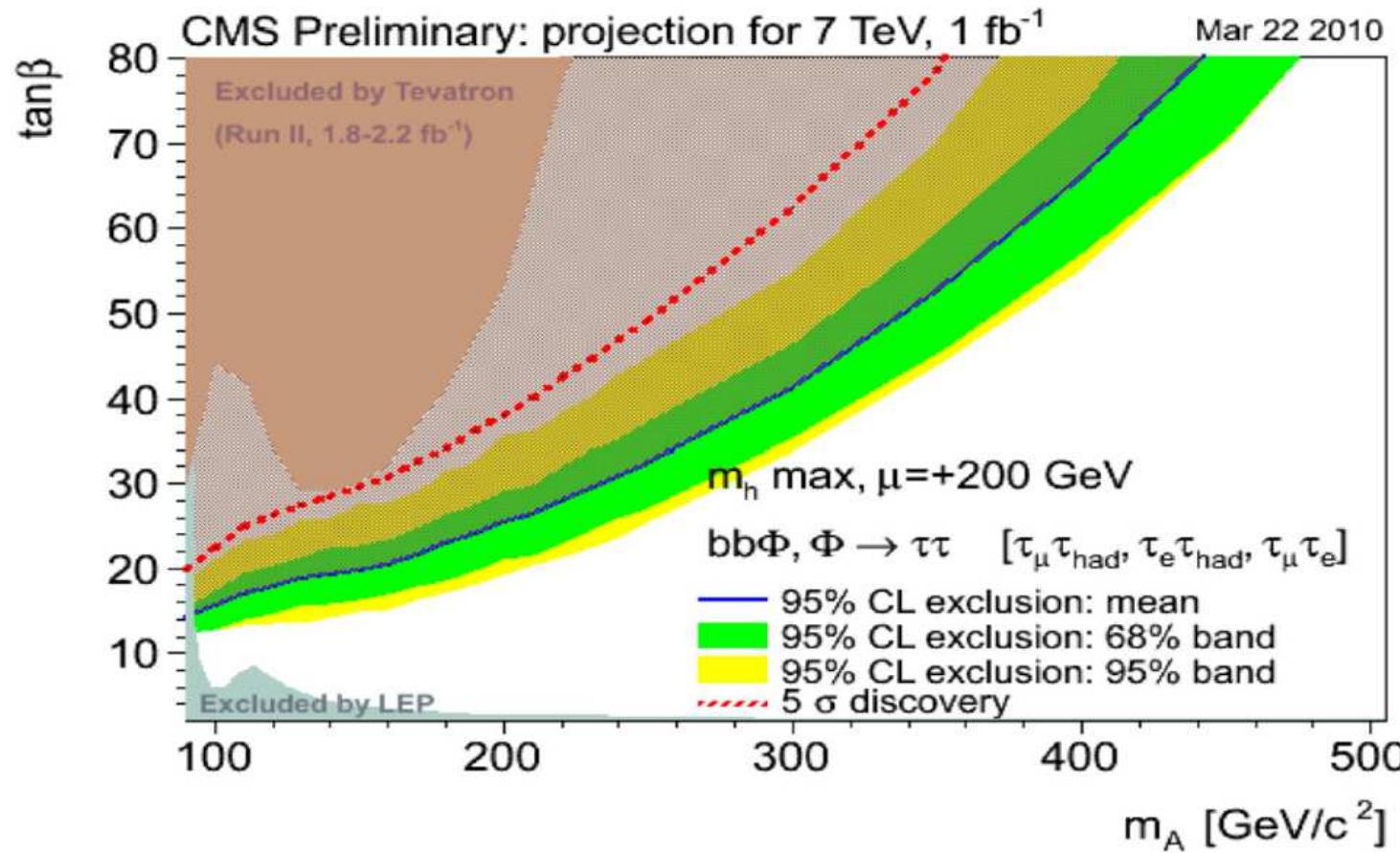
[O. Buchmueller, R. Cavanaugh, A. De Roeck, J. Ellis, H. Flächer, S. Heinemeyer, G. Isidori, K. Olive, P. Paradisi, F. Ronga, G. W. '10]



⇒ Good prospects for early discovery! Get hint in first run?

# Prospects for SUSY Higgs search at the LHC with $1 \text{ fb}^{-1}$ at 7 TeV

[CMS Collaboration '10]

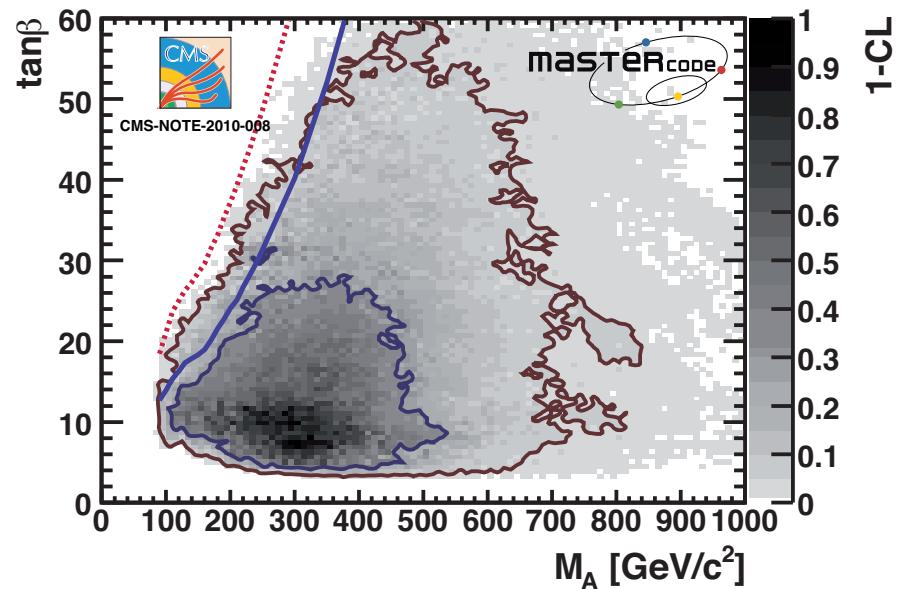
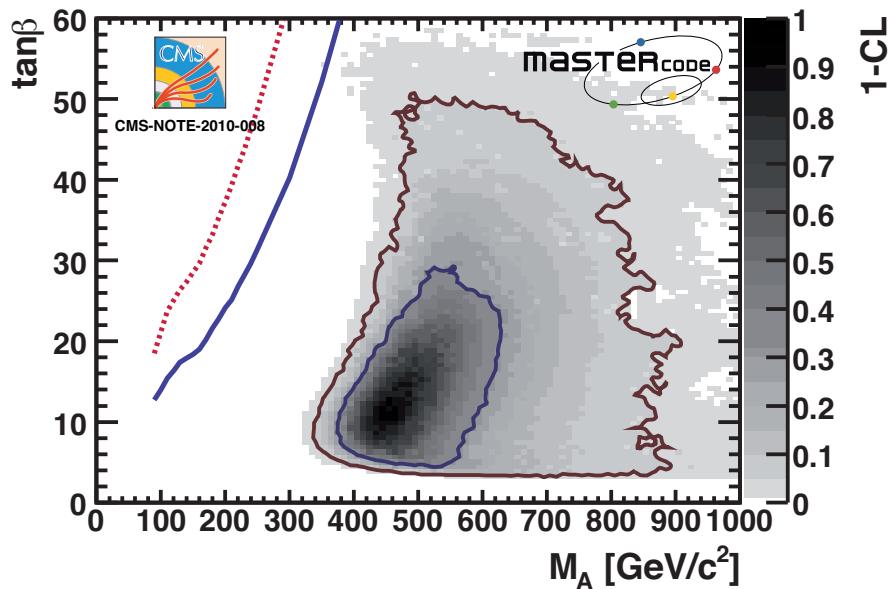


⇒ Higgs searches with early LHC data have a chance to discover the heavy MSSM Higgses  $H, A$  before a light SM-like Higgs  $h$  is found

# *Early LHC data: sensitivity for SUSY Higgs*

## *searches vs. preferred regions in CMSSM, NUHM1*

[O. Buchmueller, R. Cavanaugh, A. De Roeck, J. Ellis, H. Flächer, S. Heinemeyer, G. Isidori, K. Olive, F. Ronga, G. W. '10]



- ⇒ Not much hope in the CMSSM and NUHM1 with the first  $1 \text{ fb}^{-1}$  at 7 TeV
- ⇒ A hint in early searches could point towards a non-minimal model

## ***Beyond CMSSM, NUHM1, ... ?***

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At present global SUSY fits based on present data on precision observables focus on constrained SUSY models (CMSSM, NUHM1, ...); fits have limited sensitivity to the structure of less minimal models

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Recent result on  $A_{sl}^b$  from D0?

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At present global SUSY fits based on present data on precision observables focus on constrained SUSY models (CMSSM, NUHM1, ...); fits have limited sensitivity to the structure of less minimal models

**MSSM with complex parameters: additional sources for  $\mathcal{CP}$ -violation ( $\leftrightarrow$  asymmetry between matter and anti-matter in the Universe)**

Recent result on  $A_{sl}^b$  from D0?

Constraints on  $\mathcal{CP}$  phases from electric dipole moments affect mainly first two generations; corresponding phases are tightly constrained or mass scales have to be very heavy

Much weaker bounds on 3rd generation phases, gluino phase

## Higgs phenomenology

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MSSM Higgs potential contains two Higgs doublets:

$$V_H = m_1^2 H_{1i}^* H_{1i} + m_2^2 H_{2i}^* H_{2i} - \epsilon^{ij} (m_{12}^2 H_{1i} H_{2j} + m_{12}^{2*} H_{1i}^* H_{2j}^*)$$

$$+ \frac{1}{8} (g_1^2 + g_2^2) (H_{1i}^* H_{1i} - H_{2i}^* H_{2i})^2 + \frac{1}{2} g_2^2 |H_{1i}^* H_{2i}|^2$$

$$\begin{pmatrix} H_{11} \\ H_{12} \end{pmatrix} = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}}(\phi_1 - i\chi_1) \\ -\phi_1^- \end{pmatrix}$$

$$\begin{pmatrix} H_{21} \\ H_{22} \end{pmatrix} = e^{i\xi} \begin{pmatrix} \phi_2^+ \\ v_2 + \frac{1}{\sqrt{2}}(\phi_2 + i\chi_2) \end{pmatrix}$$

Complex phases  $\arg(m_{12}^2)$ ,  $\xi$  can be rotated away

⇒ Higgs sector is  $\mathcal{CP}$ -conserving at tree level

# ***Higher-order corrections in the MSSM Higgs sector***

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- Quartic couplings in the Higgs sector are given by the gauge couplings,  $g_1, g_2$  (SM: free parameter)  
 $\Leftrightarrow$  Upper bound on the lightest Higgs mass
  - Large higher-order corrections from Yukawa sector:  
 $\Rightarrow$  Leading corr.:  $\Delta m_h^2 \sim G_\mu m_t^4$   
Can be of  $\mathcal{O}(100\%)$
- $\Rightarrow$  Higher-order corrections are phenomenologically very important (constraints on parameter space from search limits / possible future measurements)
- Can induce  $\mathcal{CP}$ -violating effects

# *Higgs physics in the MSSM with complex parameters*

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Five physical states; tree level:  $h^0, H^0, A^0, H^\pm$

Complex parameters enter via (often large) loop corrections:

- $\mu$ : Higgsino mass parameter
- $A_{t,b,\tau}$ : trilinear couplings
- $M_{1,2}$ : gaugino mass parameter (one phase can be eliminated)
- $M_3$ : gluino mass  $m_{\tilde{g}}$  + complex phase

⇒  $\mathcal{CP}$ -violating mixing between neutral Higgs bosons  $h_1, h_2, h_3$

Lowest-order Higgs sector has two free parameters

⇒ choose  $\tan \beta \equiv \frac{v_2}{v_1}$ ,  $M_{H^\pm}$  as input parameters

# Higgs propagator-type corrections

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Mixing between  $h, H, A$

⇒ loop-corrected masses obtained from propagator matrix

$$\Delta_{hHA}(p^2) = -\left(\hat{\Gamma}_{hHA}(p^2)\right)^{-1}, \quad \hat{\Gamma}_{hHA}(p^2) = i [p^2 \mathbb{1} - M_n(p^2)]$$

where (up to sub-leading two-loop corrections)

$$M_n(p^2) = \begin{pmatrix} m_h^2 - \hat{\Sigma}_{hh}(p^2) & -\hat{\Sigma}_{hH}(p^2) & -\hat{\Sigma}_{hA}(p^2) \\ -\hat{\Sigma}_{hH}(p^2) & m_H^2 - \hat{\Sigma}_{HH}(p^2) & -\hat{\Sigma}_{HA}(p^2) \\ -\hat{\Sigma}_{hA}(p^2) & -\hat{\Sigma}_{HA}(p^2) & m_A^2 - \hat{\Sigma}_{AA}(p^2) \end{pmatrix}$$

$$\Rightarrow \text{Higgs propagators: } \Delta_{ii}(p^2) = \frac{i}{p^2 - m_i^2 + \hat{\Sigma}_{ii}^{\text{eff}}(p^2)}$$

# **Determination of the masses from the complex poles**

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$$\hat{\Sigma}_{ii}^{\text{eff}}(p^2) = \hat{\Sigma}_{ii}(p^2) - i \frac{2\hat{\Gamma}_{ij}(p^2)\hat{\Gamma}_{jk}(p^2)\hat{\Gamma}_{ki}(p^2) - \hat{\Gamma}_{ki}^2(p^2)\hat{\Gamma}_{jj}(p^2) - \hat{\Gamma}_{ij}^2(p^2)\hat{\Gamma}_{kk}(p^2)}{\hat{\Gamma}_{jj}(p^2)\hat{\Gamma}_{kk}(p^2) - \hat{\Gamma}_{jk}^2(p^2)}$$

Complex pole  $\mathcal{M}^2$  of each propagator is determined from

$$\mathcal{M}_i^2 - m_i^2 + \hat{\Sigma}_{ii}^{\text{eff}}(\mathcal{M}_i^2) = 0,$$

where

$$\mathcal{M}^2 = M^2 - iM\Gamma,$$

Expansion around the real part of the complex pole:

$$\hat{\Sigma}_{jk}(\mathcal{M}_{h_a}^2) \approx \hat{\Sigma}_{jk}(M_{h_a}^2) + i \operatorname{Im} [\mathcal{M}_{h_a}^2] \hat{\Sigma}'_{jk}(M_{h_a}^2)$$

$j, k = h, H, A, a = 1, 2, 3$

# **Wave function normalisation (finite) for amplitudes with external Higgs bosons**

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Finite wave-function normalisation factors ensure the correct on-shell properties of the S matrix

$$Z_h = \frac{1}{\left. \frac{\partial}{\partial p^2} \left( \frac{i}{\Delta_{hh}(p^2)} \right) \right|_{p^2=\mathcal{M}_{h_a}^2}}$$

$$Z_H = \frac{1}{\left. \frac{\partial}{\partial p^2} \left( \frac{i}{\Delta_{HH}(p^2)} \right) \right|_{p^2=\mathcal{M}_{h_b}^2}}$$

$$Z_A = \frac{1}{\left. \frac{\partial}{\partial p^2} \left( \frac{i}{\Delta_{AA}(p^2)} \right) \right|_{p^2=\mathcal{M}_{h_c}^2}}$$

$$Z_{hH} = \left. \frac{\Delta_{hH}}{\Delta_{hh}} \right|_{p^2=\mathcal{M}_{h_a}^2}$$

$$Z_{Hh} = \left. \frac{\Delta_{hH}}{\Delta_{HH}} \right|_{p^2=\mathcal{M}_{h_b}^2} \quad Z_{Ah} = \left. \frac{\Delta_{hA}}{\Delta_{AA}} \right|_{p^2=\mathcal{M}_{h_c}^2}$$

$$Z_{hA} = \left. \frac{\Delta_{hA}}{\Delta_{hh}} \right|_{p^2=\mathcal{M}_{h_a}^2}$$

$$Z_{HA} = \left. \frac{\Delta_{HA}}{\Delta_{HH}} \right|_{p^2=\mathcal{M}_{h_b}^2} \quad Z_{AH} = \left. \frac{\Delta_{HA}}{\Delta_{AA}} \right|_{p^2=\mathcal{M}_{h_c}^2}$$

# Wave function normalisation for amplitudes with external Higgs bosons

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WF constants can be written as (non-unitary) matrix  $\hat{\mathbf{Z}}$ ,

$$\hat{\mathbf{Z}} = \begin{pmatrix} \sqrt{Z_h} & \sqrt{Z_h}Z_{hH} & \sqrt{Z_h}Z_{hA} \\ \sqrt{Z_H}Z_{Hh} & \sqrt{Z_H} & \sqrt{Z_H}Z_{HA} \\ \sqrt{Z_A}Z_{Ah} & \sqrt{Z_A}Z_{AH} & \sqrt{Z_A} \end{pmatrix}, \quad \begin{pmatrix} \hat{\Gamma}_{h_a} \\ \hat{\Gamma}_{h_b} \\ \hat{\Gamma}_{h_c} \end{pmatrix} = \hat{\mathbf{Z}} \cdot \begin{pmatrix} \hat{\Gamma}_h \\ \hat{\Gamma}_H \\ \hat{\Gamma}_A \end{pmatrix}$$

Fulfils the conditions

$$\lim_{p^2 \rightarrow \mathcal{M}_{h_a}^2} -\frac{i}{p^2 - \mathcal{M}_{h_a}^2} \left( \hat{\mathbf{Z}} \cdot \hat{\Gamma}_2 \cdot \hat{\mathbf{Z}}^T \right)_{hh} = 1$$

$$\lim_{p^2 \rightarrow \mathcal{M}_{h_b}^2} -\frac{i}{p^2 - \mathcal{M}_{h_b}^2} \left( \hat{\mathbf{Z}} \cdot \hat{\Gamma}_2 \cdot \hat{\mathbf{Z}}^T \right)_{HH} = 1$$

$$\lim_{p^2 \rightarrow \mathcal{M}_{h_c}^2} -\frac{i}{p^2 - \mathcal{M}_{h_c}^2} \left( \hat{\mathbf{Z}} \cdot \hat{\Gamma}_2 \cdot \hat{\mathbf{Z}}^T \right)_{AA} = 1$$

# ***Expansion of the propagator around the complex pole***

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$$\Delta_{ii}(p^2) = \frac{i}{p^2 - \mathcal{M}_{h_i}^2} Z_i + \dots$$

where  $\mathcal{M}_{h_i}^2 = M_i^2 - i M_i \Gamma_i$

⇒ Propagator in the vicinity of the complex pole is given by a Breit–Wigner factor with constant width,

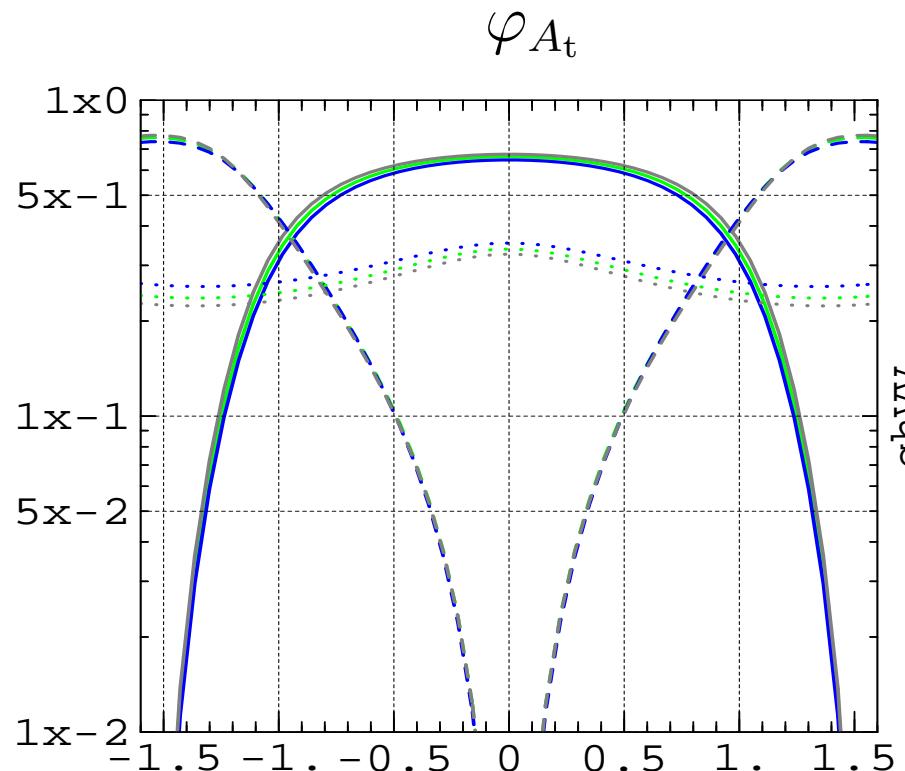
$$\Delta_i^{\text{BW}}(p^2) = \frac{i}{p^2 - M_i^2 + i M_i \Gamma_i}$$

weighted by a wave function normalisation factor  $Z_i$  evaluated at the complex pole

# ***Impact of complex phases***

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Example:  $g_{hVV}^2$  for  $h_1, h_2, h_3$ : [M. Frank, S. Heinemeyer, W. Hollik, G. W. '03]



full:  $h_1$ , dashed:  $h_2$ , dotted:  $h_3$

Parameters:

$M_{\text{SUSY}} = 500 \text{ GeV}$ ,

$M_2 = 500 \text{ GeV}$ ,

$\mu = 2000 \text{ GeV}$ ,

$|A_t| = 1000 \text{ GeV}$ ,

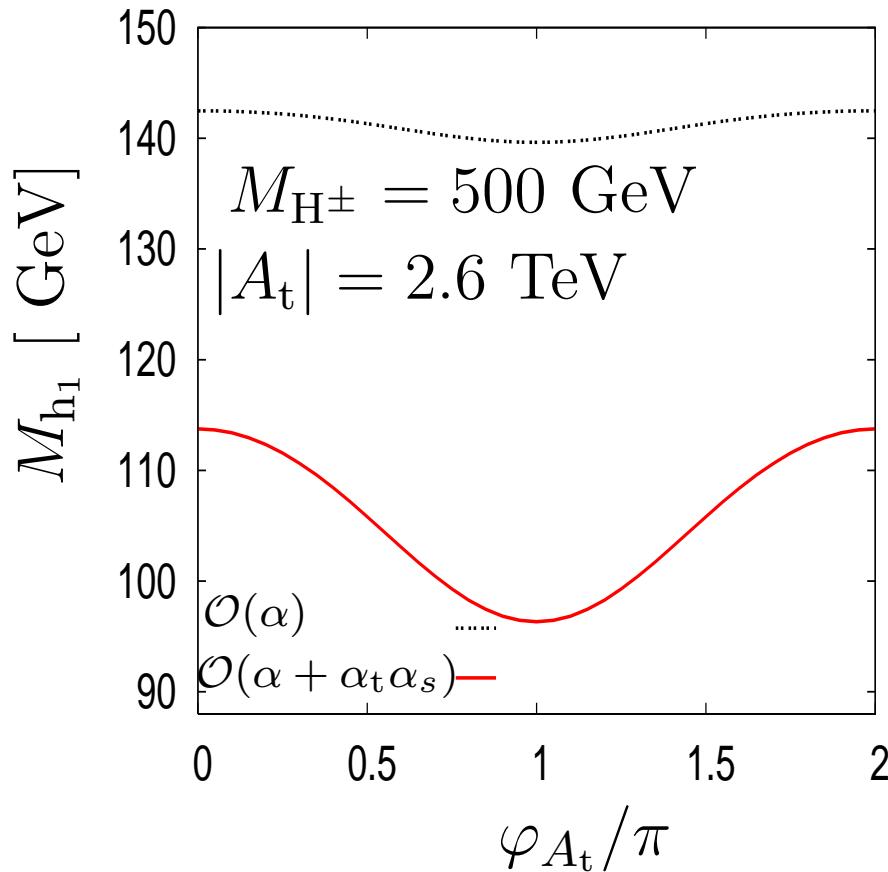
$M_{H^\pm} = 150 \text{ GeV}$ ,  $\tan \beta = 5$

⇒ Complex phases can have large effects on Higgs couplings

# ***Dependence of prediction for $M_{h_1}$ on $\varphi_{A_t}$ : one-loop vs. two-loop***

[S. Heinemeyer, W. Hollik, H. Rzehak, G. W. '07]

$\mu = 1 \text{ TeV}, \tan \beta = 10$



⇒ Two-loop corrections significantly enhance the effects of the complex phase  $\varphi_{A_t}$ , sizable effects for large  $|A_t|$

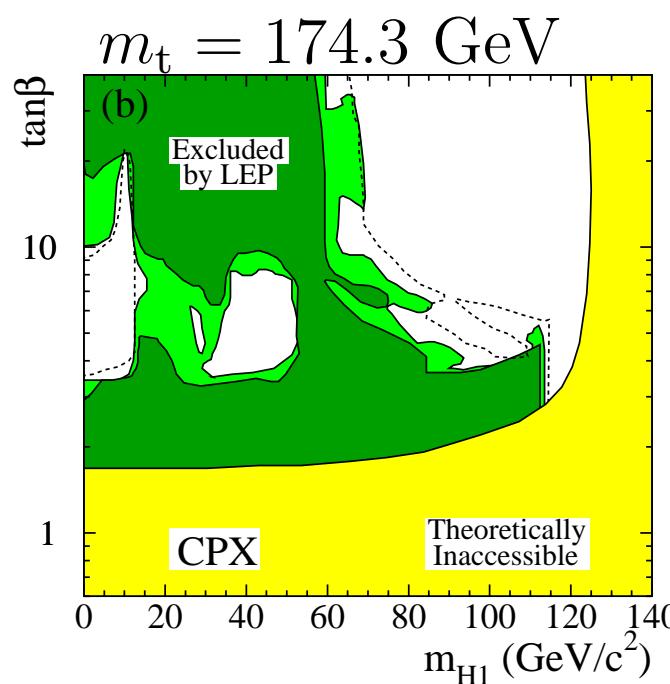
# **MSSM with complex parameters: a very light SUSY Higgs?**

MSSM with  $\mathcal{CP}$ -violating phases (CPX scenario):

Light Higgs,  $h_1$ : strongly suppressed  $h_1 VV$  couplings

Second-lightest Higgs,  $h_2$ , possibly within LEP reach (with reduced  $VVh_2$  coupling),  $h_3$  beyond LEP reach

Large  $\text{BR}(h_2 \rightarrow h_1 h_1) \Rightarrow$  difficult final state

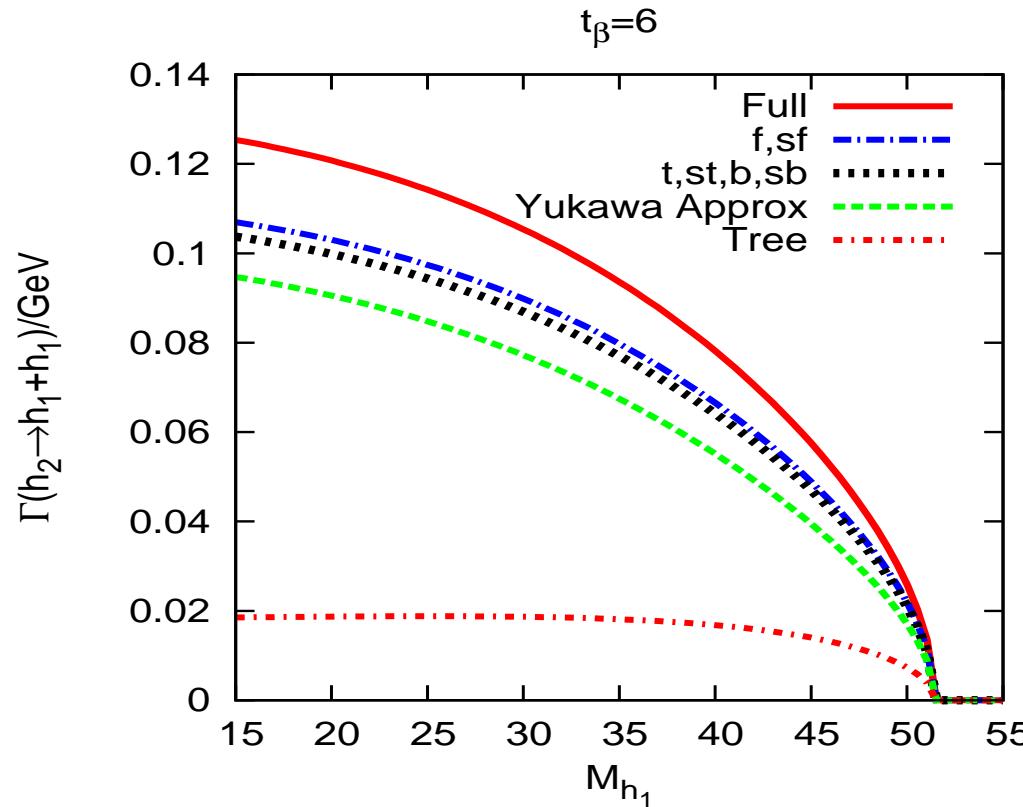


[LEP Higgs WG '06]

⇒ Light SUSY Higgs not ruled out!

# ***Impact of higher-order corrections on prediction for $\Gamma(h_2 \rightarrow h_1 h_1)$***

Complete 1-loop result for  $(h_2 h_1 h_1)$  vertex contribution in the MSSM with complex parameters [K. Williams, G. W. '07]  
+ 2-loop propagator corrections; CPX benchmark scenario  
[S. Heinemeyer, W. Hollik, H. Rzehak, G. W. '07]

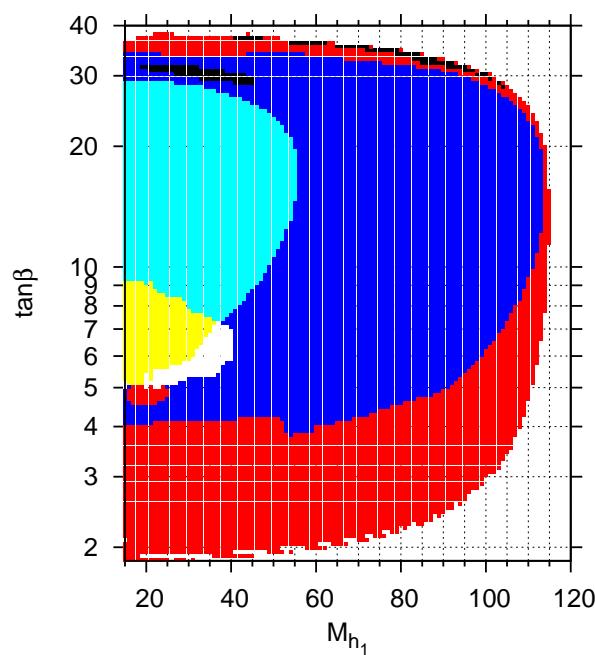


⇒ Huge effect from corrections to genuine  $(h_2 h_1 h_1)$  vertex

# *Analysis of LEP coverage with improved theoretical prediction*

*HiggsBounds [P. Bechtle, O. Brein, S. Heinemeyer, G. W., K. Williams '08]*

Use cross section limits (expected and observed) from LEP and the Tevatron; determine for every parameter point the search channel with the highest statistical sensitivity for setting an exclusion; comparison of prediction for this channel with observed limit yields 95% C.L. exclusion contour



## Channels:

$$(\blacksquare) = (h_1 Z) \rightarrow (b\bar{b}Z)$$

$$(\square) = (h_2 Z) \rightarrow (b\bar{b}Z)$$

$$(\square) = (h_2 Z) \rightarrow (h_1 h_1 Z) \rightarrow (b\bar{b}b\bar{b}Z)$$

$$(\blacksquare) = (h_2 h_1) \rightarrow (b\bar{b}b\bar{b})$$

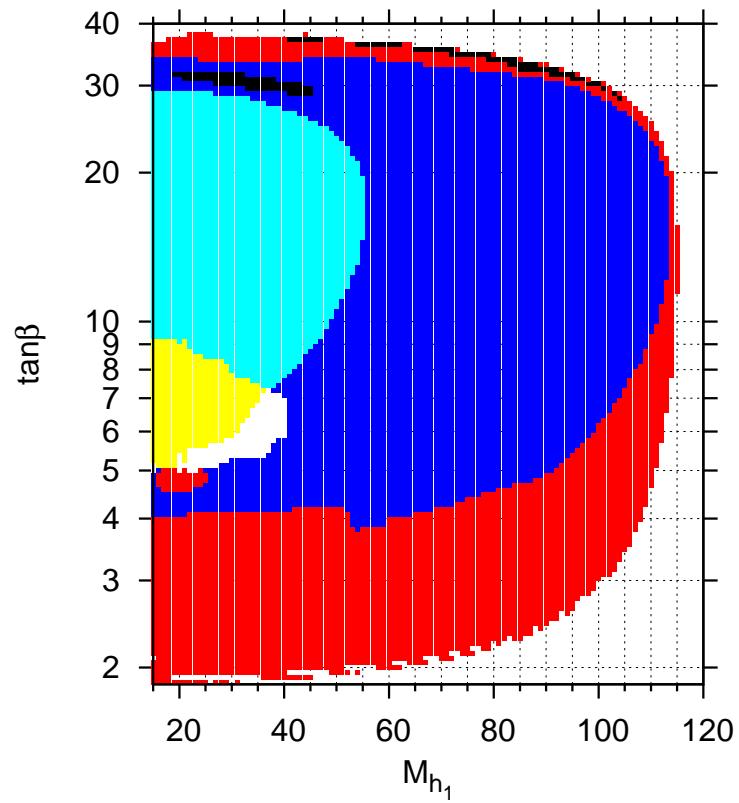
$$(\blacksquare) = (h_2 h_1) \rightarrow (h_1 h_1 h_1) \rightarrow (b\bar{b}b\bar{b}b\bar{b})$$

$$(\blacksquare) = \text{other channels}$$

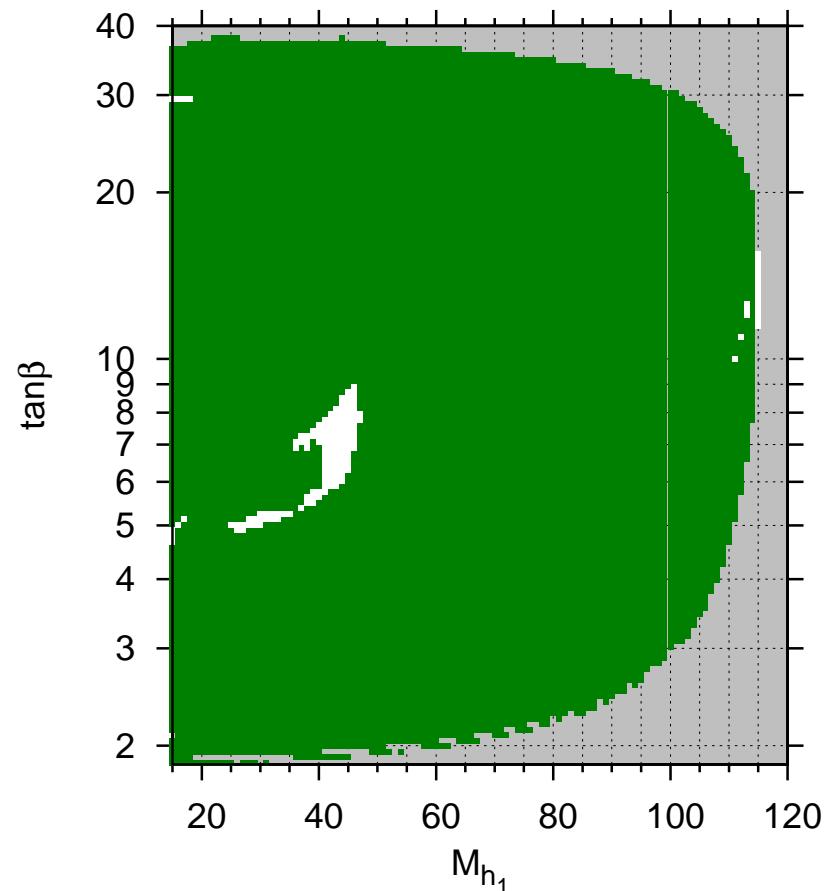
# **Impact on exclusion bounds from the LEP Higgs searches, CPX scenario, $m_t = 170.9$ GeV**

Channels (*HiggsBounds*)

( $\square$ ) :  $(h_2 Z) \rightarrow (h_1 h_1 Z) \rightarrow (b\bar{b} b\bar{b} Z)$



Excluded region from LEP,  
95% C.L. [K. Williams, G. W. '07]



- ⇒ Confirmation of the “hole” in the LEP coverage
- ⇒ Very light Higgs boson is not excluded

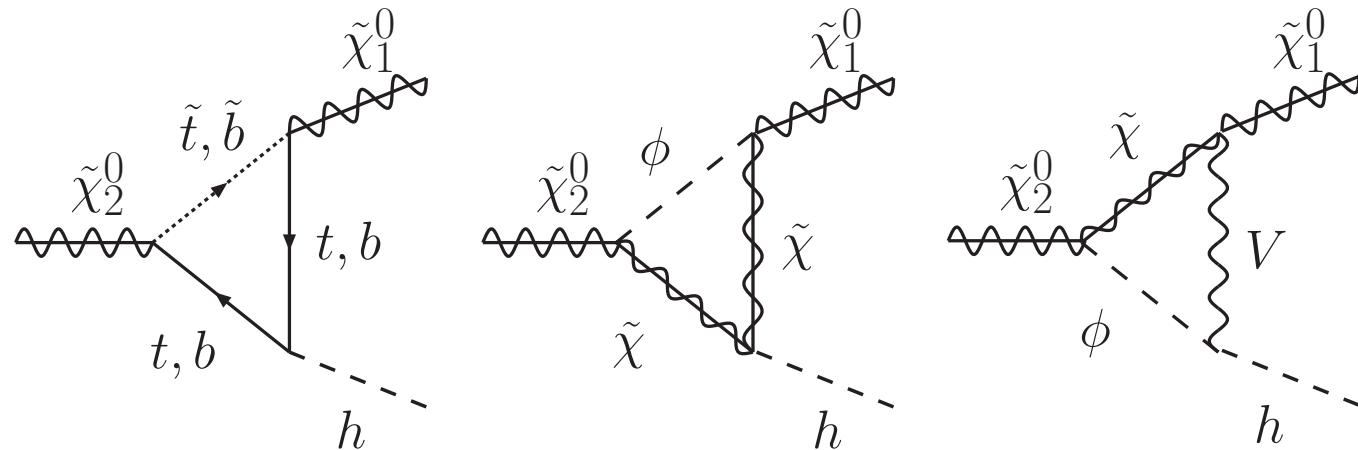
# Higgs production in SUSY cascade decays

SUSY cascade decays could be a promising Higgs source

E.g.  $\mathcal{CP}$ -violating scenario: very light Higgs,  $M_{h_1} \approx 40$  GeV  
not excluded by LEP, difficult to cover with standard search channels at the LHC

$\Rightarrow \tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 h$  can dominate over  $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 l\bar{l}$

[A. Fowler, G. W. '09]



$\Rightarrow$  CPX scenario: 13% of the gluinos decay into  $h_1$

# *Renormalisation for complex parameters and unstable particles*

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Occurrence of imaginary parts:

- From complex parameters
- From absorptive parts of loop integrals  
 $\leftrightarrow$  unstable particles

$\Rightarrow$  MSSM with complex parameters:

absorptive parts of loop integrals can contribute to real part  
of 1-loop quantities

## ***Comparison: Standard Model case***

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- CKM matrix renormalisation: intense discussion in the literature (gauge invariance, . . . )
- Mass renormalisation: difference between mass renormalisation according to complex pole of the propagator / pole of the real part of the propagator occurs at 2-loop order

Need to define the mass according to the real part of the complex pole in order to obtain a gauge-invariant mass counterterm

# ***Renormalisation in the chargino / neutralino sector of the MSSM with complex parameters***

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Allow field renormalisations to be different for in- and outgoing fermions:

$$\begin{aligned}\omega_L \tilde{\chi}_i^- &\rightarrow (1 + \frac{1}{2} \delta Z_-^L)_{ij} \omega_L \tilde{\chi}_j^-, & \overline{\tilde{\chi}_i^-} \omega_R &\rightarrow \overline{\tilde{\chi}_i^-} (1 + \frac{1}{2} \delta \bar{Z}_-^L)_{ij} \omega_R, \\ \omega_R \tilde{\chi}_i^- &\rightarrow (1 + \frac{1}{2} \delta Z_-^R)_{ij} \omega_R \tilde{\chi}_j^-, & \overline{\tilde{\chi}_i^-} \omega_L &\rightarrow \overline{\tilde{\chi}_i^-} (1 + \frac{1}{2} \delta \bar{Z}_-^R)_{ij} \omega_L,\end{aligned}$$

In  $\mathcal{CP}$ -conserving case: can choose a scheme where hermiticity relation holds (up to purely imaginary terms that do not contribute to squared matrix elements at 1-loop)

$$\delta \bar{Z}_{ij} = \delta Z_{ij}^\dagger$$

Decomposition of fermion self-energies:

$$\Sigma_{ij}(p^2) = p' \omega_L \Sigma_{ij}^L(p^2) + p' \omega_R \Sigma_{ij}^R(p^2) + \omega_L \Sigma_{ij}^{SL}(p^2) + \omega_R \Sigma_{ij}^{SR}(p^2)$$

# **Field renormalisation conditions (1-loop)**

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Vanishing of off-diagonal contributions and unit residues:

$$\hat{\Gamma}_{ij}^{(2)} \tilde{\chi}_i(p) \Big|_{p^2=m_{\tilde{\chi}_j}^2} = 0, \quad \bar{\tilde{\chi}}_i(p) \hat{\Gamma}_{ij}^{(2)} \Big|_{p^2=m_{\tilde{\chi}_i}^2} = 0$$
$$\lim_{p^2 \rightarrow m_{\tilde{\chi}_i}^2} \frac{1}{p' - m_{\tilde{\chi}_i}} \hat{\Gamma}_{ii}^{(2)} \tilde{\chi}_i(p) = i \tilde{\chi}_i, \quad \lim_{p^2 \rightarrow m_{\tilde{\chi}_i}^2} \bar{\tilde{\chi}}_i(p) \hat{\Gamma}_{ii}^{(2)} \frac{1}{p' - m_{\tilde{\chi}_i}} = i \bar{\tilde{\chi}}_i$$

**Additional conditions needed for the general case with  $\mathcal{CP}$  violation:**

Loop-corrected propagator should have the same Lorentz structure in the on-shell as at tree level ( $\rightarrow$  vanishing of  $\gamma_5$  contributions)

$$\hat{\Sigma}_{ii}^{SL}(m_{\tilde{\chi}_i}^2) = \hat{\Sigma}_{ii}^{SR}(m_{\tilde{\chi}_i}^2)$$

**Exploit additional freedom:**  $\delta Z_{ii}^R - \delta \bar{Z}_{ii}^R = \delta \bar{Z}_{ii}^L - \delta Z_{ii}^L$

# **Result for chargino field renormalisation constants (left-handed, diagonal)**

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$$\begin{aligned}\delta Z_{-,ii}^L = & - \Sigma_{-,ii}^L(m_{\tilde{\chi}_i^\pm}^2) - m_{\tilde{\chi}_i^\pm}^2 [\Sigma_{-,ii}^{L'}(m_{\tilde{\chi}_i^\pm}^2) + \Sigma_{-,ii}^{R'}(m_{\tilde{\chi}_i^\pm}^2)] \\ & - m_{\tilde{\chi}_i^\pm} [\Sigma_{-,ii}^{SL'}(m_{\tilde{\chi}_i^\pm}^2) + \Sigma_{-,ii}^{SR'}(m_{\tilde{\chi}_i^\pm}^2)] \\ & + \frac{1}{2m_{\tilde{\chi}_i^\pm}} [\Sigma_{-,ii}^{SL}(m_{\tilde{\chi}_i^\pm}^2) - \Sigma_{-,ii}^{SR}(m_{\tilde{\chi}_i^\pm}^2) + (V\delta X^\dagger U^T)_{ii} - (U^* \delta X V^\dagger)_{ii}]\end{aligned}$$

$$\begin{aligned}\delta \bar{Z}_{-,ii}^L = & - \Sigma_{-,ii}^L(m_{\tilde{\chi}_i^\pm}^2) - m_{\tilde{\chi}_i^\pm}^2 [\Sigma_{-,ii}^{L'}(m_{\tilde{\chi}_i^\pm}^2) + \Sigma_{-,ii}^{R'}(m_{\tilde{\chi}_i^\pm}^2)] \\ & - m_{\tilde{\chi}_i^\pm} [\Sigma_{-,ii}^{SL'}(m_{\tilde{\chi}_i^\pm}^2) + \Sigma_{-,ii}^{SR'}(m_{\tilde{\chi}_i^\pm}^2)] \\ & - \frac{1}{2m_{\tilde{\chi}_i^\pm}} [\Sigma_{-,ii}^{SL}(m_{\tilde{\chi}_i^\pm}^2) - \Sigma_{-,ii}^{SR}(m_{\tilde{\chi}_i^\pm}^2) + (V\delta X^\dagger U^T)_{ii} - (U^* \delta X V^\dagger)_{ii}]\end{aligned}$$

# **Result for chargino field renormalisation constants (left-handed, off-diagonal)**

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$$\begin{aligned} \delta Z_{-,ij}^L = & \frac{2}{m_{\tilde{\chi}_i^\pm}^2 - m_{\tilde{\chi}_j^\pm}^2} \left[ m_{\tilde{\chi}_j^\pm}^2 \Sigma_{-,ij}^L(m_{\tilde{\chi}_j^\pm}^2) + m_{\tilde{\chi}_i^\pm} m_{\tilde{\chi}_j^\pm} \Sigma_{-,ij}^R(m_{\tilde{\chi}_j^\pm}^2) \right. \\ & + m_{\tilde{\chi}_i^\pm} \Sigma_{-,ij}^{SL}(m_{\tilde{\chi}_j^\pm}^2) + m_{\tilde{\chi}_j^\pm} \Sigma_{-,ij}^{SR}(m_{\tilde{\chi}_j^\pm}^2) \\ & \left. - m_{\tilde{\chi}_i^\pm} (U^* \delta X V^\dagger)_{ij} - m_{\tilde{\chi}_j^\pm} (V \delta X^\dagger U^T)_{ij} \right] \end{aligned}$$

$$\begin{aligned} \delta \bar{Z}_{-,ij}^L = & \frac{2}{m_{\tilde{\chi}_j^\pm}^2 - m_{\tilde{\chi}_i^\pm}^2} \left[ m_{\tilde{\chi}_i^\pm}^2 \Sigma_{-,ij}^L(m_{\tilde{\chi}_i^\pm}^2) + m_{\tilde{\chi}_i^\pm} m_{\tilde{\chi}_j^\pm} \Sigma_{-,ij}^R(m_{\tilde{\chi}_i^\pm}^2) \right. \\ & + m_{\tilde{\chi}_i^\pm} \Sigma_{-,ij}^{SL}(m_{\tilde{\chi}_i^\pm}^2) + m_{\tilde{\chi}_j^\pm} \Sigma_{-,ij}^{SR}(m_{\tilde{\chi}_i^\pm}^2) \\ & \left. - m_{\tilde{\chi}_i^\pm} (U^* \delta X V^\dagger)_{ij} - m_{\tilde{\chi}_j^\pm} (V \delta X^\dagger U^T)_{ij} \right] \end{aligned}$$

Additional rel. for neutralinos (Majorana part.):  $\delta Z_{0,ij}^{L/R} = \delta \bar{Z}_{0,ji}^{R/L}$

# *Chargino / neutralino field renormalisation in the MSSM with complex parameters*

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- In general  $\delta\bar{Z}_{ij} \neq \delta Z_{ij}^\dagger$ 
  - ⇒ Hermiticity relation does not hold, but it can be shown that the CPT theorem is fulfilled

# *Chargino / neutralino field renormalisation in the MSSM with complex parameters*

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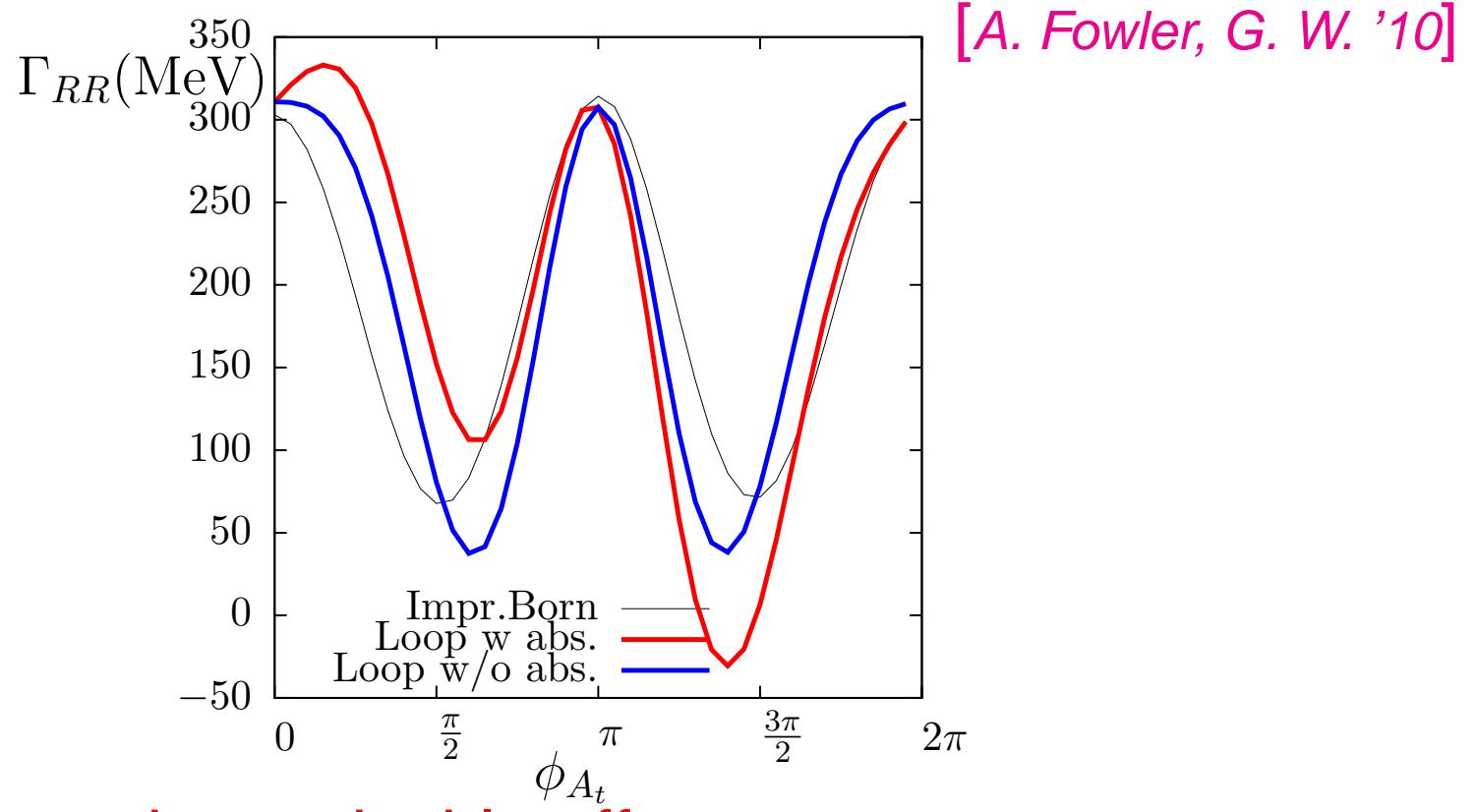
- In general  $\delta\bar{Z}_{ij} \neq \delta Z_{ij}^\dagger$ 
  - ⇒ Hermiticity relation does not hold, but it can be shown that the CPT theorem is fulfilled
- If absorptive parts of the loop integrals are discarded from the field renormalisation constants
  - ⇒ Hermiticity relation is restored, but correct on-shell properties are spoiled
  - ⇒ Additional mixing contributions needed to obtain the correct on-shell properties

# **Numerical relevance of absorptive parts**

Consider Higgs decays into neutralinos at the LHC:  $h_2 \rightarrow \tilde{\chi}_3^0 \tilde{\chi}_2^0$   
⇒ Sensitivity at the LHC in search for 4-lepton final states

[M. Bisset et al. '09]

Partial decay width into right-handed neutralinos:



[A. Fowler, G. W. '10]

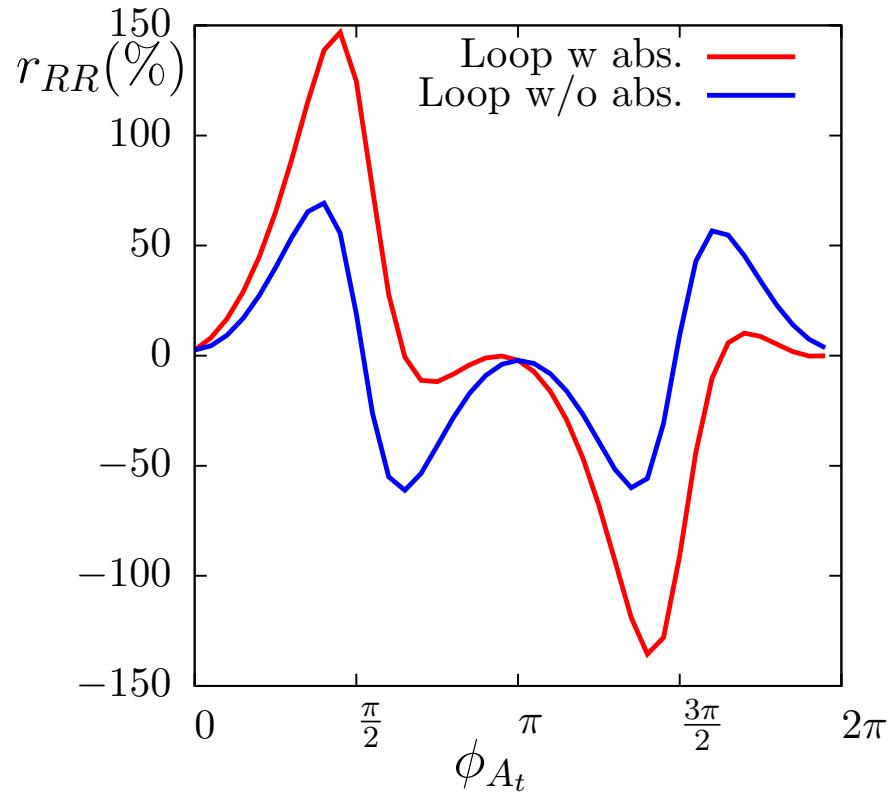
⇒ Absorptive parts have sizable effect

# ***Relative size of the loop corrections***

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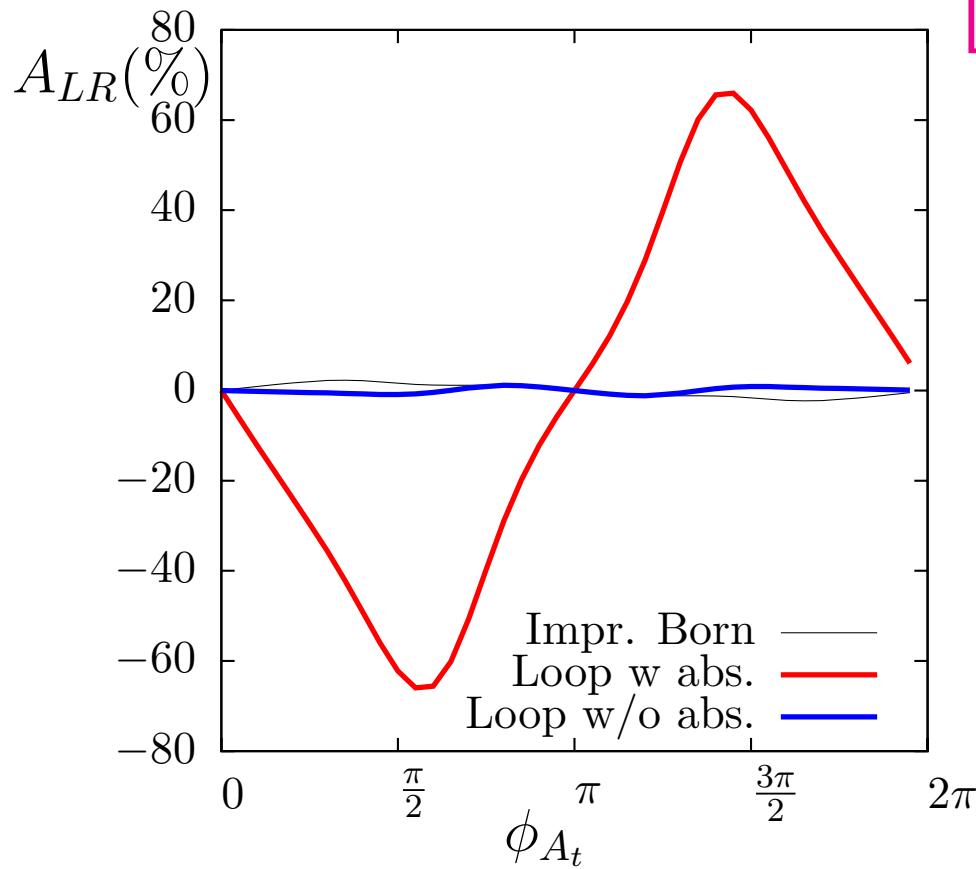
$M_{H^\pm} = 800 \text{ GeV}$ ,  $\mu = 500 \text{ GeV}$ ,  $M_2 = 200 \text{ GeV}$ ,  $m_{\tilde{g}} = 1 \text{ TeV}$ ,  
 $M_{\text{SUSY}} = 500 \text{ GeV}$ ,  $M_{\tilde{l}} = 200 \text{ GeV}$ ,  $M_{\tilde{\tau}} = 300 \text{ GeV}$ ,  
 $|A_t| = 1200 \text{ GeV}$ ,  $\tan \beta = 20$

[A. Fowler, G. W. '10]



⇒ Genuine vertex corrections are large, incorporation of absorptive parts is crucial

# $\mathcal{CP}$ -violating asymmetry



[A. Fowler, G. W. '10]

⇒ Large asymmetries possible

Condition for sizable asymmetries:

$\mathcal{CP}$  violation (complex parameters) + absorptive parts

# **Interference effects and narrow-width approximation**

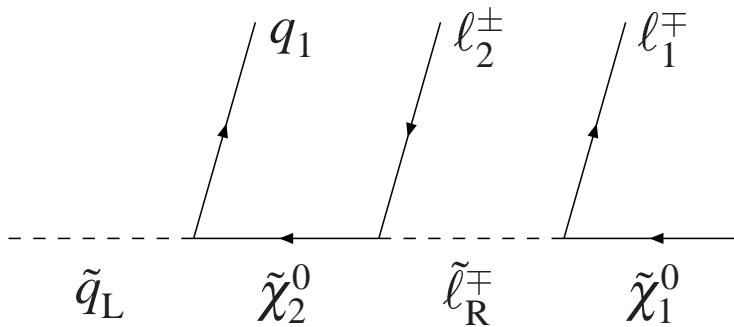
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The usual factorisation of processes into

(production cross section)  $\times$  (branching ratio)

is based on the **narrow width approximation**

- $\sigma(gg \rightarrow h_i) \times \text{BR}(h_i \rightarrow \tau^+ \tau^-)$
- SUSY cascade decays



written as  $\sigma_{\text{prod}} \times \text{BR}_1 \times \text{BR}_2 \dots$



...

# Interference effects

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For cases with mass degeneracies

$$|M_i - M_j| \lesssim \Gamma_i, \Gamma_j$$

⇒ Narrow width approximation no longer valid  
Resonance-type behaviour possible

In MSSM with complex parameters: mass degeneracy between  $M_{h_2}$ ,  $M_{h_3}$  occurs generically for  $M_{H^\pm} \gg M_Z$ , large mixing effects possible

[A. Pilaftsis '97, '98] [J. Ellis, J.S. Lee, A. Pilaftsis '04] [S.Y. Choi, J. Kalinowski, Y. Liao, P. Zerwas '05] [M. Frank, T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak, G. W. '07] [H. Dreiner, O. Kittel, F. von der Pahlen '07] [D. Berdine, N. Kauer, D. Rainwater '07] [M. Gigg, P. Richardson '08] [G. Cacciapaglia, A. Deandrea, S. De Curtis '09] [...]

# **Treatment of interference effects**

## **example: $2 \times 2$ mixing case**

Write total matrix element in terms of production and decay matrix elements and resonant Breit–Wigner type propagators:

$$\begin{aligned}\mathcal{M}(ab \rightarrow cef) = & \mathcal{M}(ab \rightarrow ch_1) \frac{1}{q^2 - M_{h_1}^2 + iM_{h_1}\Gamma_{h_1}} \mathcal{M}(h_1 \rightarrow ef) \\ & + \mathcal{M}(ab \rightarrow ch_2) \frac{1}{q^2 - M_{h_2}^2 + iM_{h_2}\Gamma_{h_2}} \mathcal{M}(h_2 \rightarrow ef)\end{aligned}$$

$$\begin{aligned}\sigma(ab \rightarrow cef) = & \frac{1}{2\pi} \frac{1}{2\lambda^{\frac{1}{2}}(s, m_a^2, m_b^2)} \int dq^2 d\text{lips}(s; p_c, q) d\text{lips}(q; p_e, p_f) \times \\ & \left( \frac{|\mathcal{M}(ab \rightarrow ch_1)|^2 |\mathcal{M}(h_1 \rightarrow ef)|^2}{(q^2 - M_{h_1}^2)^2 + M_{h_1}^2 \Gamma_{h_1}^2} + \frac{|\mathcal{M}(ab \rightarrow ch_2)|^2 |\mathcal{M}(h_2 \rightarrow ef)|^2}{(q^2 - M_{h_2}^2)^2 + M_{h_2}^2 \Gamma_{h_2}^2} \right. \\ & \left. + 2\text{Re} \left[ \frac{\mathcal{M}(ab \rightarrow ch_1) \mathcal{M}^*(ab \rightarrow ch_2) \mathcal{M}(h_1 \rightarrow ef) \mathcal{M}^*(h_2 \rightarrow ef)}{(q^2 - M_{h_1}^2 + iM_{h_1}\Gamma_{h_1})(q^2 - M_{h_2}^2 - iM_{h_2}\Gamma_{h_2})} \right] \right)\end{aligned}$$

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## **example: $2 \times 2$ mixing case**

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Observation: as long as one is sufficiently far away from thresholds, it works well to use an **on-shell approximation**

$$\Rightarrow \sigma(ab \rightarrow cef) \equiv \sigma_{\text{tot}} = \sigma_1 \times \text{BR}_1 + \sigma_2 \times \text{BR}_2 \\ + C \left( 2\text{Re} \int dq^2 \frac{1}{(q^2 - M_{h_1}^2 + iM_{h_1}\Gamma_{h_1})(q^2 - M_{h_2}^2 - iM_{h_2}\Gamma_{h_2})} \right)$$

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$\Rightarrow$  Interference contribution is obtained from **coefficient  $C$** , which contains the dependence on the on-shell matrix elements, and a **universal integral factor over Breit–Wigner propagators**

## ***Contribution of on-shell matrix elements***

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$$\frac{\mathcal{M}(ab \rightarrow ch_1)\mathcal{M}^*(ab \rightarrow ch_2)\mathcal{M}(h_1 \rightarrow ef)\mathcal{M}^*(h_2 \rightarrow ef)}{|\mathcal{M}(ab \rightarrow ch_1)|^2 |\mathcal{M}(h_1 \rightarrow ef)|^2} = x_1$$

$$\frac{\mathcal{M}(ab \rightarrow ch_1)\mathcal{M}^*(ab \rightarrow ch_2)\mathcal{M}(h_1 \rightarrow ef)\mathcal{M}^*(h_2 \rightarrow ef)}{|\mathcal{M}(ab \rightarrow ch_2)|^2 |\mathcal{M}(h_2 \rightarrow ef)|^2} = x_2$$

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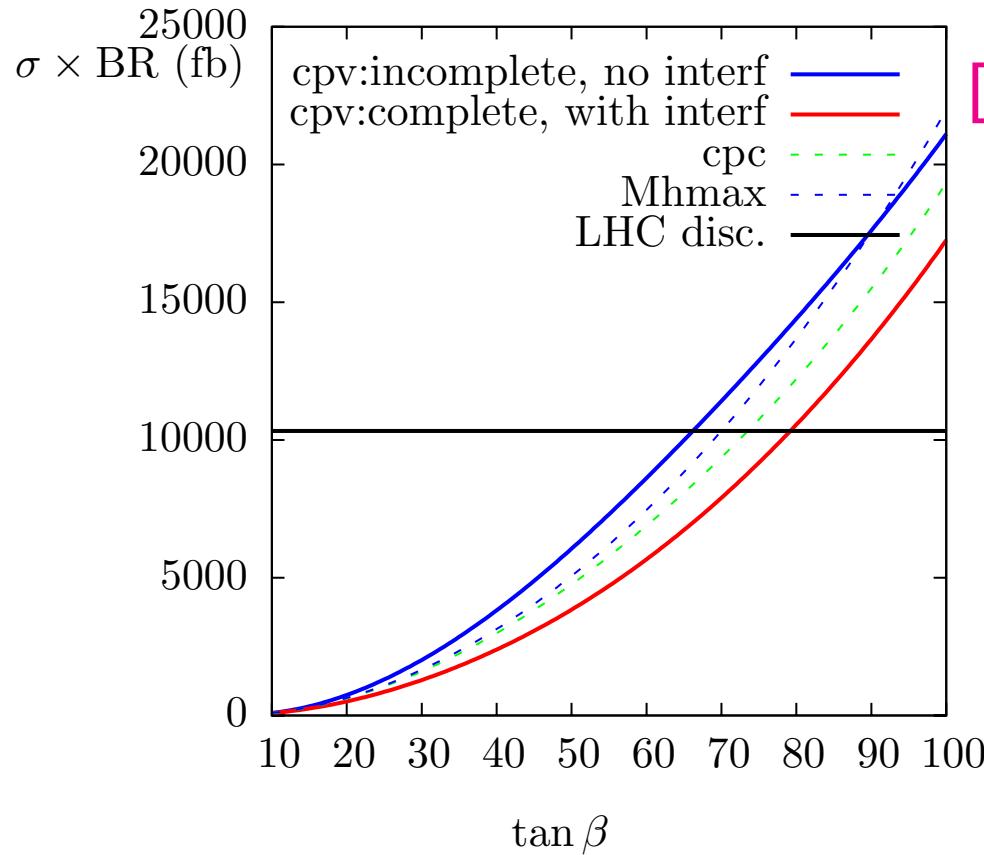
With appropriate approximation for  $x_1, x_2$

- ⇒ Result can be expressed in terms of cross sections, branching ratios and a universal integral factor over Breit–Wigner propagators
- ⇒ Convenient way to incorporate interference effects

# ***Impact of interference effects on Higgs***

## ***production processes at Tevatron and LHC***

CPV scenario:  $|A_t| = 1200$  GeV,  $\phi_{A_t} = \phi_{A_b} = \phi_{A_\tau} = \pi/5$ ,  
 $M_{H^\pm} = 340$  GeV,  $M_{\text{SUSY}} = 500$  GeV,  $\mu = 200$  GeV,  $M_2 = 200$  GeV,  
 $m_{\tilde{g}} = 1$  TeV

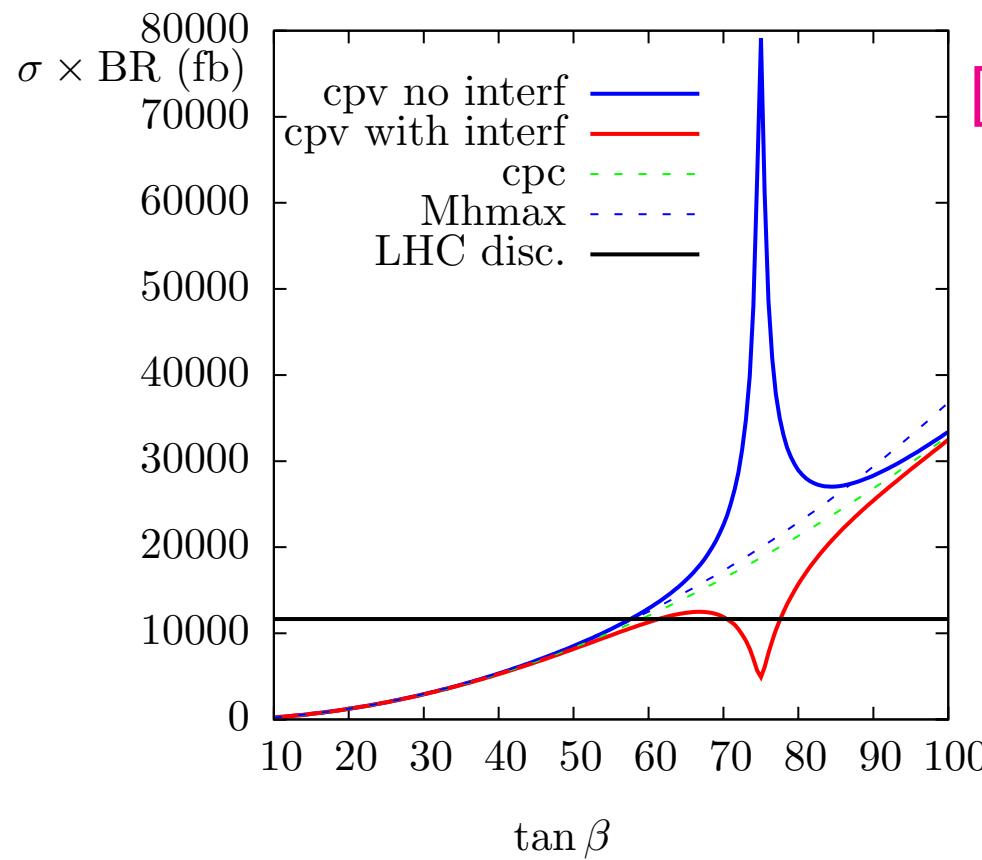


[A. Fowler, G. W. '10]

⇒ Interference effect yields sizable reduction of  $\sigma \times \text{BR}$   
Can be important for interpretation of Higgs search results

# "Resonance-type" scenario

CPV scenario:  $|A_t| = 800$  GeV,  $\phi_{A_t} = \phi_{A_b} = \phi_{A_\tau} = \pi/30$ ,  
 $M_{H^\pm} = 300$  GeV,  $M_{\text{SUSY}} = 500$  GeV,  $\mu = 200$  GeV,  $M_2 = 200$  GeV,  
 $m_{\tilde{g}} = 1$  TeV



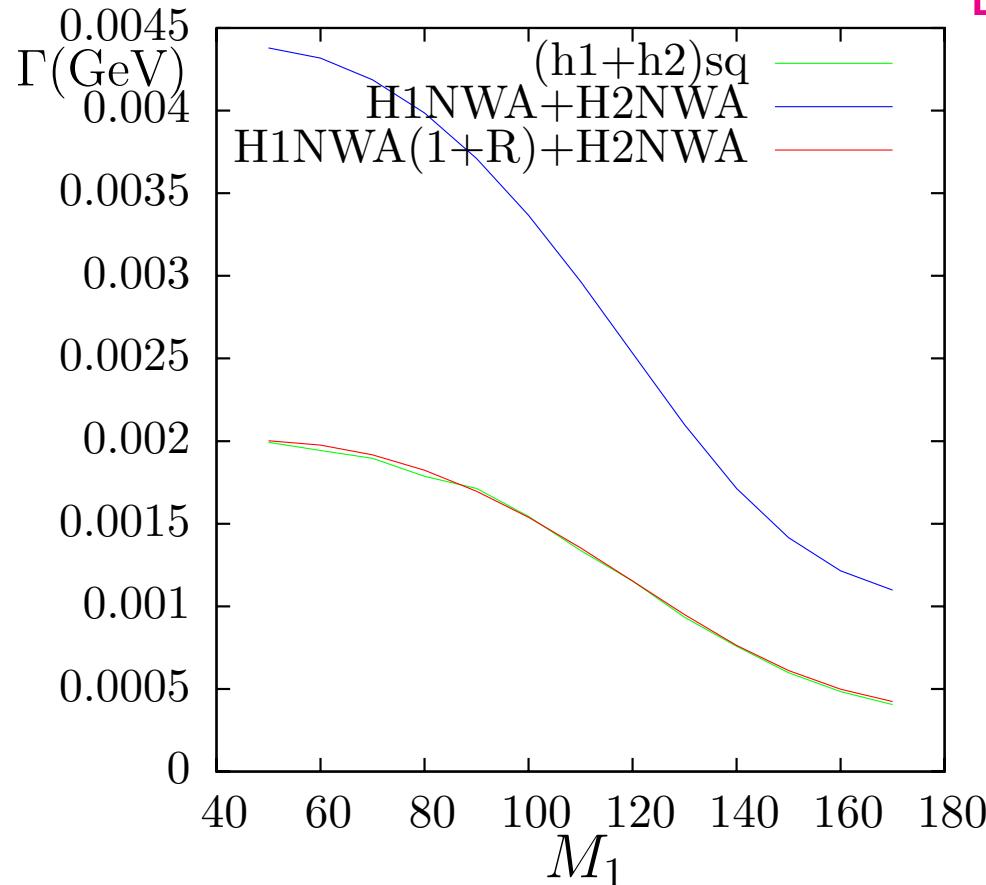
[A. Fowler, G. W. '10]

⇒ Large destructive interference contribution needs to be taken into account

# *Example of interference effect in the $\mathcal{CP}$ -conserving case*

Decay  $\tilde{\chi}_4^0 \rightarrow \tilde{\chi}_1^0 \tau^+ \tau^-$ , contributions from  $h$  and  $H$  exchange,  
 $M_h^{\max}$  scenario,  $\tan \beta = 40$ ,  $M_{H^\pm} = 170$  GeV

[A. Fowler, G. W. '10]



⇒ Interference effects have large impact

## ***Conclusions***

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- MSSM with complex parameters offers many interesting features both conceptually and phenomenologically

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Absorptive parts play an important role in particular in  $\mathcal{CP}$  asymmetries

# *Conclusions*

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- MSSM with complex parameters offers many interesting features both conceptually and phenomenologically
- Careful treatment of imaginary parts from complex parameters and from absorptive parts of loop integrals is necessary  
Absorptive parts play an important role in particular in  $\mathcal{CP}$  asymmetries
- Mass degeneracies occur generically in the MSSM with or without complex parameters  
⇒ Interference effects can be sizable  
**The narrow width approximation can be extended to incorporate interference effects**